SMARANDACHE SEMI-AUTOMATON AND AUTOMATON

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Abstract: *In this paper we study the Smarandache Semi-Automaton and Automaton using Smarandache free groupoids.*

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Definition 1: Let S be a non-empty set. Then $\langle S \rangle$ denotes the *free groupoid generated by the set S as a basis*.
We assume the free semigroup generated by S is also contained in the free groupoid generated by S.

Remark: Even $a(bc) \neq (ab)c$ for $a, b, c \in S$. So unlike a free semigroup where the operation is associative in case of free groupoid we do not assume the associativity while placing them in the juxtaposition.

Definition [W.B.Vasantha Kandasamy]: A groupoid G is said to be a *Smarandache groupoid*, if G contains a non-empty proper subset S such that S is a semigroup under the operations of the groupoid G.

Theorem 2: Every free groupoid is a Smarandache free groupoid.

Proof: By the very definition of the free groupoid we have the above theorem to be true.

Definition [R. Lidl, G. Pilz]: A *Semi-Automaton* is a triple $\mathcal{S} = (Z, A, \delta)$ consisting of two non-empty sets Z and A and a function $\delta: Z \times A \to Z$, Z is called the set of states, A the input alphabet, and δ the next state function of $\mathcal{S}$.

Definition [R. Lidl, G. Pilz]: An *Automaton* is a quintuple $\mathcal{A} = (Z, A, B, \delta, \lambda)$, where (Z, A, δ) is a semi automaton, B is a non-empty set called the output alphabet and $\lambda: Z \times A \to B$ is the output function.

Now it is important and interesting to note that Z, A, and B are only non-empty sets. They have no algebraic operation defined on them. The automatons and semi automatons defined in this manner do not help to perform sequential operations. Thus, it is reasonable to consider the set of all finite sequences of elements of A including the empty sequence Λ. In other words, in our study of automaton we extend the input set A to the free monoid $\overline{A}$ and similarly for B. We also extend δ and λ from $Z \times A$ to $Z \times \overline{A}$ by defining $z \in Z$ and $a_1, \ldots, a_n \in \overline{A}$ by

$$
\begin{aligned}
\overline{\delta}(z, \Lambda) &= z \\
\overline{\delta}(z, a_1) &= \delta(z, a_1) \\
\overline{\delta}(z, a_1 a_2) &= \delta(\overline{\delta}(z, a_1), a_2) \\
&\vdots \\
\overline{\delta}(z, a_1 a_2 \ldots a_n) &= \delta(\overline{\delta}(z, a_1 a_2 \ldots a_{n-1}), a_n)
\end{aligned}
$$

and

$\lambda : Z \times A \to B$ by $\overline{\lambda} : Z \times \overline{A} \to \overline{B}$

by

$$
\begin{aligned}
\overline{\lambda}(z, \Lambda) &= \Lambda \\
\overline{\lambda}(z, a_1) &= \lambda(z, a_1) \\
\overline{\lambda}(z, a_1 a_2) &= \lambda(z, a_1) \lambda(\delta(z, a_1), a_2) \\
&\vdots \\
\overline{\lambda}(z, a_1 a_2 \ldots a_n) &= \lambda(z, a_1) \overline{\lambda}(\delta(z, a_1), a_2 \ldots a_r)
\end{aligned}
$$

The semi-automaton $\mathcal{S}$ = (Z, A, δ) and automaton $\mathcal{A}$ = (Z, A, B, δ, λ) are thus generalized to the new semi-automaton $\mathcal{S}$ = (Z, $\overline{A}, \overline{\delta}$) and new automaton $\mathcal{A}$ = (Z, $\overline{A}, \overline{B}, \overline{\delta}, \overline{\lambda}$).

Definition 3: $\mathcal{S}_s$ = (Z, $\overline{A}_s, \overline{\delta}_s$) is said to be a *Smarandache semi-automaton* if $\overline{A} = \langle A \rangle$ is the free groupoid generated by A, with Λ the unit element adjoined with it. Thus the Smarandache semi-automaton contains $\mathcal{S}$ = (Z, $\overline{A}, \overline{\delta}$) as a new semi-automaton which is a proper sub-structure of $\mathcal{S}_s$.

