Relativity without Lorentz Invariance Solves the Twin Paradox

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Abstract

The Twin Paradox is one of the most fascinating paradoxes in physics. In Special Relativity, the paradox arises due to the nonexistence of a preferred frame of reference, resulting in both twins observing that he or she is younger than the other twin. Nonetheless, it is commonly agreed that the "traveling" twin returns younger than the "staying" twin. The prevailing solution is obtained by deviating from the relativity principle and assuming that the "staying" twin's frame is preferred over the "traveling" twin's frame. Here I show that a relativity theory without the Lorentz Invariance Principle yields a commonsensical solution to the paradox, according to which the twins should age equally. The proposed solution is in full agreement with the essence of relativity, which prescribes the nonexistence of a preferred frame of reference.

Keywords: Twin Paradox, Relativity.

Introduction

The Twin Paradox is undoubtedly the most famous thought experiment in physics. The enormous literature about it renders any attempt to review it almost impossible. In the Twin Paradox, one of two twins stays on Earth while the other twin travels at near the speed of light to a distant star and returns to earth. According to Special Relativity, the twin who stayed on earth will measure a time dilation given by:

\[
\frac{t}{t'} = \frac{1}{\sqrt{1-(\frac{v}{c})^2}} \quad \cdots \cdots (1)
\]

Where \(t\) and \(t'\) are the durations of the round trip from earth and back, measured in the internal frames of the remaining and the traveling twins, respectively; \(v\) is the spaceship velocity and \(c\) is the velocity of light. The paradox lies in the fact that from the point of view of the traveling twin, the "staying" twin is the one experienced as distancing away and then returning back with the same relative velocity \(v\). Thus, the traveling twin will measure an equal time dilation of:
\[
\frac{t'}{t} = \frac{1}{\sqrt{1-(\frac{v}{c})^2}} = \frac{t}{t'}
\] ....... (2)

Hence the paradox, since upon their reunion, each of the twins will find the other one younger than him- or herself. It is worth noting that a similar argument was proposed more than half a century ago by the late Herbert Dingle, who served as president of the Royal Astronomical Society. In a paper published in 1962 in *Nature* [1], and in several subsequent writings [2-4], Dingle argued that the theory of Special Relativity leads to inconsistency. According to Dingle, "Einstein deduced, from the basic ideas of his theory that a moving clock works slower than a stationary one. By a similar line of reasoning I deduced from the same basic ideas that the same moving clock works faster than the same stationary one. Hence the theory, since it entails with equal validity two incompatible conclusions, must be false". ([2], p. 41). Dingle posited that the inconsistency of Special Relativity stems from Einstein's attempt to reconcile his theory with Lorentz's electrodynamics. In Dingle's words, "Einstein, in 1905, proposed an amendment of mechanics, the effects of which, however, would be perceptible only at velocities far beyond practical realization. If the amendment were justified it would succeed in making the electromagnetic equations, like those of mechanics, relativistic, and so remove the incompatibility; but, clearly, the only possible test of such a theory was a mechanical one. It was framed in order to justify electromagnetic theory, so that to use electromagnetic theory to justify it would be to argue in a circle" ([2], p. 49). Dingle concludes that "The alternative, that the laws of electromagnetism need reformulation, thus appears almost inescapable, and indeed, quantum phenomena have long been telling us this—though, in view of the apparent justification of the Maxwell-Lorentz theory by special relativity, attempts have naturally been concentrated (without success) on the attempt to reconcile it with such phenomena instead of on the formulation of fundamentally new laws" ([2], p. 59).

Dingle's critique was countered by many physicists and was eventually ignored. The prevailing solution of the paradox is one which prescribes that the "traveling" twin returns younger than the "staying" twin. This solution was proposed by Albert Einstein himself, first within the framework of Special Relativity, and later within the framework of General Relativity. In his famous 1905 paper [5], although calling SR's answer a ‘peculiar consequence’ (*eigent ümliche Konsequenz*), Einstein stated that the traveling brother is the one to become younger. According to Einstein, this solution is independent of whether the travel-path is comprised of straight lines or of a closed curve of any shape. In Einstein's words: ‘If there are two synchronous clocks at A, and one of them is moved along a closed curve with constant velocity \([v]\) until it has returned to
A, which takes, say t seconds, then this clock will lag on its arrival at A by \( \frac{1}{2} t \left( \frac{v}{c} \right)^2 \) seconds behind the clock that has not been moved' [5].

