SMARANDACHE DUAL SYMMETRIC FUNCTIONS AND CORRESPONDING NUMBERS OF THE TYPE OF STIRLING NUMBERS OF THE FIRST KIND

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In the rising factorial $(x+1)(x+2)(x+3)\ldots(x+n)$, the coefficients of different powers of x are the absolute values of the Stirling numbers of the first kind. REF[1].

Let $x_1, x_2, x_3, \ldots x_n$ be the roots of the equation

$(x+1)(x+2)(x+3)\ldots(x+n) = 0.$

Then the elementry symmetric functions are

$x_1 + x_2 + x_3 +, \ldots, + x_n = \Sigma x_1$, (sum of all the roots)

$x_1x_2 + x_1x_3 + \ldots x_{n-1}x_n = \Sigma x_1x_2$. (sum of all the products of the roots taking two at a time)

$\Sigma x_1x_2x_3\ldots x_r$ = (sum of all the products of the roots taking r at a time) .

In the above we deal with sums of products. Now we define **Smarandache Dual symmetric functions** as follows.

We take the product of the sums instead of the sum of the products. The duality is evident. As an example we take only 4 variables say $\mathbf{x_1}$, $\mathbf{x_2}$, $\mathbf{x_3}$, $\mathbf{x_4}$. Below is the chart of both types of functions

Elementry symmetic funcions	**Smarandache Dual Symmetric functions**
(sum of the products)	**(Product of the sums)**
$x_1 + x_2 + x_3 + x_4$	$x_1x_2x_3x_4$
$x_1x_2 + x_1x_3 + x_1x_4 + x_2x_3 + x_2x_4 + x_3x_4$	$(x_1 + x_2)(x_1+x_3)(x_1 + x_4)(x_2+x_3)(x_2+x_4)(x_3+x_4)$
$x_1x_2x_3 + x_1x_2x_4 + x_1x_3x_4 + x_2x_3x_4$	$(x_1 + x_2 + x_3)(x_1+x_2 + x_4)(x_1 +x_3 + x_4)(x_2+x_3 +x_4)$
$x_1x_2x_3x_4$	$x_1 + x_2 + x_3 + x_4$

We define for convenience the product of sums of taking **none** at a time as 1.

Now if we take $x_r = r$ in the above we get the absolute values of the Stirling numbers of the first kind. For the firs column.

24, 50, 35, 10 ,1.

The corresponding numbers for the second column are **10 , 3026, 12600, 24, 1.**

The Triangle of the absolute values of Stirling numbers of the first kind is

1				
1	**1**			
2	**3**	**1**		
6	**11**	**6**	**1**	
24	**50**	**35**	**10**	**1**

The corresponding **Smarandache dual symmetric Triangle is**

1				
1	**1**			
3	**2**	**1**		
6	**60**	**6**	**1**	
10	**3026**	**12600**	**24**	**1**

The next row (5^{th}) numbers are

15, 240240 , 2874009600, 4233600, 120 , 1.

Following propertiesof the above triangle are visible:

(1) The leading diagonal contains unity.

(2) The r^{th} row element of the second leading diagonal contains $r!$.

(3) The First column entries are the corresponding triangular numbers.

Readers are invited to find relations between the two triangles.

Application: Smarandache Dual Symmetric functions give us another way of generalising the **Arithmetic Mean Geometric Mean Inequality.** One can prove easily that

$$(x_1x_2x_3x_4)^{1/4} \leq [\{ (x_1 + x_2) (x_1+ x_3)(x_1 + x_4)(x_2+ x_3)(x_2 + x_4)(x_3+ x_4)\}^{1/6}] / 2 \leq$$

$$[\{(x_1 + x_2 +x_3)(x_1+ x_2 + x_4)(x_1 +x_3 + x_4)(x_2+ x_3 +x_4) \}^{1/4}] / 3 \leq \{x_1 + x_2 + x_3 + x_4 \} / 4$$

The above inequality is generally true can also be established easily.