Raising the many-dimensional vector spaces to the rational power M/L.

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Abstract

If N - dimensional vector space W can be represented as the tensor product of L identical n – dimensional vector spaces V, then we can say, that V is the W raised to the power 1/L. If we take the tensor product of M vector spaces V, then we get the vector space R. And we can say that R is the W raised to the power M/L.

1) Vector spaces W and V.

Let us consider W – N-dimensional generalization of our 4-dimentional vector space. And we call W as basic vector space. If we choose N such as n^L, then we can represent W as the tensor product of L identical n-dimensional vector spaces V:

$$W = {}_1V \otimes {}_2V \otimes \ldots \otimes {}_LV \qquad (1)$$

It can be written so: $$W = V^L \qquad (2)$$

Or: $$V = W^{\frac{1}{L}} \qquad (3)$$

2) Metric tensors.

If $\vec{e}_\mu$ - basis of W, $\vec{n}_\alpha$ - basis of V, then their connection can be expressed so:

$$\vec{e}_\mu = e_\mu^{\overbrace{\alpha\beta\ldots\gamma}} \cdot \vec{n}_\alpha \otimes \vec{n}_\beta \otimes \ldots \otimes \vec{n}_\gamma \qquad (4)$$

and so:

$$\vec{n}_\alpha \otimes \vec{n}_\beta \otimes \ldots \otimes \vec{n}_\gamma = \vec{e}_\mu \cdot e^\mu{}_{\underbrace{\alpha\beta\ldots\gamma}_L} \qquad (5)$$

$$\vec{n}_\delta \otimes \vec{n}_k \otimes \ldots \otimes \vec{n}_s = \vec{e}_\nu \cdot e^\nu{}_{\underbrace{\delta k\ldots s}_L} \qquad (6)$$

If $(\vec{e}_\mu, \vec{e}_\nu) = g_{\mu\nu}$ - the metric tensor for W, and $(\vec{n}_\alpha, \vec{n}_\beta) = q_{\alpha\beta}$ - the metric tensor for V, then the scalar multiplication of (5) and (6) gives :

$$q_{\alpha\delta} \cdot q_{\beta k} \cdot \ldots \cdot q_{\gamma s} = g_{\mu\nu} \cdot e^\mu{}_{\underbrace{\alpha\beta\ldots\gamma}_L} \cdot e^\nu{}_{\underbrace{\delta k\ldots s}_L} \qquad (7)$$

And we can write:

$$g_{\mu\nu} \cdot e^\mu{}_{\underbrace{\alpha\alpha\ldots\alpha}_L} \cdot e^\nu{}_{\underbrace{\delta\delta\ldots\delta}_L} = \underbrace{q_{\alpha\delta} \cdot q_{\alpha\delta} \cdot \ldots \cdot q_{\alpha\delta}}_L = (q_{\alpha\delta})^L \qquad (8)$$

And

$$q_{\alpha\delta} = (g_{\mu\nu} \cdot e^\mu{}_{\underbrace{\alpha\alpha\ldots\alpha}_L} \cdot e^\nu{}_{\underbrace{\delta\delta\ldots\delta}_L})^{\frac{1}{L}} \qquad (9)$$

3) Algebraic tensors.

Algebraic tensor defines the algebra of basis vectors of vector space. Let us introduce the algebraic tensors for W and for V by this way:

$$[\vec{e}_{\mu} \times \vec{e}_{\nu}] = \vec{e}_{\sigma} \cdot F^{\sigma}{}_{\mu\nu} \qquad (10)$$

$$[\vec{n}_{\alpha} \times \vec{n}_{\beta}] = \vec{n}_{\gamma} \cdot f^{\gamma}{}_{\alpha\beta} \qquad (11)$$

From (4), (10), (11) we can derive:

$$f^{\beta}{}_{\alpha\gamma} = (e_{\sigma}{}^{\overbrace{\beta\beta\ldots\beta}} \cdot e^{\mu}{}_{\underbrace{\alpha\alpha\ldots\alpha}_{L}} \cdot e^{\nu}{}_{\underbrace{\gamma\gamma\ldots\gamma}_{L}} \cdot F^{\sigma}{}_{\mu\nu})^{\frac{1}{L}} \qquad (12)$$

4) Vector space R.

