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Super Fibonacci Graceful Labeling of Some Special Class of Graphs

R.Sridevi[1], S.Navaneethakrishnan[2] and K.Nagarajan[1]

1. Department of Mathematics, Sri S.R.N.M.College, Sattur - 626 203, Tamil Nadu, India

2. Department of Mathematics, V.O.C.College, Tuticorin - 628 008, Tamil Nadu, India

E-mail: r.sridevi_2010@yahoo.com, snk.voc@gmail.com, k_nagarajan_srnmc@yahoo.co.in

Abstract: A Smarandache-Fibonacci Triple is a sequence $S(n)$, $n \geq 0$ such that $S(n) = S(n-1) + S(n-2)$, where $S(n)$ is the Smarandache function for integers $n \geq 0$. Certainly, it is a generalization of Fibonacci sequence. A Fibonacci graceful labeling and a super Fibonacci graceful labeling on graphs were introduced by Kathiresan and Amutha in 2006. Generally, let G be a (p, q)-graph and $S(n)|n \geq 0$ a Smarandache-Fibonacci Triple. An bijection $f : V(G) \to \{S(0), S(1), S(2), \dots, S(q)\}$ is said to be a super Smarandache-Fibonacci graceful graph if the induced edge labeling $f^*(uv) = |f(u) - f(v)|$ is a bijection onto the set $\{S(1), S(2), \dots, S(q)\}$. Particularly, if $S(n)$, $n \geq 0$ is just the Fibonacci sequence F_i, $i \geq 0$, such a graph is called a super Fibonacci graceful graph. In this paper, we show that some special class of graphs namely F_n^t, C_n^t and $S_{m,n}^t$ are super fibonacci graceful graphs.

Key Words: Smarandache-Fibonacci triple, graceful labeling, Fibonacci graceful labeling, super Smarandache-Fibonacci graceful graph, super Fibonacci graceful graph.

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§1. Introduction

By a graph, we mean a finite undirected graph without loops or multiple edges. A path of length n is denoted by P_{n+1}. A cycle of length n is denoted by C_n. G^+ is a graph obtained from the graph G by attaching pendant vertex to each vertex of G. Graph labelings, where the vertices are assigned certain values subject to some conditions, have often motivated by practical problems. In the last five decades enormous work has been done on this subject [1].

The concept of graceful labeling was first introduced by Rosa [6] in 1967. A function f is a graceful labeling of a graph G with q edges if f is an injection from the vertices of G to the set $\{0, 1, 2, \dots, q\}$ such that when each edge uv is assigned the label $|f(u) - f(v)|$, the resulting edge labels are distinct.

The notion of Fibonacci graceful labeling and Super Fibonacci graceful labeling were introduced by Kathiresan and Amutha [5]. We call a function f, a fibonacci graceful labeling of a graph G with q edges if f is an injection from the vertices of G to the set $\{0, 1, 2, \dots, F_q\}$, where

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F_q is the q^{th} fibonacci number of the fibonacci series $F_1 = 1$,$F_2 = 2$,$F_3 = 3$, $F_4 = 5$,..., such that each edge uv is assigned the labels $|f(u) - f(v)|$, the resulting edge labels are $F_1, F_2, \ldots, \ldots F_q$. An injective function $f : V(G) \to \{F_0, F_1, \ldots, F_q\}$, where F_q is the q^{th} fibonacci number, is said to be a super fibonacci graceful labeling if the induced edge labeling $|f(u) - f(v)|$ is a bijection onto the set $\{F_1, F_2, \ldots, F_q\}$. In the labeling problems the induced labelings must be distinct. So to introduce fibonacci graceful labelings we assume $F_1 = 1, F_2 = 2, F_3 = 3, F_4 = 5, \ldots,$ as the sequence of fibonacci numbers instead of $0, 1, 2, \cdots$ [3].

Generally, a Smarandache-Fibonacci Triple is a sequence $S(n)$, $n \geq 0$ such that $S(n) = S(n-1) + S(n-2)$, where $S(n)$ is the Smarandache function for integers $n \geq 0$ ([2]). A (p, q)-graph G is a super Smarandache-Fibonacci graceful graph if there is an bijection $f : V(G) \to \{S(0), S(1), S(2), \ldots, S(q)\}$ such that the induced edge labeling $f^*(uv) = |f(u) - f(v)|$ is a bijection onto the set $\{S(1), S(2), \ldots, S(q)\}$. So a super Fibonacci graceful graph is a special type of Smarandache-Fibonacci graceful graph by definition.

We have constructed some new types of graphs namely $F_n \oplus K^+_{1,m}$, $C_n \oplus P_m$, $K_{1,n} \oslash K_{1,2}$, $F_n \oplus P_m$ and $C_n \oplus K_{1,m}$ and we proved that these graphs are super fibonacci graceful labeling in [7]. In this paper, we prove that F^t_n, C^t_n and $S^t_{m,n}$ are super fibonacci graceful graphs.

§2. Main Results

In this section, we show that some special class of graphs namely F^t_n, C^t_n and $S^t_{m,n}$ are super fibonacci graceful graphs.

Definition 2.1 *Let G be a (p, q) graph. An injective function $f : V(G) \to \{F_0, F_1, F_2, \cdots, F_q\}$, where F_q is the q^{th} fibonacci number, is said to be a super fibonacci graceful graphs if the induced edge labeling $f^*(uv) = |f(u) - f(v)|$ is a bijection onto the set $\{F_1, F_2, \ldots, F_q\}$.*

Definition 2.2 *The one point union of t copies of fan F_n is denoted by F^t_n.*

The following theorem shows that the graph F^t_n is a super fibonacci graceful graph.

Theorem 2.3 *F^t_n is a super fibonacci graceful graph for all $n \geq 2$.*

Proof Let u_0 be the center vertex of F^t_n and u^j_i, where $i = 1, 2, \ldots, t$, $j = 1, 2, \ldots, n$ be the other vertices of F^t_n. Also, $|V(G)| = nt + 1$ and $|E(G)| = 2nt - t$. Define $f : V(F^t_n) \to \{F_0, F_1, \ldots, F_q\}$ by $f(u_0) = F_0$, $f(u^j_1) = F_{2j-1}$, $1 \leq j \leq n$. For $i = 2, 3, \ldots, t$, $f(u^j_i) = F_{2n(i-1)+2(j-1)-(i-2)}$, $1 \leq j \leq n$. We claim that all these edge labels are distinct. Let $E_1 = \{f^*(u_0 u^j_1) : 1 \leq j \leq n\}$. Then

$$\begin{aligned}
E_1 &= \{|f(u_0) - f(u^j_1)| : 1 \leq j \leq n\} \\
&= \{|f(u_0) - f(u^1_1)|, |f(u_0) - f(u^2_1)|, \ldots, |f(u_0) - f(u^{n-1}_1)|, |f(u_0) - f(u^n_1)|\} \\
&= \{|F_0 - F_1|, |F_0 - F_3|, \ldots, |F_0 - F_{2n-3}|, |F_0 - F_{2n-1}|\} \\
&= \{F_1, F_3, \ldots, F_{2n-3}, F_{2n-1}\}.
\end{aligned}$$

