EFFECTIVE ISO-RADIUS OF DYNAMIC ISO-SPHERE HOLOGRAPHIC RINGS

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November 28, 2013

Abstract

In this work, we introduce the “effective iso-radius” for dynamic iso-sphere Inopin holographic rings (IHR) as the iso-radius varies, which facilitates a heightened characterization of these emerging, cutting-edge iso-spheres as they vary in size and undergo “iso-transitions” between “iso-states”. The initial results of this exploration fuel the construction of a new “effective iso-state” platform with a potential for future scientific application, but this emerging dynamic iso-architecture warrants further development, scrutiny, collaboration, and hard work in order to advance it as such.

Keywords: Santilli iso-number; Inopin holographic ring; Santilli iso-sphere; Dynamic iso-sphere; Iso-radius; Effective iso-state.
1 Introduction

Santilli’s iso-mathematics [1, 2, 3, 4, 5] has sparked a revolution in universal number classification and topology. Recently, the triplex numbers and Inopin’s dual 4D space-time IHR topology [6, 7] were iso-topically lifted [1, 2, 3, 4, 5] to construct the iso-triplex numbers and the iso-dual 4D space-time IHR topology for iso-fractals [8]. Subsequently, these emerging developments [8] were deployed to propose a topological iso-string theory [9] and assemble Mandelbrot iso-sets [10]. Furthermore, such implementations facilitated the dynamic iso-topic lifting of iso-spaces to install dynamic iso-spaces [11], which built the foundation of dynamic iso-sphere IHRs with exterior and interior iso-duality [12].

In this assignment, we focus on advancing the representation of dynamic iso-sphere IHRs [12] by forging the effective iso-radius platform to launch the encoding of their characteristic “iso-transitions” between “iso-states” as they vary in size. The effective iso-radius concept introduced in this paper was originally inspired by the “effective radius” from Corda’s new black hole effective state framework [13, 14, 15, 16]. However, this paper is devoted to iso-mathematics rather than physics; thus, the effective representation proposed here targets spherically-symmetric iso-mathematical structures rather than spherically-symmetric physical quantities. Hence, for now, we limit our investigation to the realm of iso-mathematics [1, 2, 3, 4, 5] but recognize a significant potential for scientific application in the near future. Thus, we conduct our investigation with Section procedure by presenting a step-by-step procedure that constructs the effective iso-radius for a dynamic iso-sphere IHR [12] with dynamic iso-topic lifting [11] in the iso-dual 4D space-time IHR topology [8]. Finally, we conclude our paper with Section 3, where we recapitulate the results of Section 2 with a brief discussion and suggest future modes of research.

2 Procedure

In the venture of this section, we assemble the effective iso-radius for a dynamic iso-sphere IHR [12] and thereby introduce the notion of “effective iso-state”.

2
2.1 Initializing the dual 4D space-time IHR topology

Here, we instantiate the dual 4D space-time IHR topology via the following procedure:

1. First, from eq. (7) of [8], let \( X = \mathbb{C} \) be the set of all complex numbers, the Euclidean complex space, and the dual 2D Cartesian-polar coordinate-vector state space, where the complex number \( \vec{x} \in X \) is a dual 2D Cartesian-polar coordinate-vector state that is defined by eq. (6) of [8] as

\[
x = \vec{x} = \vec{x}_R + \vec{x}_I = (|\vec{x}|, \langle \vec{x} \rangle)_P = (\vec{x}_R, \vec{x}_I)_C, \quad \forall \vec{x} \in X.
\]

In eq. (1), \((\vec{x}_R, \vec{x}_I)_C\) is a 2D Cartesian coordinate-vector state in the 2D Cartesian coordinate-vector state space \( X_C \) so \((\vec{x}_R, \vec{x}_I)_C \in X_C\), while \((|\vec{x}|, \langle \vec{x} \rangle)_P\) is a 2D polar coordinate-vector state in the 2D polar coordinate-vector state space \( X_P \) so \((|\vec{x}|, \langle \vec{x} \rangle)_P \in X_P\), where \( X_C \) and \( X_P \) are iso-morphic, dual, synchronized, and interlocking in \( X \) [8]. Thus, eq. (1) complies with the constraints imposed by eqs. (8–13) of [8]—see Figure 1.