Now we form new vector space R so: $R = {}_1V \otimes {}_2V \otimes \ldots \otimes {}_M V$ (13)
M here is any integer number. Then dimension of R is n^{M}. And we can write:

$$R = W^{\frac{M}{L}} \qquad (14)$$

If we denote the basis of R as $\vec{m}_{d}$, then

$$\vec{m}_{d} = E_{d}{}^{\alpha_1 \alpha_2 \ldots \alpha_M} \cdot \vec{n}_{\alpha_1} \otimes \vec{n}_{\alpha_2} \otimes \ldots \otimes \vec{n}_{\alpha_M} \qquad (15)$$

Metric tensor in R is r_{db}:

$$(\vec{m}_{d}, \vec{m}_{b}) = r_{db} \qquad (16)$$

Algebraic tensor in R is $Z^{c}{}_{db}$:

$$[\vec{m}_{d} \times \vec{m}_{b}] = \vec{m}_{c} \cdot Z^{c}{}_{db} \qquad (17)$$

If we define $E_{d}{}^{\alpha}$ as:

$$E_{d}{}^{\alpha} \cdot E^{b}{}_{\underbrace{\alpha\alpha\ldots\alpha}_{M}} = \delta_{d}{}^{b} \qquad (18)$$

then we can find the metric and algebraic tensors for R:

$$r_{db} = E_{d}{}^{\alpha} \cdot E_{b}{}^{\beta} \cdot (q_{\alpha\beta})^{M} \qquad (19)$$

$$Z^{c}{}_{db} = E^{c}{}_{\beta} \cdot E_{d}{}^{\alpha} \cdot E_{b}{}^{\gamma} \cdot (f^{\beta}{}_{\alpha\gamma})^{M} \qquad (20)$$

5) Curved W.

Let W be curved. And η_{ab} - metric tensor in uncurved space. Then

$$g_{\mu\nu} = h_{\mu}{}^{a} \cdot h_{\nu}{}^{b} \cdot \eta_{ab} \qquad (21)$$

and if $\varphi^{a}{}_{bc}$-algebraic tensor in uncurved space, then

$$F^{\sigma}{}_{\mu\nu} = h^{\sigma}{}_{d} \cdot h_{\mu}{}^{b} \cdot h_{\nu}{}^{c} \cdot \varphi^{d}{}_{bc} \qquad (22)$$

If metric tensor asymmetric and space is curved, then we use another formula - (27) – (more general) for the algebraic tensor.

$$[\vec{e}_{m} \times \vec{e}_{n}] = \vec{e}_{l} \cdot F^{l}{}_{mn} \qquad (23) \qquad \partial_{\rho}\overrightarrow{e_{\mu}} = \overrightarrow{e_{\mu,\rho}} = \vec{e}_{\sigma} \cdot \Gamma^{\sigma}{}_{\mu\rho} \quad (24)$$

$$\partial_{\rho}(23) : F^{s}{}_{\sigma\nu} \cdot \Gamma^{\sigma}{}_{\mu\rho} + F^{s}{}_{\mu\sigma} \cdot \Gamma^{\sigma}{}_{\nu\rho} = \Gamma^{s}{}_{\lambda\rho} \cdot F^{\lambda}{}_{\mu\nu} + F^{s}{}_{\mu\nu,\rho} \quad (25)$$

$$\partial_k(25): F^s{}_{\sigma\nu,k}\cdot\Gamma^\sigma{}_{\mu\rho}+F^s{}_{\sigma\nu}\cdot\Gamma^\sigma{}_{\mu\rho,k}+F^s{}_{\mu\sigma,k}\cdot\Gamma^\sigma{}_{\nu\rho}+F^s{}_{\mu\sigma}\cdot\Gamma^\sigma{}_{\nu\rho,k}=$$
$$=\ \Gamma^s{}_{\lambda\rho,k}\cdot F^\lambda{}_{\mu\nu}+\Gamma^s{}_{\lambda\rho}\cdot F^\lambda{}_{\mu\nu,k}+F^s{}_{\mu\nu,\rho k}\quad (26)$$

Let us contract (26) by s and k :

$$F^s{}_{\sigma\nu,s}\cdot\Gamma^\sigma{}_{\mu\rho}+F^s{}_{\sigma\nu}\cdot\Gamma^\sigma{}_{\mu\rho,s}+F^s{}_{\mu\sigma,s}\cdot\Gamma^\sigma{}_{\nu\rho}+F^s{}_{\mu\sigma}\cdot\Gamma^\sigma{}_{\nu\rho,s}=$$
$$=\ \Gamma^s{}_{\lambda\rho,s}\cdot F^\lambda{}_{\mu\nu}+\Gamma^s{}_{\lambda\rho}\cdot F^\lambda{}_{\mu\nu,s}+F^s{}_{\mu\nu,\rho s}\qquad (27)$$

Christoffel symbols for asymmetric metric tensors you can take from
http://vixra.org/abs/1302.0072

6) New term.

And the question of naming. $e_\mu^{\overbrace{\alpha\beta\kappa\gamma}}$ - COeffitient of BASIses Connection- we will name as “COBASIC”.

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Part of the item 5 is added by : (23) – (27) at 27/02-2014