Let $E_2 = \{f^*(u_1^j u_1^{j+1}) : 1 \leq j \leq n-1\}$. Then

$$\begin{aligned}
E_2 &= \{|f(u_1^j) - f(u_1^{j+1})| : 1 \leq j \leq n-1\} \\
&= \{|f(u_1^1) - f(u_1^2)|, |f(u_1^2) - f(u_1^3)|, \ldots, |f(u_1^{n-2}) - f(u_1^{n-1})|, |f(u_1^{n-1}) - f(u_1^n)|\} \\
&= \{|F_1 - F_3|, |F_3 - F_5|, \ldots, |F_{2n-5} - F_{2n-3}|, |F_{2n-3} - F_{2n-1}|\} \\
&= \{F_2, F_4, \ldots, F_{2n-4}, F_{2n-2}\}.
\end{aligned}$$

For $i = 2$, we know that

$$\begin{aligned}
E_3 &= \{f^*(u_0 u_2^j) : 1 \leq j \leq n\} = \{|f(u_0) - f(u_2^j)| : 1 \leq j \leq n\} \\
&= \{|f(u_0) - f(u_2^1)|, |f(u_0) - f(u_2^2)|, \ldots, |f(u_0) - f(u_2^{n-1})|, |f(u_0) - f(u_2^n)|\} \\
&= \{|F_0 - F_{2n}|, |F_0 - F_{2n+2}|, \ldots, |F_0 - F_{4n-4}|, |F_0 - F_{4n-2}|\} \\
&= \{F_{2n}, F_{2n+2}, \ldots, F_{4n-4}, F_{4n-2}\},
\end{aligned}$$

$$\begin{aligned}
E_4 &= \{f^*(u_2^j u_2^{j+1}) : 1 \leq j \leq n-1\} \\
&= \{|f(u_2^j) - f(u_2^{j+1})| : 1 \leq j \leq n-1\} \\
&= \{|f(u_2^1) - f(u_2^2)|, |f(u_2^2) - f(u_2^3)|, \ldots, |f(u_2^{n-2}) - f(u_2^{n-1})|, |f(u_2^{n-1}) - f(u_2^n)|\} \\
&= \{|F_{2n} - F_{2n+2}|, |F_{2n+2} - F_{2n+4}|, \ldots, |F_{4n-6} - F_{4n-4}|, |F_{4n-4} - F_{4n-2}|\} \\
&= \{F_{2n+1}, F_{2n+3}, \ldots, F_{4n-5}, F_{4n-3}\}.
\end{aligned}$$

For $i = 3$, let $E_5 = \{f^*(u_0 u_3^j) : 1 \leq j \leq n\}$. Then

$$\begin{aligned}
E_5 &= \{|f(u_0) - f(u_3^j)| : 1 \leq j \leq n\} \\
&= \{|f(u_0) - f(u_3^1)|, |f(u_0) - f(u_3^2)|, \ldots, |f(u_0) - f(u_3^{n-1})|, |f(u_0) - f(u_3^n)|\} \\
&= \{|F_0 - F_{4n-1}|, |F_0 - F_{4n+1}|, \ldots, |F_0 - F_{6n-5}|, |F_0 - F_{6n-3}|\} \\
&= \{F_{4n-1}, F_{4n+1}, \ldots, F_{6n-5}, F_{6n-3}\}.
\end{aligned}$$

Let $E_6 = \{f^*(u_3^j u_3^{j+1}) : 1 \leq j \leq n-1\}$. Then

$$\begin{aligned}
E_6 &= \{|f(u_3^j) - f(u_3^{j+1})| : 1 \leq j \leq n-1\} \\
&= \{|f(u_3^1) - f(u_3^2)|, |f(u_3^2) - f(u_3^3)|, \ldots, |f(u_3^{n-2}) - f(u_3^{n-1})|, |f(u_3^{n-1}) - f(u_3^n)|\} \\
&= \{|F_{4n-1} - F_{4n+1}|, |F_{4n+1} - F_{4n+3}|, \ldots, |F_{6n-7} - F_{6n-5}|, |F_{6n-5} - F_{6n-3}|\} \\
&= \{F_{4n}, F_{4n+2}, \ldots, F_{6n-6}, F_{6n-4}\}
\end{aligned}$$

$$\cdots\cdots\cdots\cdots\cdots ,$$

Now, for $i = t-1$, let $E_{t-1} = \{f^*(u_0 u_{t-1}^j) : 1 \leq j \leq n\}$. Then

$$\begin{aligned}
E_{t-1} &= \{|f(u_0) - f(u_{t-1}^j)| : 1 \leq j \leq n\} \\
&= \{|f(u_0) - f(u_{t-1}^1)|, |f(u_0) - f(u_{t-1}^2)|, \ldots, |f(u_0) - f(u_{t-1}^{n-1})|, |f(u_0) - f(u_{t-1}^n)|\} \\
&= \{|F_0 - F_{2nt-4n-t+3}|, |F_0 - F_{2nt-4n-t+5}|, \ldots, |F_0 - F_{2nt-2n-t-1}|, |F_0 - F_{2nt-2n-t+1}|\} \\
&= \{F_{2nt-4n-t+3}, F_{2nt-4n-t+5}, \ldots, F_{2nt-2n-t-1}, F_{2nt-2n-t+1}\}.
\end{aligned}$$

Let $E_{t-1} = \{f^*(u_{t-1}^j u_{t-1}^{j+1}) : 1 \leq j \leq n-1\}$. Then

$$
\begin{aligned}
E_{t-1} &= \{|f(u_{t-1}^j) - f(u_{t-1}^{j+1})| : 1 \leq j \leq n-1\} \\
&= \{|f(u_{t-1}^1) - f(u_{t-1}^2)|, |f(u_{t-1}^2) - f(u_{t-1}^3)|, \\
&\quad \ldots, |f(u_{t-1}^{n-2}) - f(u_{t-1}^{n-1})|, |f(u_{t-1}^{n-1}) - f(u_{t-1}^n)|\} \\
&= \{|F_{2nt-4n-t+3} - F_{2nt-4n-t+5}|, |F_{2nt-4n-t+5} - F_{2nt-4n-t+7}|, \\
&\quad \ldots, |F_{2nt-2n-t-3} - F_{2nt-2n-t-1}|, |F_{2nt-2n-t-1} - F_{2nt-2n-t+1}|\} \\
&= \{F_{2nt-4n-t+4}, F_{2nt-4n-t+6}, \ldots, F_{2nt-2n-t-2}, F_{2nt-2n-t}\}.
\end{aligned}
$$

F_4^4 :