Several studies [e.g., 6, 7] has pointed that the essence of the Twin Paradox is the impossibility of simultaneity between the clocks of the two twins. But why should time dilation work in favor of the traveling twin, who becomes younger? This question could not be explained only by reference to the impossibility of simultaneity. Einstein justified his solution using an example in which one observer is located on the Earth's equator and the other is located at one of Earth's poles. According to Einstein, "a balance-wheel clock (Unruhuhr) that is located at the Earth’s equator must be very slightly slower than an absolutely identical clock, subjected to otherwise identical conditions, that is located at one of the Earth’s poles" [8]. This solution of the paradox assumes arbitrarily that the clock at the pole is "stationary", while the clock at the equator is the "moving" one. Such assumption is in complete contradiction with the principle of relativity, according to which an observer at the internal frame of the equator, will observe that the equator's clock is "stationary", while the pole's clock is "moving". In fact, the example brought by Einstein is irrelevant to the twin paradox, since in the paradox the two clocks should start from one location, from which the "traveling" clock moves in a closed curve and returns to the internal frame of the "staying" clock.

Einstein's confidence in his solution of the paradox, namely that the twin in the spaceship will return younger, has made him go as far as to speculate about the possibility of utilizing the time dilation on earth for a possible construction of a time-dilation machine. In a speech delivered in 1911 at the Naturforschende Gesellschaft in Zurich, Einstein is quoted to have said: "Were we, for example, to place a living organism in a box and make it perform the same to-and-fro motion as the clock discussed above, it would be possible to have this organism return to its original starting point after an arbitrarily long flight having undergone an arbitrarily small change, while identically constituted organisms that remained at rest at the point of origin have long since given way to new generations" [9].

The main point here is that the Twin Paradox is unsolvable within the framework of special Relativity, unless we make the assumption of a preferred frame of reference, which stands in diametrical opposition to the mere principle of relativity. In the Earth's pole-equator example, relativity implies that while an observer located at the pole will observe that the observer at the equator is rotating with an angular velocity \( w \) (or velocity \( v = w R \), where \( R \) is the Earth radius at the equator), the observer located at the equator will observe that the one at the pole is rotating in an opposite direction with the same velocity.
Other attempts to solve the twin paradox evoke the relativity of accelerating frames. As mentioned before, Einstein himself, after developing General Relativity, resorted to this explanation in 1918, when he argued that since one of the clocks is in an accelerated frame of reference, the postulates of the Special Theory of Relativity do not apply to it and so ‘no contradictions in the foundations of the theory can be construed’ [6, 10]. More recent attempts which evoke General Relativity are aplenty [e.g., 11-14]. The acceleration argument, although reverted to by Einstein, could be easily dismissed by making the distance between Earth and the remote star long enough to render the acceleration effect arbitrarily small [15]. Moreover, any solution based on an acceleration effect could be dismissed on the bases of symmetry. In the absence of a preferred frame of reference, the reversal of the direction of movement is also relative, with no way to determine who turns around and who does not. This implies that any possible effect of acceleration should be canceled out.

Other attempts to solve the paradox in the framework of circular [e.g., 16] or another closed-curve motion will not be reviewed here. In fact, solution of the paradox in a linear motion seems sufficient, since its extension to the case of angular motion is quite straightforward.

**Relativity without the Lorentz Invariance**

Here I propose a solution to the paradox, based on a new theory of relativity for inertial systems, termed Complete Relativity [17-20]. The theory is based on the following propositions: 1. The laws of physics are the same in all inertial frames of reference. 2. The magnitudes of all physical entities, as measured by an observer, depend on the relative motion of the observer with respect to the rest frame of the measured entities. 3. The transformations of all physical entities, from one frame of reference to another, may depend on the methods used for their measurement. 4. All translations of information from one frame of reference to another are carried by light or electromagnetic waves of equal velocity. A comprehensive presentation of Complete Relativity, including its time, distance, mass and energy transformations, is detailed in [17]. What is worth stressing here is that Complete Relativity abandons the Lorentz Invariance (and the corresponding constancy of the velocity of light).