2. Second, from eq. (16) of [8] we have

\[
T^1 = \{ \vec{x} \in X : |\vec{x}| = r \},
\]

where \( T^1 \subset X \) is the 1-sphere IHR of amplitude-radius \( r > 0 \) (with corresponding curvature \( \kappa = \frac{1}{r} \)) that is centered on the origin \( O \in X \); \( T^1 \) is the multiplicative group of all non-zero complex numbers with amplitude-radius \( r \), which is iso-metrically embedded in \( X \) and is simultaneously dual to the two complex sub-spaces \( X_- \) and \( X_+ \) [8, 6, 7]—see Figure 2.

3. Third, from eq. (18) of [8], let \( Y = \mathbb{T} \) be the set of all triplex numbers, the Euclidean triplex space, and the dual 3D Cartesian-spherical coordinate-vector state space, where the triplex number \( \vec{y} \in Y \) is a dual 3D Cartesian-spherical coordinate-vector state that is defined by eq. (17) of [8] as

\[
y = \vec{y} = \vec{y}_R + \vec{y}_I + \vec{y}_Z = (\vec{y}) = (|\vec{y}|, \langle \vec{y} \rangle, [\vec{y}])_S = (\vec{y}_R, \vec{y}_I, \vec{y}_Z)_C, \quad \forall \vec{y} \in Y.
\]
Fig. 1: Complex components for the dual 2D Cartesian-polar coordinate-vector state $\vec{x}$ in the dual 2D Cartesian-polar coordinate-vector state space (and Euclidean complex space) $X$, such that $\vec{x} \in X$, where $\vec{x}$ is simultaneously treated as a complex number, 2D polar coordinate-vector, and 2D Cartesian coordinate-vector [8]. Specifically, $(\vec{x}_R, \vec{x}_I)_C$ is a 2D Cartesian coordinate-vector state in the 2D Cartesian coordinate-vector state space $X_C$ so $(\vec{x}_R, \vec{x}_I)_C \in X_C$, while $(|\vec{x}|, \langle \vec{x} \rangle)_P$ is a 2D polar coordinate-vector state in the 2D polar coordinate-vector state space $X_P$ so $(|\vec{x}|, \langle \vec{x} \rangle)_P \in X_P$, where $X_C$ and $X_P$ are iso-morphic, dual, synchronized, and interlocking in $X$ [8]. Note that $\vec{x}_R$ and $\vec{x}_I$ are treated as vectors (with axis-dependent magnitude and direction) so the vector sum is $\vec{x} = \vec{x}_R + \vec{x}_I$ with amplitude $|\vec{x}|$ and direction $\langle \vec{x} \rangle$ [8].
Fig. 2: The dual 3D space-time 1-sphere IHR topology for the dual 2D Cartesian-polar coordinate-vector state space (and Euclidean complex space) $X$, where the topological 1-sphere IHR $T^1 \subset X$ is simultaneously dual to two spatial 2-branes [8, 6]: the “2D micro sub-space zone” $X_- \subset X$ and the “2D macro sub-space zone” $X_+ \subset X$ for interior and exterior dynamical systems, respectively [8].
In eq. (3), \((\vec{y}_R, \vec{y}_I, \vec{y}_Z)_C\) is a 3D Cartesian coordinate-vector state in the 3D Cartesian coordinate-vector state space \(Y_C\) so \((\vec{y}_R, \vec{y}_I, \vec{y}_Z)_C \in Y_C\), while \((|\vec{y}|, \langle \vec{y} \rangle, [\vec{y}])_S\) is a 3D spherical coordinate-vector state in the 3D spherical coordinate-vector state space \(Y_S\) so \((|\vec{y}|, \langle \vec{y} \rangle, [\vec{y}])_S \in Y_S\), where \(Y_C\) and \(Y_S\) are iso-morphic, dual, synchronized, and interlocking in \(Y\) \([8]\). Thus, eq. (3) complies with the constraints imposed by eqs. (19–28) of \([8]\)—see Figures 3 and 4.