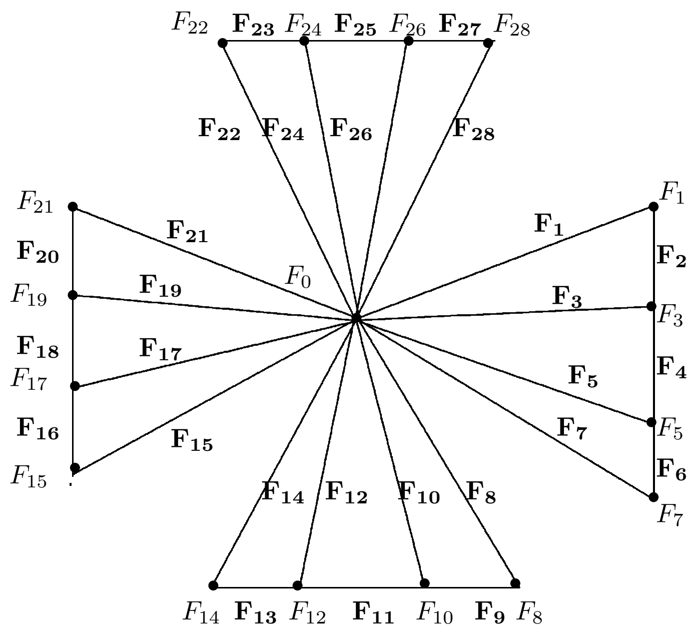

Fig.1

For $i = t$, let $E_t = \{f^*(u_0 u_t^j) : 1 \leq j \leq n\}$. Then

$$
\begin{aligned}
E_t &= \{|f(u_0) - f(u_t^j)| : 1 \leq j \leq n\} \\
&= \{|f(u_0) - f(u_t^1)|, |f(u_0) - f(u_t^2)|, \ldots, |f(u_0) - f(u_t^{n-1})|, \\
&\quad |f(u_0) - f(u_t^n)|\} \\
&= \{|F_0 - F_{2nt-2n-t+2}|, |F_0 - F_{2nt-2n-t+4}|, \ldots, |F_0 - F_{2nt-t-2}|, |F_0 - F_{2nt-t}|\} \\
&= \{F_{2nt-2n-t+2}, F_{2nt-2n-t+4}, \ldots, F_{2nt-t-2}, F_{2nt-t}\}.
\end{aligned}
$$

Let $E_t = \{f^*(u_t^j u_t^{j+1}) : 1 \leq j \leq n-1\}$. Then

$$\begin{aligned} E_t &= \{|f(u_t^j) - f(u_t^{j+1})| : 1 \leq j \leq n-1\} \\ &= \{|f(u_t^1) - f(u_t^2)|, |f(u_t^2) - f(u_t^3)|, \ldots, |f(u_t^{n-2}) - f(u_t^{n-1})|, |f(u_t^{n-1}) - f(u_t^n)|\} \\ &= \{|F_{2nt-2n-t+2} - F_{2nt-2n-t+4}|, |F_{2nt-2n-t+4} - F_{2nt-2n-t+6}|, \\ &\quad \ldots, |F_{2nt-t-4} - F_{2nt-t-2}|, |F_{2nt-t-2} - F_{2nt-t}|\} \\ &= \{F_{2nt-2n-t+3}, F_{2nt-2n-t+5}, \ldots, F_{2nt-t-3}, F_{2nt-t-1}\}. \end{aligned}$$

Therefore, $E = E_1 \cup E_2 \cup, \ldots, \cup E_{t-1} \cup E_t = \{F_1, F_2, \ldots, F_{2nt-t}\}$ Thus, the edge labels are distinct. Therefore, F_n^t admits super fibonacci graceful labeling. □

For example the super fibonacci graceful labeling of F_4^4 is shown in Fig.1.

Definition 2.4 *The one point union of t cycles of length n is denoted by C_n^t.*

Theorem 2.5 *C_n^t is a super fibonacci graceful graph for $n \equiv 0(mod 3)$.*

Proof Let u_0 be the one point union of t cycles and $u_1, u_2, \ldots, u_{t(n-1)}$ be the other vertices of C_n^t. Also, $|V(G)| = t(n-1)+1, |E(G)| = nt$. Define $f : V(C_n^t) \to \{F_0, F_1, \ldots, F_q\}$ by $f(u_0) = F_0$. For $i = 1, 2, \ldots, t$, $f(u_{(n-1)(i-1)+(j-1)+1}) = F_{nt-n(i-1)-2(j-1)}$, $1 \leq j \leq 2$. For $s = 1, 2, \cdots, \frac{n-3}{3}$, $i = 1, 2, \ldots, t$, $f(u_{(n-1)(i-1)+j}) = F_{nt-1-n(i-1)-2(j-s-2)+(s-1)}$, $3s \leq j \leq 3s+2$. Next, we claim that the edge labels are distinct.

We find the edge labeling between the vertex u_0 and starting vertex of each copy of $(u_{(n-1)(i-1)+1})$. Let $E_1 = \{f^*(u_0 u_{(n-1)(i-1)+1}) : 1 \leq i \leq t\}$. Then

$$\begin{aligned} E_1 &= \{|f(u_0) - f(u_{(n-1)(i-1)+1})| : 1 \leq i \leq t\} \\ &= \{|f(u_0) - f(u_1)|, |f(u_0) - f(u_n)|, \ldots, |f(u_0) - f(u_{nt-2n-t+3})|, \\ &\quad |f(u_0) - f(u_{nt-n-t+2})|\} \\ &= \{|F_0 - F_{nt}|, |F_0 - F_{nt-n}|, \ldots, |F_0 - F_{2n}|, |F_0 - F_n|\} \\ &= \{F_{nt}, F_{nt-n}, \ldots, F_{2n}, F_n\} \end{aligned}$$

Now we determine the edge labelings between the vertex $u_{(n-1)(i-1)+1}$ and the vertex $u_{(n-1)(i-1)+2}$ of each copy. Let $E_2 = \{f^*(u_{(n-1)(i-1)+1} u_{(n-1)(i-1)+2}) : 1 \leq i \leq t\}$. Then

$$\begin{aligned} E_2 &= \{|f(u_{(n-1)(i-1)+1}) - f(u_{(n-1)(i-1)+2})| : 1 \leq i \leq t\} \\ &= \{|f(u_1) - f(u_2)|, |f(u_n) - f(u_{n+1})|, \ldots, \\ &\quad |f(u_{nt-2n-t+3}) - f(u_{nt-2n-t+4})|, |f(u_{nt-n-t+2}) - f(u_{nt-n-t+3})|\} \\ &= \{|F_{nt} - F_{nt-2}|, |F_{nt-n} - F_{nt-n-2}|, \ldots, |F_{2n} - F_{2n-2}|, |F_n - F_{n-2}|\} \\ &= \{F_{nt-1}, F_{nt-n-1}, \ldots, F_{2n-1}, F_{n-1}\} \end{aligned}$$

We calculate the edge labeling between the vertex $u_{(n-1)(i-1)+2}$ and starting vertex $u_{(n-1)(i-1)+3}$

of the first loop. Let $E_3 = \{f^*(u_{(n-1)(i-1)+2}u_{(n-1)(i-1)+3}) : 1 \leq i \leq t\}$. Then

$$\begin{aligned}
E_3 &= \{|f(u_{(n-1)(i-1)+2}) - f(u_{(n-1)(i-1)+3})| : 1 \leq i \leq t\} \\
&= \{|f(u_2) - f(u_3)|, |f(u_{n+1}) - f(u_{n+2})|, |f(u_{2n}) - f(u_{2n+1})|, \ldots, \\
&\quad |f(u_{nt-2n-t+4}) - f(u_{nt-2n-t+5})|, |f(u_{nt-n-t+3}) - f(u_{nt-n-t+4})|\} \\
&= \{|F_{nt-2} - F_{nt-1}|, |F_{nt-n-2} - F_{nt-n-1}|, |F_{nt-2n-2} - F_{nt-2n-1}|, \\
&\quad \ldots, |F_{2n-2} - F_{2n-1}|, |F_{n-2} - F_{n-1}|\} \\
&= \{F_{nt-3}, F_{nt-n-3}, F_{nt-2n-3}, \ldots, F_{2n-3}, F_{n-3}\}.
\end{aligned}$$

