

THE PRIMES IN THE SMARANDACHE POWER PRODUCT SEQUENCES OF THE FIRST KIND

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Abstract. In this paper we prove that if $r > 1$ and r is not a power of 2, then the Smarandache r -power product sequence of the first kind contains only one prime 2.

Key words . Smarandache power product sequence, first kind, prime.

For any positive integers n, r with $r > 1$, let $P(n, r)$ be the n -th power of degree r . Further, let

$$(1) \quad V(n, r) = \prod_{k=1}^n P(k, r) + 1.$$

Then the sequence $V(r) = \{V(n, r)\}_{n=1}^{\infty}$ is called the Smarandache r -power product sequence of the first kind. In [2], Russo proposed the following question.

Question . How many terms in $V(2)$ and $V(3)$ are primes?

In fact, Le and Wu [1] showed that if r is odd, then $V(r)$ contains only one prime 2. It implies that $V(3)$ contains only one prime 2. In this paper we prove a general result as follows.

Theorem . If r is not a power of 2, then $V(r)$ contains only one prime 2.

Proof. Since $r > 1$, if r is not a power of 2, then r has an odd prime divisor p . By (1), we get

$$V(n, r) = (n!)^r + 1 = ((n!)^{r/p} + 1)((n!)^{r(p-1)/p} - (n!)^{r(p-2)/p} + \dots - (n!)^{r/p} + 1),$$

Where r/p is a positive integer. Notice that if $n > 1$, then $(n!)^{r/p} + 1 > 1$ and $(n!)^{r(p-1)/p} \dots + 1 > 1$. Therefore, we see from (2) that if $n > 1$, then $V(n, r)$ is not a prime. Thus, the sequence $V(r)$ contains only one prime $V(1, r) = 2$. The theorem is proved.

References

- [1] M.-H. Le and K.-J. Wu, The primes in Smarandache power product sequences, Smarandache Notions J. 9(1998), 97-98.
- [2] F. Ruso, Some results about four Smarandache U-product sequences, Smarandache Notions J. 11(2000), 42-49.

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