Or equivalently, we define a Smarandache semi-automaton as one which has a new semi-automaton as a sub-structure.

The advantages of the Smarandache semi-automaton is: if the triple $\mathcal{S}$ = (Z, A, δ) is a semi-automaton with Z, the set of states, and $\delta : Z \times A \to Z$ is the next state function, and when we generate the Smarandache free groupoid by A and adjoin with it the empty alphabet Λ then we are sure that $\overline{A}$ has all free semigroups. Thus each free semigroup will give a new semi-automaton. Thus by choosing a suitable A we can get several new semi-automatons using a single Smarandache semi-automaton.

Definition 4: $\overline{\mathcal{A}}_s = (Z, \overline{A}_s, \overline{B}_s, \overline{\delta}_s, \overline{\lambda}_s)$ is defined to be a Smarandache automaton if $\overline{\mathcal{A}} = (Z, \overline{A}, \overline{B}, \overline{\delta}, \overline{\lambda})$ is the new automaton, and $\overline{A}_s$ and $\overline{B}_s$ the Smarandache free groupoids so that $\overline{\mathcal{A}} = (Z, \overline{A}, \overline{B}, \overline{\delta}, \overline{\lambda})$, the new automaton got from $\mathcal{A}$ and $\overline{\mathcal{A}}$, is strictly contained in $\overline{\mathcal{A}}_s$.

Thus Smarandache Automaton enables us to adjoin some more elements that are present in A and freely generated by A, as a free groupoid; that will be the case when the compositions may not be associative. Secondly, by using Smarandache Automaton we can couple several automatons as:

$$Z = Z_1 \cup Z_2 \cup \ldots \cup Z_n$$
$$A = A_1 \cup A_2 \cup \ldots \cup A_n$$
$$B = B_1 \cup B_2 \cup \ldots \cup B_n$$
$$\lambda = \lambda_1 \cup \lambda_2 \cup \ldots \cup \lambda_n$$
$$\delta = \delta_1 \cup \delta_2 \cup \ldots \cup \delta_n$$

where the union of $\lambda_i \cup \lambda_j$ and $\delta_i \cup \delta_j$ denote only extension maps as '$\cup$' has no meaning in the composition of maps, where $\mathcal{A}_i = (Z_i, A_i, B_i, \delta_i, \lambda_i)$ for i = 1, 2, 3, ..., n and $\overline{\mathcal{A}} = \overline{\mathcal{A}}_1 \cup \overline{\mathcal{A}}_2 \cup \ldots \cup \overline{\mathcal{A}}_n$. Now $\overline{\mathcal{A}}_n = (\overline{Z}_s, \overline{A}_s, \overline{B}_s, \overline{\lambda}_s, \overline{\delta}_s)$ is the Smarandache Automaton. A machine equipped with this Smarandache Automaton can use any new automaton as per need.

Definition 5: $\overline{\mathcal{A}}_s' = (Z_1, \overline{A}_s, \overline{B}_s, \overline{\delta}_s', \overline{\lambda}_s')$ is called *Smarandache sub-automaton* of $\overline{\mathcal{A}}_s = (Z_2, \overline{A}_s, \overline{B}_s, \overline{\delta}_s, \overline{\lambda}_s)$ denoted by $\overline{\mathcal{A}}_s' \leq \overline{\mathcal{A}}_s$ if $Z_1 \subseteq Z_2$ and $\overline{\delta}_s'$ and $\overline{\lambda}_s'$ are the restriction of $\overline{\delta}_s$ and $\overline{\lambda}_s$ respectively on $Z_1 \times \overline{A}_s$ and $\overline{\mathcal{A}}_s'$ has a proper subset $\overline{H} \subset \overline{\mathcal{A}}_s'$ such that $\overline{H}$ is a new automaton.