4. Fourth, from eq. (33) of \([8]\) we have

\[
T^2 = \{\vec{y} \in Y : |\vec{y}| = r\},
\]

where \(T^2 \subset Y\) is the 2-sphere IHR of amplitude-radius \(r > 0\) (the same as \(T^1\)) that is centered on the origin \(O \in X, Y\); \(T^2\) is the multiplicative group of all non-zero triplex numbers with amplitude-radius \(r\), which is iso-metrically embedded in \(Y\) and is simultaneously dual to the two triplex sub-spaces \(Y_-\) and \(Y_+\) \([8, 6, 7]\). Here, given \(X \subset Y\), \(T^1 \subset T^2\), \(X, Y\) is the great circle of \(T^2\), such that \(T^1 = X \cap T^2\) \([8, 6, 7]\)—see Figure 5.

At this point, we’ve initialized the dual 4D space-time IHR topology \([8, 6, 7]\). Therefore, we are ready to explore the proposed the effective iso-radius encoding platform of Section 2.2.

2.2 Constructing the effective iso-radius for the iso-dual 4D space-time IHR topology

Here, we assemble the effective iso-radius encoding platform for representing iso-sphere IHR \([12]\) iso-states and iso-transitions in the iso-dual 4D space-time topology \([8]\) via the following procedure:

1. First, in conventional mathematics, the number 1 is the multiplicative identity that satisfies the original number field axioms \([18]\). Thus, the number 1 plays important and diverse roles throughout mathematics in general such as, for example, normalization in statistics. Therefore, we start by setting the amplitude-radius \(r = 1\) for \(T^1\) and \(T^2\).

2. Second, in iso-mathematics \([1, 2, 3, 4, 5, 8]\), Santilli demonstrated that the multiplicative unit is not limited to the number 1 and can
Fig. 3: Triplex components for the dual 3D Cartesian-spherical coordinate-vector state \( \vec{y} \) in the dual 3D Cartesian-polar coordinate-vector state space (and Euclidean triplex space) \( Y \), such that \( \vec{y} \in Y \), where \( \vec{y} \) is simultaneously treated as a triplex number, 3D spherical coordinate-vector, and 3D Cartesian coordinate-vector [8]. Specifically, \((\vec{y}_R, \vec{y}_I, \vec{y}_Z)_C\) is a 3D Cartesian coordinate-vector state in the 3D Cartesian coordinate-vector state space \( Y_C \) so \((\vec{y}_R, \vec{y}_I, \vec{y}_Z)_C \in Y_C\), while \((|\vec{y}|, \langle \vec{y} \rangle, [\vec{y}])_S\) is a 3D spherical coordinate-vector state in the 3D spherical coordinate-vector state space \( Y_S \) so \((|\vec{y}|, \langle \vec{y} \rangle, [\vec{y}])_S \in Y_S\), where \( Y_C \) and \( Y_S \) are iso-morphic, dual, synchronized, and inter-locking in \( Y \) [8]. Note that \( vec\vec{y}_R, \vec{y}_I, \) and \( \vec{y}_Z \) are treated as vectors (with axis-dependent magnitude and direction) so the vector sum is \( \vec{y} = \vec{y}_R + \vec{y}_I + \vec{y}_Z \) with amplitude \(|\vec{y}|\) and two directions \( \langle \vec{y} \rangle \) and \([\vec{y}] \) [8].
Fig. 4: Aligned perspectives of $\vec{y} \in Y$ from the $\mathbb{RI}$-plane (top) and the $\mathbb{RZ}$-plane (bottom) [8].
Fig. 5: The dual 4D space-time 2-sphere IHR topology in the dual 3D Cartesian-spherical coordinate-vector state space (and Euclidean triplex space) $Y$, where the topological 2-sphere IHR $T^2 \subset Y$ is simultaneously dual to two spatial 3-branes [8, 6]: the “3D micro sub-space zone” $Y^- \subset Y$ and the “3D macro sub-space zone” $Y^+ \subset Y$ for interior and exterior dynamical systems, respectively [8]. Here, $T^2$ is depicted as M. C. Escher’s famous reflecting sphere [17].
therefore be replaced with the positive-definite iso-multiplicative iso-unit \( \hat{r} > 0 \) with corresponding inverse \( \hat{\kappa} = \frac{1}{\hat{r}} > 0 \) for iso-numbers. Hence, for some \( \hat{r} \), we employ Santilli’s iso-methodology \([1, 2, 3, 4, 5]\) to iso-topically lift \( T^2 \) via

\[
\vec{y}_{\hat{r}} \equiv \vec{y} \times \hat{r}, \quad \forall \vec{y} \in T^2 \rightarrow \forall \vec{y}_{\hat{r}} \in T^2_{\hat{r}},
\]

