THE PRIMES IN SMARANDACHE POWER PRODUCT SEQUENCES

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Abstract

For any positive integer k, let A_k be the Smarandache k-power product sequence. In this paper we prove that if k is an odd integer, with $k>1$, then A_k contains only one prime 2.

In [1], Iacobescu defined the sequence $\{1+c_1c_2...c_n\}_{n=1}^{\infty}$; is the Smarandache cubic product sequence, where c_n is the n-th cubic number. Simultaneous, he posed the following question:

Question: Hou many primes are in the sequence $\{1+c_1c_2...c_n)_{n=1}^{\infty}$?

We nou give a general definition as follows:

For any positive integers k, n let

(1) $$a_k(n) = 1+1^k 2^k ... n^k,$$

and let $A_k = \{a_k(n)\}_{n=1}^{\infty}$. Then A_k is called the Smarandache k-power product sequence. In this paper we prove the following result:

Theorem. If k is an odd integer, with $k>1$, then A_k contains only one prime 2.

Clearly, the above result completely solves Iacobescu's question.

Proof of Theorem. We see from (1) that

(2) $$a_k(n) = 1 + (n!)^k.$$

If k is an odd integer, with $k>1$, then from (2) we get

(3) $$a_k(n) = 1^k + (n!)^k$$
$$= (1+n!)(1 - n! + (n!)^2 - ... - (n!)^{k-2} + (n!)^{k-1}).$$

When $n = 1$, $a_k(1) = 2$ is a prime.

When $n > 1$, since

$1 + n! > 1$ and $1 - n! + (n!)^2 - \ldots - (n!)^{k-2} + (n!)^{k-1} =$

$$((n!)^{k-1} - (n!)^{k-2}) + \ldots + ((n!)^2 - n!) + 1 > 1,$$

we find from (3) that $a_k(n)$ is not a prime. Thus, the sequence A_k contains only one prime 2. The theorem is proved.

Reference:

1. F. Iacobescu, "Smarandache partition type and other sequences", Bulletin of Pure and applied Sciences, 16E(1997), No.2, 237-240.