Now, for $s = 1$, let $E_4 = \bigcup\limits_{i=1}^{t} \{f^*(u_{(n-1)(i-1)+j}u_{(n-1)(i-1)+j+1}) : 3 \leq j \leq 4\}$. Then

$$\begin{aligned}
E_4 &= \cup_{i=1}^{t} \{|f(u_{(n-1)(i-1)+j}) - f(u_{(n-1)(i-1)+j+1})| : 3 \leq j \leq 4\} \\
&= \{|f(u_3) - f(u_4)|, |f(u_4) - f(u_5)|\} \\
&\quad \cup \{|f(u_{n+2}) - f(u_{n+3})|, |f(u_{n+3}) - f(u_{n+4})|\} \cup, \ldots, \\
&\quad \cup \{|f(u_{nt-2n-t+5}) - f(u_{nt-2n-t+6})|, |f(u_{nt-2n-t+6}) - f(u_{nt-2n-t+7})|\} \\
&\quad \cup \{|f(u_{nt-n-t+4}) - f(u_{nt-n-t+5})|, |f(u_{nt-n-t+5}) - f(u_{nt-n-t+6})|\} \\
&= \{|F_{nt-1} - F_{nt-3}|, |F_{nt-3} - F_{nt-5}|\} \\
&\quad \cup \{|F_{nt-n-1} - F_{nt-n-3}|, |F_{nt-n-3} - F_{nt-n-5}|\} \\
&\quad \cup, \ldots, \cup \{|F_{2n-1} - F_{2n-3}|, |F_{2n-3} - F_{2n-5}|\} \\
&\quad \cup \{|F_{n-1} - F_{n-3}|, |F_{n-3} - F_{n-5}|\} \\
&= \{F_{nt-2}, F_{nt-4}\} \cup \{F_{nt-n-2}, F_{nt-n-4}\} \cup, \ldots, \\
&\quad \cup \{F_{2n-2}, F_{2n-4}\} \cup \{F_{n-2}, F_{n-4}\}
\end{aligned}$$

For the edge labeling between the end vertex $(u_{(n-1)(i-1)+5})$ of the first loop and starting vertex $(u_{(n-1)(i-1)+6})$ of the second loop, calculation shows that

$$\begin{aligned}
E_4^1 &= \{|f(u_{(n-1)(i-1)+5}) - f(u_{(n-1)(i-1)+6})| : 1 \leq i \leq t\} \\
&= \{|f(u_5) - f(u_6)|, |f(u_{n+4}) - f(u_{n+5})|, \ldots, \\
&\quad |f(u_{nt-2n-t+7}) - f(u_{nt-2n-t+8})|, |f(u_{nt-n-t+6}) - f(u_{nt-n-t+7})|\} \\
&= \{|F_{nt-5} - F_{nt-4}|, |F_{nt-n-5} - F_{nt-n-4}|, \ldots, |F_{2n-5} - F_{2n-4}|, |F_{n-5} - F_{n-4}|\} \\
&= \{F_{nt-6}, F_{nt-n-6}, \ldots, F_{2n-6}, F_{n-6}\}
\end{aligned}$$

For $s = 2$, let $E_5 = \cup_{i=1}^{t} \{f^*(u_{(n-1)(i-1)+j}u_{(n-1)(i-1)+j+1}) : 6 \leq j \leq 7\}$. Then

$$\begin{aligned}
E_5 &= \cup_{i=1}^{t}\{|f(u_{(n-1)(i-1)+j}) - f(u_{(n-1)(i-1)+j+1})| : 6 \le j \le 7\} \\
&= \{|f(u_6) - f(u_7)|, |f(u_7) - f(u_8)|\} \\
&\quad \cup\{|f(u_{n+5}) - f(u_{n+6})|, |f(u_{n+6}) - f(u_{n+7})|\}\cup, \ldots, \\
&\quad \cup|f(u_{nt-2n-t+8}) - f(u_{nt-2n-t+9})|, |f(u_{nt-2n-t+9}) - f(u_{nt-2n-t+10})|\} \\
&\quad \cup\{|f(u_{nt-n-t+7}) - f(u_{nt-n-t+8})|, |f(u_{nt-n-t+8}) - f(u_{nt-n-t+9})|\} \\
&= \{|F_{nt-4} - F_{nt-6}|, |F_{nt-6} - F_{nt-8}|\} \\
&\quad \cup\{|F_{nt-n-4} - F_{nt-n-6}|, |F_{nt-n-6} - F_{nt-n-8}|\} \\
&\quad \cup, \ldots, \cup\{|F_{2n-4} - F_{2n-6}|, |F_{2n-6} - F_{2n-8}|\} \\
&\quad \cup\{|F_{n-4} - F_{n-6}|, |F_{n-6} - F_{n-8}|\} \\
&= \{F_{nt-5}, F_{nt-7}\} \cup \{F_{nt-n-5}, F_{nt-n-7}\}\cup, \ldots, \\
&\quad \cup\{F_{2n-5}, F_{2n-7}\} \cup \{F_{n-5}, F_{n-7}\}
\end{aligned}$$

Similarly, for finding the edge labeling between the end vertex $(u_{(n-1)(i-1)+8})$ of the second loop and starting vertex $(u_{(n-1)(i-1)+9})$ of the third loop, calculation shows that

$$\begin{aligned}
E_5^1 &= \{|f(u_{(n-1)(i-1)+8}) - f(u_{(n-1)(i-1)+9})| : 1 \le i \le t\} \\
&= \{|f(u_8) - f(u_9)|, |f(u_{n+7}) - f(u_{n+8})|, \ldots, \\
&\quad |f(u_{nt-2n-t+10}) - f(u_{nt-2n-t+11})|, |f(u_{nt-n-t+9}) - f(u_{nt-n-t+10})|\} \\
&= \{|F_{nt-8} - F_{nt-7}|, |F_{nt-n-8} - F_{nt-n-7}|, \ldots, |F_{2n-8} - F_{2n-7}|, \\
&\quad |F_{n-8} - F_{n-7}|\} \\
&= \{F_{nt-9}, F_{nt-n-9}, \ldots, F_{2n-9}, F_{n-9}\},
\end{aligned}$$

$$\cdots\cdots\cdots\cdots,$$

For $s = \dfrac{n-3}{3} - 1$, let $E_{\frac{n-3}{3}-1} = \cup_{i=1}^{t}\{f^*(u_{(n-1)(i-1)+j}u_{(n-1)(i-1)+j+1}) : n-6 \le j \le n-5\}$. Then