(5)

for the transition and its inverse

\[
f(\hat{r}, T^2) : T^2 \rightarrow T^2_{\hat{r}}
\]

\[
f^{-1}(\hat{r}, T^2_{\hat{r}}) : T^2_{\hat{r}} \rightarrow T^2
\]

(6)

to identify the iso-2-sphere IHR of iso-radius (or “iso-amplitude-radius”) \( \hat{r} \) from \([8]\), so \( T^2 \) and \( T^2_{\hat{r}} \) are locally iso-morphic and are both centered on the origin \( O \in X, Y \). Here, note that in addition to being the iso-radius of \( T^1 \) and \( T^2 \), \( \hat{r} \) also serves as the iso-unit for Santilli’s iso-multiplication \([1, 2, 3, 4, 5, 8]\), where the iso-unit inverse \( \hat{\kappa} \) is also the iso-curvature of \( T^1 \) and \( T^2 \).

3. Third, given the dynamic iso-topic lifting and dynamic iso-spheres of \([11, 12]\), we furthermore define \( T^2 \)'s iso-radius as an iso-function in the positive-definite form

\[
T^2_{\hat{r}(m)} : \hat{r} \equiv \hat{r}(m) \equiv ma + b \equiv \frac{1}{\hat{\kappa}(m)} > 0,
\]

(7)

where \( \hat{r}(m) \) is the dynamic iso-radius iso-function (or “dynamic iso-unit iso-function”) with the parameter \( m \) and \( \hat{\kappa}(m) \) is the corresponding dynamic iso-curvature iso-function, such that \( m \) is some mathematical quantity while \( a \) and \( b \) are coefficients. Thus, eq. (5) is rewritten as

\[
\vec{y}_{\hat{r}(m)} \equiv \vec{y} \times \hat{r}(m), \quad \forall \vec{y} \in T^2 \rightarrow \forall \vec{y}_{\hat{r}(m)} \in T^2_{\hat{r}(m)}
\]

(8)

so eq. (6) becomes

\[
f(\hat{r}(m), T^2) : T^2 \rightarrow T^2_{\hat{r}(m)}
\]

\[
f^{-1}(\hat{r}(m), T^2_{\hat{r}(m)}) : T^2_{\hat{r}(m)} \rightarrow T^2
\]

(9)
4. Fourth, given that eq. (7) is a *dynamic* iso-unit iso-function, we wish to show that \( \hat{r}(m) \) is characterized by constant change as \( m \) varies and takes on values from some positive-definite sequence \( M \), such that \( m \in M \) as \( m \to \infty \). In [11, 12], there are *two* distinct types of dynamic iso-unit iso-functions:

- **continuous dynamic iso-unit iso-functions**, so \( M \) may be a *continuous* sequence of positive-definite values such as, for example, the case of \( M \equiv \mathbb{R}_+ \) for the positive-definite interval of *real numbers*

  \[
  M_{\mathbb{R}+} = (0, \infty_{\mathbb{R}+}), \ m \in M_{\mathbb{R}+}, \ m \to \infty_{\mathbb{R}+} ; \tag{10}
  \]

  and

- **discrete dynamic iso-unit iso-functions**, so \( M \) may be a *discrete* sequence of positive-definite values such as, for example, the case of \( M \equiv \mathbb{N} \) for the positive-definite set of *natural numbers*

  \[
  M_{\mathbb{N}} = \{1, 2, 3, 4, 5, \ldots\}, \ m \in M_{\mathbb{N}}, \ m \to \infty_{\mathbb{N}} \tag{11}
  \]

  or in the case of \( M \equiv M_{\text{Fib}} \) for the positive-definite set of *Fibonacci numbers*

  \[
  M_{\text{Fib}} = \{1, 1, 2, 3, 5, \ldots\}, \ m \in M_{\text{Fib}}, \ m \to \infty_{\text{Fib}}. \tag{12}
  \]

