A recurrent formula inspired by Rowland’s formula and based on Smarandache function which might be a criterion for primality

Marius Coman
Bucuresti, Romania
email: mariuscoman13@gmail.com

Abstract. Studying the two well known recurrent relations with the exceptional property that they generate only values which are equal to 1 or are primes, id est the formula which belongs to Eric Rowland and the one that belongs to Benoit Cloitre, I managed to discover a formula based on Smarandache function, from the same family of recurrent relations, which, instead to give a prime value for any input, seems to give the same value, 2, if and only if the value of the input is a prime. I name this relation the Coman-Smarandache criterion for primality and the exceptions from this rule, if they exist, Coman-Smarandache pseudoprimes.

Conjecture
Let \( f(1) = 1 \) and \( f(n) = S(f(n - 1)) + \text{lcm}[n, S(f(n - 1))] \), where \( S \) is the Smarandache function and \( \text{lcm} \) the least common multiple. Then the value of the function \( g(n) = f(n)/S(f(n - 1)) \) is equal to 2 if and only if \( n \) is an odd prime.

Verifying the conjecture (up to \( n = 17 \))

\[
\begin{align*}
 f(2) &= 1 + \text{lcm}[2, 1] = 3; \quad \text{then } g(2) = 3/1 = 3; \\
 f(3) &= 3 + \text{lcm}[3, 3] = 6; \quad \text{then } g(3) = 6/3 = 2; \\
 f(4) &= 3 + \text{lcm}[4, 3] = 15; \quad \text{then } g(4) = 15/3 = 5; \\
 f(5) &= 5 + \text{lcm}[5, 5] = 10; \quad \text{then } g(5) = 10/5 = 2; \\
 f(6) &= 5 + \text{lcm}[6, 5] = 35; \quad \text{then } g(6) = 35/5 = 7; \\
 f(7) &= 7 + \text{lcm}[7, 7] = 14; \quad \text{then } g(7) = 14/7 = 2; \\
 f(8) &= 7 + \text{lcm}[8, 7] = 63; \quad \text{then } g(8) = 63/7 = 9; \\
 f(9) &= 7 + \text{lcm}[9, 7] = 70; \quad \text{then } g(9) = 70/7 = 10; \\
 f(10) &= 7 + \text{lcm}[10, 7] = 77; \quad \text{then } g(10) = 77/7 = 11; \\
 f(11) &= 11 + \text{lcm}[11, 11] = 22; \quad \text{then } g(11) = 22/11 = 2; \\
 f(12) &= 11 + \text{lcm}[12, 11] = 143; \quad \text{then } g(12) = 143/11 = 13; \\
 f(13) &= 13 + \text{lcm}[13, 13] = 26; \quad \text{then } g(13) = 26/13 = 2; \\
 f(14) &= 13 + \text{lcm}[14, 13] = 195; \quad \text{then } g(14) = 195/13 = 15; \\
 f(15) &= 13 + \text{lcm}[15, 13] = 208; \quad \text{then } g(15) = 208/13 = 16; \\
 f(16) &= 13 + \text{lcm}[16, 13] = 221; \quad \text{then } g(16) = 221/13 = 17; \\
 f(17) &= 17 + \text{lcm}[17, 17] = 17; \quad \text{then } g(17) = 34/17 = 2.
\end{align*}
\]
Note

It can be seen that, in the verified cases, the value of $g(n)$ is equal to 2 if and only if $n$ is odd prime; the value of $g(n)$ in any other case (for any other $n$) beside $f(1) = 1$ and $f(p) = 2$, where $p$ is odd prime, is equal to $n + 1$.

Note

The function $g(n) = \frac{f(n)}{S(f(n - 1))} - 1$, where $f(n) = f(n - 1) + \text{lcm}[n, f(n - 1)]$ might also be interesting to study as a prime generating formula, as it gives prime values (i.e. 5, 17, 23, 191, 383) for the following consecutive values of $n$: 4, 5, 6, 7, 8; however, for $n = 9$ the value obtained is a semiprime and for $n = 10$ is not even obtained an integer value, because $m$ is not always divisible by $S(m)$ so $f(n)$, which is always divisible by $f(n - 1)$, is not always divisible by $S(f(n - 1))$.

References:

1. Rowland, Eric, A simple prime-generating recurrence;
2. Peterson, Ivars, A new formula for generating primes;