$$\begin{aligned}
E_{\frac{n-3}{3}-1} &= \cup_{i=1}^{t}\{|f(u_{(n-1)(i-1)+j}) - f(u_{(n-1)(i-1)+j+1})| : n-6 \le j \le n-5\} \\
&= \{|f(u_{n-6}) - f(u_{n-5})|, |f(u_{n-5}) - f(u_{n-4})|\} \\
&\quad \cup\{|f(u_{2n-7}) - f(u_{2n-6})|, |f(u_{2n-6}) - f(u_{2n-5})|\}\cup, \ldots, \\
&\quad \cup\{|f(u_{nt-n-t-4}) - f(u_{nt-n-t-3})|, |f(u_{nt-n-t-3}) - f(u_{nt-n-t-2})|\} \\
&\quad \cup\{|f(u_{nt-t-5}) - f(u_{nt-t-4})|, |f(u_{nt-t-4}) - f(u_{nt-t-3})|\} \\
&= \{|F_{nt-n+8} - F_{nt-n+6}|, |F_{nt-n+6} - F_{nt-n+4}|\} \cup \{|F_{nt-2n+8} - F_{nt-2n+6}|, \\
&\quad |F_{nt-2n+6} - F_{nt-2n+4}|\}\cup, \ldots, \cup\{|F_{n+8} - F_{n+6}|, |F_{n+6} - F_{n+4}|\} \\
&\quad \cup\{|F_8 - F_6|, |F_6 - F_4|\} \\
&= \{F_{nt-n+7}, F_{nt-n+5}\} \cup \{F_{nt-2n+7}, F_{nt-2n+5}\}\cup, \ldots, \cup\{F_{n+7}, F_{n+5}\} \\
&\quad \cup\{F_7, F_5\}
\end{aligned}$$

We calculate the edge labeling between the end vertex $(u_{(n-1)(i-1)+n-4})$ of the $(\frac{n-3}{3}-1)^{th}$ loop and starting vertex $(u_{(n-1)(i-1)+n-3})$ of the $(\frac{n-3}{3})^{rd}$ loop as follows.

$$\begin{aligned}
E^1_{\frac{n-3}{3}-1} &= \{|f(u_{(n-1)(i-1)+n-4}) - f(u_{(n-1)(i-1)+n-3})| : 1 \leq i \leq t\} \\
&= \{|f(u_{n-4}) - f(u_{n-3})|, |f(u_{2n-5}) - f(u_{2n-4})|, \ldots, \\
&\quad |f(u_{nt-n-t+2}) - f(u_{nt-n-t-1})|, |f(u_{nt-t-3}) - f(u_{nt-t-2})|\} \\
&= \{|F_{nt-n+6} - F_{nt-n+5}|, |F_{nt-2n+4} - F_{nt-2n+5}|, \ldots, \\
&\quad |F_{n+4} - F_{n+5}|, |F_4 - F_5|\} \\
&= \{F_{nt-n+4}, F_{nt-2n+3}, \ldots, F_{n+3}, F_3\}
\end{aligned}$$

For $s = \dfrac{n-3}{3}$, let $E_{\frac{n-3}{3}} = \bigcup_{i=1}^{t} \{f^*(u_{(n-1)(i-1)+j}u_{(n-1)(i-1)+j+1}) : n-3 \leq j \leq n-2\}$. Then

$$\begin{aligned}
E_{\frac{n-3}{3}} &= \cup_{i=1}^{t}\{|f(u_{(n-1)(i-1)+j}) - f(u_{(n-1)(i-1)+j+1})| : n-3 \leq j \leq n-2\} \\
&= \{|f(u_{n-3}) - f(u_{n-2})|, |f(u_{n-2}) - f(u_{n-1})|\} \\
&\quad \cup\{|f(u_{2n-4}) - f(u_{2n-3})|, |f(u_{2n-3}) - f(u_{2n-2})|\}\cup, \ldots, \\
&\quad \cup|f(u_{nt-n-t-1}) - f(u_{nt-n-t})|, |f(u_{nt-n-t}) - f(u_{nt-n-t+1})|\} \\
&\quad \cup\{|f(u_{nt-t-2}) - f(u_{nt-t-1})|, |f(u_{nt-t-1}) - f(u_{nt-t})|\} \\
&= \{|F_{nt-n+5} - F_{nt-n+3}|, |F_{nt-n+3} - F_{nt-n+1}|\} \\
&\quad \cup\{|F_{nt-2n+5} - F_{nt-2n+3}|, |F_{nt-2n+3} - F_{nt-2n+1}|\}\cup, \ldots, \\
&\quad \cup\{|F_{n+5} - F_{n+3}|, |F_{n+3} - F_{n+1}|\} \cup \{|F_5 - F_3|, |F_3 - F_1|\} \\
&= \{F_{nt-n+4}, F_{nt-n+2}\} \cup \{F_{nt-2n+4}, F_{nt-2n+2}\}\cup, \ldots, \\
&\quad \cup\{F_{n+4}, F_{n+2}\} \cup \{F_4, F_2\}
\end{aligned}$$

Calculation shows the edge labeling between the end vertex $(u_{(n-1)(i-1)+n-1})$ of the $(\frac{n-3}{3})^{rd}$ loop and the vertex u_0 are

$$\begin{aligned}
E^* &= \{|f(u_{(n-1)(i-1)+n-1}) - f(u_0)| : 1 \leq i \leq t\} \\
&= \{|f(u_{n-1}) - f(u_0)|, |f(u_{2n-2}) - f(u_0)|, \ldots, \\
&\quad |f(u_{nt-n-t+1}) - f(u_0)|, |f(u_{nt-t}) - f(u_0)|\} \\
&= \{|F_{nt-n+1} - F_0|, |F_{nt-2n+1} - F_0|, \ldots, |F_{n+1} - F_0|, |F_1 - F_0|\} \\
&= \{F_{nt-n+1}, F_{nt-2n+1}, \ldots, F_{n+1}, F_1\}.
\end{aligned}$$

Therefore,

$$\begin{aligned}
E &= (E_1 \cup E_2\cup, \ldots, \cup E_{\frac{n-3}{3}}) \cup (E^1_4 \cup E^1_5\cup, \ldots, \cup E^1_{\frac{n-3}{3}-1}) \cup E^* \\
&= \{F_1, F_2, \ldots, F_{nt}\}
\end{aligned}$$

Thus, the edge labels are distinct. Therefore, C_n^t admits a super fibonacci graceful labeling. $\square$

C_9^2 :

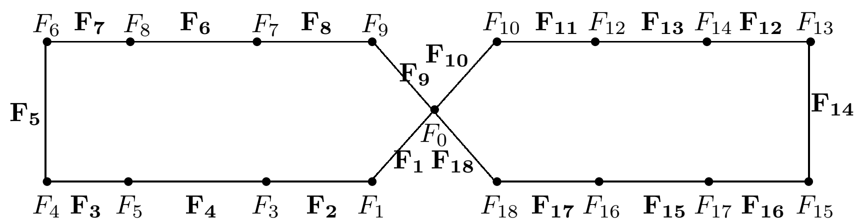

Fig.2

For example the super fibonacci graceful labeling of C_9^2 is shown in Fig.2.