Definition 6: Let $\overline{\mathcal{A}}_1$ and $\overline{\mathcal{A}}_2$ be any two Smarandache Automatons where $\overline{\mathcal{A}}_1 = (Z_1, \overline{A}_1, \overline{B}_1, \overline{\delta}_1, \overline{\lambda}_1)$ and $\overline{\mathcal{A}}_2 = (Z_2, \overline{A}_2, \overline{B}_2, \overline{\delta}_2, \overline{\lambda}_2)$. A map ϕ: $\overline{\mathcal{A}}_1$ to $\overline{\mathcal{A}}_2$ is a *Smarandache Automaton homomorphism* if ϕ is an automaton homomorphism from $\overline{\mathcal{A}}_1$ and $\overline{\mathcal{A}}_2$.

And ϕ is called a *monomorphism (epimorphism or isomorphism)* if ϕ is an automaton isomorphism from $\overline{A}_1$ and $\overline{A}_2$.

Definition 7: Let $\overline{A}_1$ and $\overline{A}_2$ be two Smarandache automatons, where $\overline{A}_1 = (Z_1, \overline{A}_1, \overline{B}_1, \overline{\delta}_1, \overline{\lambda}_1)$ and $\overline{A}_2 = (Z_2, \overline{A}_2, \overline{B}_2, \overline{\delta}_2, \overline{\lambda}_2)$. The *Smarandache Automaton direct product* of $\overline{A}_1$ and $\overline{A}_2$ denoted by $\overline{A}_1 \times \overline{A}_2$ is defined as the direct product of the automaton $\mathcal{A}_1 = (Z_1, A_1, B_1, \delta_1, \lambda_1)$ and $\mathcal{A}_2 = (Z_2, A_2, B_2, \delta_2, \lambda_2)$ with $\mathcal{A}_1 \times \mathcal{A}_2. = (Z_1 \times Z_2, A_1 \times A_2, B_1 \times B_2, \delta, \lambda)$ with $\delta((z_1, z_2), (a_1, a_2)) = (\delta_1(z_1, a_2), \delta_2(z_2, a_2))$, $\lambda((z_1, z_2), (a_1, a_2)) = (\lambda_1(z_1, a_2), \lambda_2(z_2, a_2))$ for all $(z_1, z_2) \in Z_1 \times Z_2$ and $(a_1, a_2) \in A_1 \times A_2$.

Remark: Here in $\overline{A}_1 \times \overline{A}_2$ we do not take the free groupoid to be generated by $A_1 \times A_2$ but only free groupoid generated by $\overline{A}_1 \times \overline{A}_2$ Thus the Smarandache Automaton direct product exists wherever an automaton direct product exists. We have made this in order to make the *Smarandache parallel composition* and *Smarandache series composition* of automaton extendable in a simple way.

Definition 8: *A Smarandache groupoid G_1 divides a Smarandache groupoid G_2* if the the groupoid G_1 divides the groupoid G_2, that is: if G_1 is a homomorphic image of a sub-groupoid of G_2. In symbols: $G_1 | G_2$. In the relation: divides is denoted by '|'.

Definition 9: Let $\overline{A}_1 = (Z_1, \overline{A}, \overline{B}, \overline{\delta}_1, \overline{\lambda}_1)$ and $\overline{A}_2 = (Z_2, \overline{A}, \overline{B}, \overline{\delta}_2, \overline{\lambda}_2)$ be two Smarandache Automaton. We say the *Smarandache Automaton $\overline{A}_1$ divides the Smarandache automaton $\overline{A}_2$* if $\overline{A}_1 = (Z_1, \overline{A}, \overline{B}, \overline{\delta}_1, \overline{\lambda}_1)$ divides $\overline{A}_2 = (Z_2, \overline{A}, \overline{B}, \overline{\delta}_2, \overline{\lambda}_2)$, i.e. if $\overline{A}_1$ is the homomorphic image of a sub-automaton of $\overline{A}_2$. Notationally $\overline{A}_1 | \overline{A}_2$.

Definition 10: Two *Smarandache Automaton $\overline{A}_1$ and $\overline{A}_2$ are* said to be *equivalent* if they divide each other. In symbols $\overline{A}_1 \sim \overline{A}_2$.

Theorem 11:

1. On any set of Smarandache Automata the relation 'divides' that is '|' is reflexive and transitive and '~' is an equivalence relation.
2. Isomorphic Smarandache Automaton are equivalent (but not conversely).

Proof: By the very definition of 'divides' (or '|') and the equivalence of two Smarandache Automaton the result follows.

References:

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