THE PRIMES IN THE SMARANDACHE POWER PRODUCT SEQUENCES OF THE SECOND KIND

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Abstract. In this paper we completely determine the primes in the Smarandache power product sequences of the second kind.

Key words . Smarandache power product sequencem , second kind, prime.

For any positive integers n,r with $r>1$, let $P(n,r)$ be the n-th power of degree r. Further, let

$$U(n,r)=\prod_{k=1}^{n} P(k,r)-1. \tag{1}$$

Then the sequence $U(r)=\left\{U(n,r)\right\}_{n=1}^{\infty}$ is called the Smarandache r-power product sequence of the second kind. In [2], Russo proposed the following question.

Question. How many terms in $U(2)$ and $U(3)$ are primes?

In this paper we completely solve the mentioned question. We prove a more strong result as follows.

Theorem. If r and 2^r-1 are both primes, then U(r) contains only one prime $U(2,r)=2^r-1$. Otherwise, $U(r)$ does not contain any prime.

Proof. Since $U(1,r)=0$,we may assume that $n>1$,By(1), we get

$$U(n,r)=(n!)^r-1=(n!-1)((n!)^{r-1}+(n!)^{r-2}+\cdots+1). \tag{2}$$

Since $n!>2$ if $n>2$, we see from (2) that $U(n,r)$ is not a prime if $n>2$. When $n=2$, we get from (2) that

(3) $$U(2,r)=2^r-1.$$

Therefore, by [1,Theorem 18], we find from (3) that $U(r)$ contains a prime if and only if r and 2^r-1 are both primes. The theorem is proved.

References

[1] G.H.Hardy and E.M.Wright, An Introduction to the Theory of Numbers, Oxford University Press, Oxford, 1937.

[2] F.Russo, Some results about four Smarandache U-product sequences, Smarandache Notions J.11(2000),42-49.

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