Definition 2.6([4]) *Let $S_{m,n}$ stand for a star with n spokes in which each spoke is a path of length m.*

Definition 2.7 *The one point union of t copies of $S_{m,n}$ is denoted by $S_{m,n}^t$.*

Next theorem shows that the graph $S_{m,n}^t$ is a super Fibonacci graceful graph.

Theorem 2.8 *$S_{m,n}^t$ is a super fibonacci graceful graph for all m, n, when $n \equiv 1(mod3)$.*

Proof Let v_0 be the center of the star and v_j^i, $i = 1, 2, \ldots, mt$, $j = 1, 2, \ldots, n$ be the other vertices of $S_{m,n}^t$. Also, $|V(G)| = mnt + 1$ and $|E(G)| = mnt$. Define $f : V(S_{m,n}^t) \to \{F_0, F_1, \ldots, F_q\}$ by $f(v_0) = F_0$. For $i = 1, 2, \ldots, mt$, $f(v_j^i) = F_{mnt-n(i-1)-2(j-1)}$, $1 \leq j \leq 2$. For $i = 1, 2, \ldots, mt$, $F(v_j^i) = F_{mnt-(2j-n-1)-n(i-1)}$, $n - 1 \leq j \leq n$. For $s = 1, 2, \ldots, \frac{n-4}{3}$, $i = 1, 2, \ldots, mt$
$f(v_j^i) = F_{mnt-1-n(i-1)-2(j-s-2)+(s-1)}$, $3s \leq j \leq 3s + 2$. We claim that all these edge labels are distinct. Let $E_1 = \{f^*(v_0 v_1^i) : 1 \leq i \leq mt\}$. Calculation shows that

$$\begin{aligned} E_1 &= \{|f(v_0) - f(v_1^i)| : 1 \leq i \leq mt\} \\ &= \{|f(v_0) - f(v_1^1)|, |f(v_0) - f(v_1^2)|, \ldots, |f(v_0) - f(v_1^{mt-1})|, \\ &\quad |f(v_0) - f(v_1^{mt})|\} \\ &= \{|F_0 - F_{mnt}|, |F_0 - F_{mnt-n}|, \ldots, |F_0 - F_{2n}|, |F_0 - F_n|\} \\ &= \{F_{mnt}, F_{mnt-n}, \ldots, F_{2n}, F_n\}. \end{aligned}$$

Let $E_2 = \{f^*(v_1^i v_2^i) : 1 \leq i \leq mt\}$. Then

$$\begin{aligned} E_2 &= \{|f(v_1^i) - f(v_2^i)| : 1 \leq i \leq mt\} \\ &= \{|f(v_1^1) - f(v_2^1)|, |f(v_1^2) - f(v_2^2)|, \ldots, |f(v_1^{mt-1}) - f(v_2^{mt-1})|, \\ &\quad |f(v_1^{mt}) - f(v_2^{mt})|\} \\ &= \{|F_{mnt} - F_{mnt-2}|, |F_{mnt-n} - F_{mnt-n-2}|, \ldots, |F_{2n} - F_{2n-2}|, |F_n - F_{n-2}|\} \\ &= \{F_{mnt-1}, F_{mnt-n-1}, \ldots, F_{2n-1}, F_{n-1}\} \end{aligned}$$

For the edge labeling between the vertex v_2^i and starting vertex v_3^i of the first loop, let $E_3 = \{f^*(v_2^i v_3^i) : 1 \leq i \leq mt\}$. Calculation shows that

$$\begin{aligned}
E_3 &= \{|f(v_2^i) - f(v_3^i)| : 1 \leq i \leq mt\} \\
&= \{|f(v_2^1) - f(v_3^1)|, |f(v_2^2) - f(v_3^2)|, \ldots, |f(v_2^{mt-1}) - f(v_3^{mt-1})|, \\
&\quad |f(v_2^{mt}) - f(v_3^{mt})|\} \\
&= \{|F_{mnt-2} - F_{mnt-1}|, |F_{mnt-n-2} - F_{mnt-n-1}|, \ldots, |F_{2n-2} - F_{2n-1}|, \\
&\quad |F_{n-2} - F_{n-1}|\} \\
&= \{F_{mnt-3}, F_{mnt-n-3}, \ldots, F_{2n-3}, F_{n-3}\}.
\end{aligned}$$

For $s = 1$, let $E_4 = \cup_{i=1}^{mt}\{f^*(v_j^i v_{j+1}^i) : 3 \leq j \leq 4\}$. Then

$$\begin{aligned}
E_4 &= \cup_{i=1}^{mt}\{|f(v_j^i) - f(v_{j+1}^i)| : 3 \leq j \leq 4\} \\
&= \{|f(v_3^1) - f(v_4^1)|, |f(v_4^1) - f(v_5^1)|\} \cup \{|f(v_3^2) - f(v_4^2)|, |f(v_4^2) - f(v_5^2)|\} \\
&\quad \cup, \ldots, \cup\{|f(v_3^{mt-1}) - f(v_4^{mt-1})|, |f(v_4^{mt-1}) - f(v_5^{mt-1})|\} \\
&\quad \cup\{|f(v_3^{mt}) - f(v_4^{mt})|, |f(v_4^{mt}) - f(v_5^{mt})|\} \\
&= \{|F_{mnt-1} - F_{mnt-3}|, |F_{mnt-3} - F_{mnt-5}|\} \\
&\quad \cup\{|F_{mnt-n-1} - F_{mnt-n-3}|, |F_{mnt-n-3} - F_{mnt-n-5}|\} \\
&\quad \cup, \ldots, \cup\{|F_{2n-1} - F_{2n-3}|, |F_{2n-3} - F_{2n-5}|\} \\
&\quad \cup\{|F_{n-1} - F_{n-3}|, |F_{n-3} - F_{n-5}|\} \\
&= \{F_{mnt-2}, F_{mnt-4}\} \cup \{F_{mnt-n-2}, F_{mnt-n-4}\}\cup, \ldots, \cup\{F_{2n-2}, F_{2n-4}\} \\
&\quad \cup\{F_{n-2}, F_{n-4}\}.
\end{aligned}$$

We find the edge labeling between the vertex v_5^i of the first loop and starting vertex v_6^i of the second loop. Let $E_4^1 = \{f^*(v_5^i v_6^i) : 1 \leq i \leq mt\}$. Then

$$\begin{aligned}
E_4^1 &= \{|f(v_5^i) - f(v_6^i)| : 1 \leq i \leq mt\} \\
&= \{|f(v_5^1) - f(v_6^1)|, |f(v_5^2) - f(v_6^2)|, \ldots, |f(v_5^{mt-1}) - f(v_6^{mt-1})|, \\
&\quad |f(v_5^{mt}) - f(v_6^{mt})|\} \\
&= \{|F_{mnt-5} - F_{mnt-4}|, |F_{mnt-n-5} - F_{mnt-n-4}|, \ldots, |F_{2n-5} - F_{2n-4}|, \\
&\quad |F_{n-5} - F_{n-4}|\} \\
&= \{F_{mnt-6}, F_{mnt-n-6}, \ldots, F_{2n-6}, F_{n-6}\}.
\end{aligned}$$