To apply Complete Relativity (CR) to the twin paradox, consider the example in Fig.1, in which one twin (Joe) stays at Earth (the "staying twin), and the other twin (Jane, the "traveling" twin) travels to a very distant star and returns back to Earth. Assume that the travel start times, relative to Earth ($\vec{t}_1$) and to the spaceship ($\vec{t}_1'$), are synchronized such that $\vec{t}_1 = \vec{t}_1'$. Furthermore assume
that upon the arrival of Jane at the distant star, a signal is sent from the star to Joe's station at Earth, indicating the arrival of Jane to the star. To solve the paradox I treat the paths Earth → Star and Star → Earth, each in turn.

**Figure 1. Twin Paradox**

1. **Earth → Star**

   The signal indicating the arrival of Jane at the star will arrive at Earth with a delay of \( \frac{d}{c} \) s., where \( d \) is the distance between Earth and the star, and \( c \) is the velocity of light (both measured at the Earth's frame).

   Denote by \( \overrightarrow{t_2} \) and \( \overrightarrow{t'_2} \) Jane's arrival times at the star, as measured by the "staying" and the "travelling" twins, respectively. We can write \( \overrightarrow{t_2} = \overrightarrow{t'_2} + \frac{d}{c} \), or:

   \[
   \overrightarrow{t_2} = \overrightarrow{t'_2} - \frac{d}{c} \quad \text{..... (3)}
   \]

   We also have

   \[
   \overrightarrow{t_1} = \overrightarrow{t'_1} \quad \text{..... (4)}
   \]

2. **Star → Earth**

   The "staying" twin receives the signal indicating that the "travelling" twin has departed from the distant star with delay of \( \frac{d}{c} \) s. This makes him conclude that his "travelling" twin has departed from the star later by \( \frac{d}{c} \) s. than the time measured by the travelling twin. Denote the return-trip's start time as measured by the "staying" and the "travelling" twins by \( \overrightarrow{t_3} \) and \( \overrightarrow{t'_3} \), respectively, and the respective arrival times to Earth by \( \overrightarrow{t_4} \) and \( \overrightarrow{t'_4} \). We can write \( \overrightarrow{t_3} = \overrightarrow{t'_3} + \frac{d}{c} \), or:

   \[
   \overrightarrow{t_3} = \overrightarrow{t'_3} - \frac{d}{c} \quad \text{..... (5)}
   \]
We also have:

\[ \vec{t}_4 = \vec{t}_4' \]  

_____ (6)

3. Earth → Star → Earth

The total time measured by the "staying" brother is:

\[ (\vec{t}_2 - \vec{t}_1) + (\vec{t}_4 - \vec{t}_3) \]  

_____ (7)

While the total time measured by the "travelling" brother is:

\[ (\vec{t}_2' - \vec{t}_1') + (\vec{t}_4' - \vec{t}_3') \]  

_____ (8)

Substituting the values of \( \vec{t}_1', \vec{t}_2', \vec{t}_3' \) and \( \vec{t}_4' \) from Eq. 3-6 in Eq. 8 we get:

\[ (\vec{t}_2 - \vec{t}_1') + (\vec{t}_4 - \vec{t}_3') = ((\vec{t}_2 - \frac{d}{c}) - \vec{t}_1) + (\vec{t}_4 - (\vec{t}_3 - \frac{d}{c})) \]

\[ = (\vec{t}_2 - \vec{t}_1) + (\vec{t}_4 - \vec{t}_3) - \frac{d}{c} + \frac{d}{c} = (\vec{t}_2 - \vec{t}_1) + (\vec{t}_4 - \vec{t}_3) \]  

_____ (9)

Thus, the twins age equally.

Concluding Remarks

I have shown that a relativity theory without the Lorentz Invariance solves the twin paradox, such that the two twins age equally. Obviously, the solution presented here is in contradiction with the widely accepted solution, according to which the "traveling" twin returns younger than the "staying" twin. Nonetheless, the solution proposed here is in full agreement with the essence of relativity, which prescribes the nonexistence of a preferred frame of reference. It is also more commonsensical than the prevailing solution, which prescribes not only that the "staying" twin should age more than the "traveling" twin, but also that all the inhabitants of Earth should age equally more!

References