For $s = 2$, let $E_5 = \cup_{i=1}^{mt}\{f^*(v_j^i v_{j+1}^i) : 6 \leq j \leq 7\}$. Then

$$\begin{aligned}
E_5 &= \cup_{i=1}^{mt}\{|f(v_j^i)-f(v_{j+1}^i)| : 6 \le j \le 7\}\\
&= \{|f(v_6^1)-f(v_7^1)|, |f(v_7^1)-f(v_8^1)|\} \cup \{|f(v_6^2)-f(v_7^2)|, |f(v_7^2)-f(v_8^2)|\}\\
&\quad \cup, \ldots, \ldots, \{|f(v_6^{mt-1})-f(v_7^{mt-1})|, |f(v_7^{mt-1})-f(v_8^{mt-1})|\}\\
&\quad \cup\{|f(v_6^{mt})-f(v_7^{mt})|, |f(v_7^{mt})-f(v_8^{mt})|\}\\
&= \{|F_{mnt-4}-F_{mnt-6}|, |F_{mnt-6}-F_{mnt-8}|\} \cup \{|F_{mnt-n-4}-F_{mnt-n-6}|,\\
&\quad |F_{mnt-n-6}-F_{mnt-n-8}|\}\cup, \ldots, \cup\{|F_{2n-4}-F_{2n-6}|, |F_{2n-6}-F_{2n-8}|\}\\
&\quad \cup\{|F_{n-4}-F_{n-6}|, |F_{n-6}-F_{n-8}|\}\\
&= \{F_{mnt-5}, F_{mnt-7}\} \cup \{F_{mnt-n-5}, F_{mnt-n-7}\}\cup, \ldots, \cup\{F_{2n-5}, F_{2n-7}\}\\
&\quad \cup\{F_{n-5}, F_{n-7}\}.
\end{aligned}$$

Let $E_5^1 = \{f^*(v_8^i v_9^i) : 1 \le i \le mt\}$. Calculation shows that the edge labeling between the vertex v_8^i of the second loop and starting vertex v_9^i of the third loop are

$$\begin{aligned}
E_5^1 &= \{|f(v_8^i)-f(v_9^i)| : 1 \le i \le mt\}\\
&= \{|f(v_8^1)-f(v_9^1)|, |f(v_8^2)-f(v_9^2)|, \ldots, |f(v_8^{mt-1})-f(v_9^{mt-1})|,\\
&\quad |f(v_8^{mt})-f(v_9^{mt})|\}\\
&= \{|F_{mnt-8}-F_{mnt-7}|, |F_{mnt-n-8}-F_{mnt-n-7}|, \ldots,\\
&\quad |F_{2n-8}-F_{2n-7}|, |F_{n-8}-F_{n-7}|\}\\
&= \{F_{mnt-9}, F_{mnt-n-9}, \ldots, F_{2n-9}, F_{n-9}\}.
\end{aligned}$$

For $s = \dfrac{n-4}{3} - 1$, let $E_{\frac{n-4}{3}-1} = \cup_{i=1}^{mt}\{f^*(v_j^i v_{j+1}^i) : n-7 \le j \le n-6\}$. Then

$$\begin{aligned}
E_{\frac{n-4}{3}-1} &= \cup_{i=1}^{mt}\{|f(v_j^i)-f(v_{j+1}^i)| : n-7 \le j \le n-6\}\\
&= \{|f(v_{n-7}^1)-f(v_{n-6}^1)|, |f(v_{n-6}^1)-f(v_{n-5}^1)|\}\\
&\quad \cup\{|f(v_{n-7}^2)-f(v_{n-6}^2)|, |f(v_{n-6}^2)-f(v_{n-5}^2)|\}\\
&\quad \cup, \ldots, \cup\{|f(v_{n-7}^{mt-1})-f(v_{n-6}^{mt-1})|, |f(v_{n-6}^{mt-1})-f(v_{n-5}^{mt-1})|\}\\
&\quad \cup\{|f(v_{n-7}^{mt})-f(v_{n-6}^{mt})|, |f(v_{n-6}^{mt})-f(v_{n-5}^{mt})|\}\\
&= \{|F_{mnt-n+9}-F_{mnt-n+7}|, |F_{mnt-n+7}-F_{mnt-n+5}|\}\\
&\quad \cup\{|F_{mnt-2n+9}-F_{mnt-2n+7}|, |F_{mnt-2n+7}-F_{mnt-2n+5}|\}\\
&\quad \cup, \ldots, \cup\{|F_{n+9}-F_{n+7}|, |F_{n+7}-F_{n+5}|\}\\
&\quad \cup\{|F_9-F_7|, |F_7-F_5|\}\\
&= \{F_{mnt-n+8}, F_{mnt-n+6}\} \cup \{F_{mnt-2n+8}, F_{mnt-2n+6}\}\cup, \ldots,\\
&\quad \cup\{F_{n+8}, F_{n+6}\} \cup \{F_8, F_6\}.
\end{aligned}$$

Similarly, for the edge labeling between the end vertex v_{n-5}^i of the $(\frac{n-4}{3}-1)^{th}$ loop and starting vertex v_{n-4}^i of the $(\frac{n-4}{3})^{rd}$ loop, let $E^1_{\frac{n-4}{3}-1} = \{f^*(v_{n-5}^i v_{n-4}^i) : 1 \le i \le mt\}$. Calculation shows that

$$\begin{aligned}
E^1_{\frac{n-4}{3}-1} &= \{|f(v^i_{n-5})-f(v^i_{n-4})| : 1 \leq i \leq mt\}\\
&= \{|f(v^1_{n-5})-f(v^1_{n-4})|, |f(v^2_{n-5})-f(v^2_{n-4})|, \ldots,\\
&\quad |f(v^{mt-1}_{n-5})-f(v^{mt-1}_{n-4})|, |f(v^{mt}_{n-5})-f(v^{mt}_{n-4})|\}\\
&= \{|F_{mnt-n+5}-F_{mnt-n+6}|, |F_{mnt-2n+5}-F_{mnt-2n+6}|, \ldots,\\
&\quad |F_{n+5}-F_{n+6}|, |F_5-F_6|\}\\
&= \{F_{mnt-n+4}, F_{mnt-2n+4}, \ldots, F_{n+4}, F_4\}.
\end{aligned}$$

Now for $s = \dfrac{n-4}{3}$, let $E_{\frac{n-4}{3}} = \bigcup\limits_{i=1}^{mt}\{f^*(v^i_j v^i_{j+1}) : n-4 \leq j \leq n-3\}$. Then

$$\begin{aligned}
E_{\frac{n-4}{3}} &= \cup_{i=1}^{mt}\{|f(v^i_j)-f(v^i_{j+1})| : n-4 \leq j \leq n-3\}\\
&= \{|f(v^1_{n-4})-f(v^1_{n-3})|, |f(v^1_{n-3})-f(v^1_{n-2})|\}\\
&\quad \cup\{|f(v^2_{n-4})-f(v^2_{n-3})|, |f(v^2_{n-3})-f(v^2_{n-2})|\}\cup\\
&\quad ,\ldots,\{|f(v^{mt-1}_{n-4})-f(v^{mt-1}_{n-3})|, |f(v^{mt-1}_{n-4})-f(v^{mt-1}_{n-3})|\}\\
&\quad \cup\{|f(v^{mt}_{n-4})-f(v^{mt}_{n-3})|, |f(v^{mt}_{n-3})-f(v^{mt}_{n-2})|\}\\
&= \{|F_{mnt-n+6}-F_{mnt-n+4}|, |F_{mnt-n+4}-F_{mnt-n+2}|\}\\
&\quad \cup\{|F_{mnt-2n+6}-F_{mnt-2n+4}|, |F_{mnt-2n+4}-F_{mnt-2n+2}|\}\\
&\quad \cup,\ldots,\cup\{|F_{n+6}-F_{n+4}|, |F_{n+4}-F_{n+2}|\}\\
&\quad \cup\{|F_6-F_4|, |F_4-F_2|\}\\
&= \{F_{mnt-n+5}, F_{mnt-n+3}\}\cup\{F_{mnt-2n+5}, F_{mnt-2n+3}\}\cup,\ldots,\\
&\quad \cup\{F_{n+5}, F_{n+3}\}\cup\{F_5, F_3\}.
\end{aligned}$$

We find the edge labeling between the end vertex v^i_{n-2} of the $(\frac{n-4}{3})^{rd}$ loop and the vertex v^i_{n-1}. Let $E^*_1 = \{f^*(v^i_{n-2}v^i_{n-1}) : 1 \leq i \leq mt\}$. Then

$$\begin{aligned}
E^*_1 &= \{|f(v^i_{n-2})-f(v^i_{n-1})| : 1 \leq i \leq mt\}\\
&= \{|f(v^1_{n-2})-f(v^1_{n-1})|, |f(v^2_{n-2})-f(v^2_{n-1})|, \ldots, |f(v^{mt-1}_{n-2})-f(v^{mt-1}_{n-1})|,\\
&\quad |f(v^{mt}_{n-2})-f(v^{mt}_{n-1})|\}\\
&= \{|F_{mnt-n+2}-F_{mnt-n+3}|, |F_{mnt-2n+2}-F_{mnt-2n+3}|, \ldots,\\
&\quad |F_{n+2}-F_{n+3}|, |F_2-F_3|\}\\
&= \{F_{mnt-n+1}, F_{mnt-2n+1}, \ldots, F_{n+1}, F_1\}.
\end{aligned}$$

Let $E^*_2 = \{f^*(v^i_{n-1}v^i_n) : 1 \leq i \leq mt\}$. Then

$$\begin{aligned}
E^*_2 &= \{|f(v^i_{n-1})-f(v^i_n)| : 1 \leq i \leq mt\}\\
&= \{|f(v^1_{n-1})-f(v^1_n)|, |f(v^2_{n-1})-f(v^2_n)|, \ldots,\\
&\quad |f(v^{mt-1}_{n-1})-f(v^{mt-1}_n)|, |f(v^{mt}_{n-1})-f(v^{mt}_n)|\}\\
&= \{|F_{mnt-n+3}-F_{mnt-n+1}|, |F_{mnt-2n+3}-F_{mnt-2n+1}|, \ldots,\\
&\quad |F_{n+3}-F_{n+1}|, |F_3-F_1|\}\\
&= \{F_{mnt-n+2}, F_{mnt-2n+2}, \ldots, F_{n+2}, F_2\}.
\end{aligned}$$

Therefore,

$$\begin{aligned} E &= (E_1 \cup E_2 \cup, \ldots, \cup E_{\frac{n-4}{3}}) \cup (E_4^1 \cup E_5^1 \cup, \ldots, \cup E^1_{\frac{n-4}{3}-1}) \cup E_1^* \cup E_2^* \\ &= \{F_1, F_2, \ldots, F_{mnt}\} \end{aligned}$$

Thus, $S^t_{m,n}$ admits a super fibonacci graceful labeling. □

For example the super fibonacci graceful labeling of $S^2_{3,7}$ is shown in Fig.3.

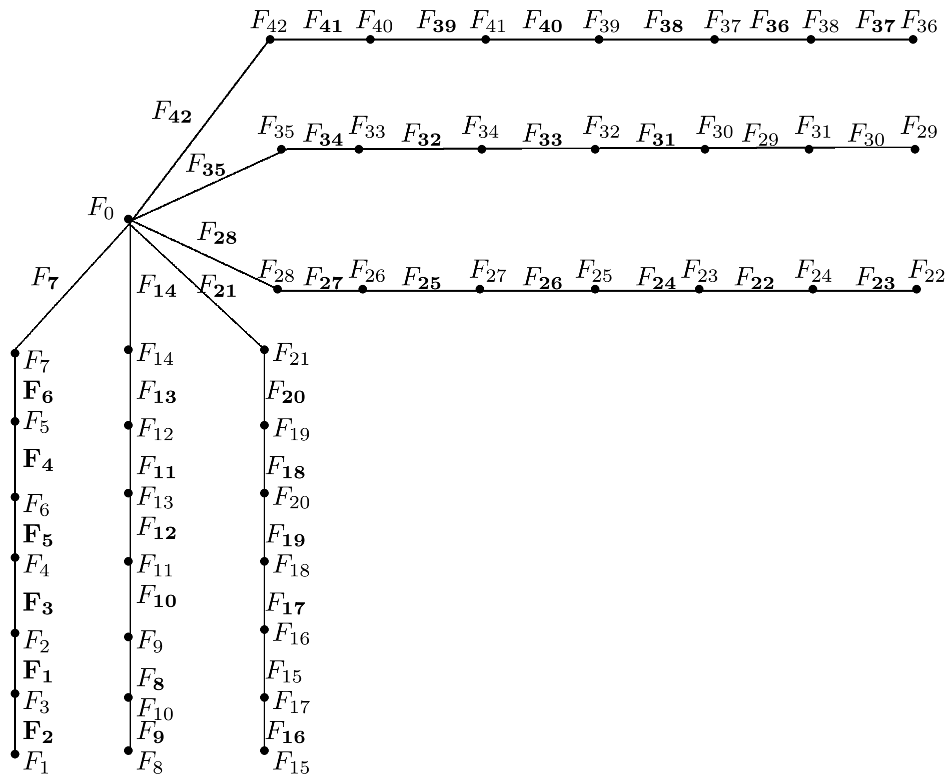

Fig.3

Theorem 2.9 *The complete graph K_n is a super fibonacci graceful graph if $n \leq 3$.*

Proof Let $\{v_0, v_1, \ldots, v_{n-1}\}$ be the vertex set of K_n. Then $v_i(0 \leq i \leq n-1)$ is adjacent to all other vertices of K_n. Let v_0 and v_1 be labeled as F_0 and F_q respectively. Then v_2 must be given F_{q-1} or F_{q-2} so that the edge v_1v_2 will receive a fibonacci number F_{q-2} or F_{q-1}. Therefore, the edges will receive the distinct labeling. Suppose not, Let v_0 and v_1 be labeled as F_1 and F_q or F_0 and F_{q-2} respectively. Then v_2 must be given F_{q-1} or F_{q-2} so that the edges v_0v_2 and v_1v_2 will receive the same edge label F_{q-2}, which is a contradiction by our definition. Hence, K_n is super fibonacci graceful graph if $n \leq 3$. □

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