5. Fifth, for this introductory investigation, consider a relatively simple case and suppose that \( a = 2 \) and \( b = 0 \), where we know that the \( \check{r}(m) > 0 \) and \( \check{\kappa}(m) > 0 \) of eq. (7) will remain positive-definite as \( m > 0 \) varies, regardless of whether the positive-definite \( M \) is continuous or discrete. Thus, eq. (7) is rewritten as

\[
T_{\check{r}(m)} : \ \hat{r} \equiv \check{r}(m) \equiv m2 + 0 \equiv 2m \equiv \frac{1}{\check{\kappa}(m)} > 0. \tag{13}
\]

In the procedure of this initial thought experiment, we will operate eq. (13) with \( \check{r}(m) = 2m \), but note that eq. (13) could be rewritten again to relate \( \hat{r} \) to additional *mathematical* quantities as long as it complies with Santilli’s positive-definite iso-unit constraint \( \check{r}(m) > 0 \) [1, 2, 3, 4, 5, 8] for the iso-topic liftings of eqs. (8–9).
6. Sixth, given the fundamental exterior and interior iso-duality establishment [12], we briefly note that

\[ T^2_{\hat{r}(m)} \equiv T^2_{\hat{r}+}(m) \]

\[ T^2_{\hat{\kappa}(m)} \equiv T^2_{\hat{r}-(m)} \]

(14)

because in this context the \( T^2_{\hat{r}(m)} \equiv T^2_{\hat{r}+}(m) \) of “outer” iso-radius \( \hat{r} \equiv \hat{r}+ \) is the exterior 2-sphere IHR that is “outside” of \( T^2 \) because \( T^2_{\hat{r}+}(m) \subset Y_+ \), while the \( T^2_{\hat{\kappa}(m)} \equiv T^2_{\hat{r}-}(m) \) of “inner” iso-radius \( \hat{\kappa} \equiv \hat{r}- \) is the interior 2-sphere IHR that is “inside” of \( T^2 \) because \( T^2_{\hat{r}-}(m) \subset Y_- \) [12]. \( T^2_{\hat{r}+}(m) \) and \( T^2_{\hat{r}-(m)} \), or equivalently \( T^2_{\hat{\kappa}(m)} \) and \( T^2_{\hat{r}(m)} \), are iso-dual [12] due to the fact that

\[ \hat{r}+ \equiv \hat{r}(m) \equiv \frac{1}{\hat{\kappa}(m)} \equiv \frac{1}{\hat{r}-} \]  

(15)

7. Seventh, given eq. (13), we define the initial iso-radius iso-state of \( T^2_{\hat{r}(m)} \) as

\[ T^2_{\hat{r}(m_0)} : \hat{r}_0 \equiv \hat{r}(m_0) \equiv 2m_0 \equiv \frac{1}{\hat{\kappa}(m_0)} > 0, \]

(16)

to identify the initial iso-2-sphere IHR iso-state \( T^2_{\hat{r}(m_0)} \), where \( \hat{r}(m_0) > 0 \) is the initial iso-radius, \( \hat{\kappa}(m_0) > 0 \) is the initial iso-curvature, and \( m_0 > 0 \) is the initial quantity, such that \( m_0 \in M \), regardless of whether the positive-definite \( M \) is continuous or discrete. Therefore, for this initial case we assign \( m = m_0 \) for eq. (8) to establish

\[ \vec{y}_{\hat{r}(m_0)} \equiv \vec{y} \times \hat{r}(m_0), \forall \vec{y} \in T^2 \rightarrow \forall \vec{y}_{\hat{r}(m_0)} \in T^2_{\hat{r}(m_0)} \]

(17)

so eq. (9) becomes

\[ f(\hat{r}(m_0), T^2) : T^2 \rightarrow T^2_{\hat{r}(m_0)} \]

\[ f^{-1}(\hat{r}(m_0), T^2_{\hat{r}(m_0)}) : T^2_{\hat{r}(m_0)} \rightarrow T^2. \]

(18)
8. Eighth, suppose that the quantity $m_0$ undergoes a change that is characterized by
\[ \delta_m : m_0 \to m_1, \] (19)
which causes
\[ \delta_{\hat{r}(m)} : \hat{r}(m_0) \to \hat{r}(m_1), \] (20)
such that
\[ m_0 = m_1 + \Delta_m. \] (21)
Thus, a second version of eq. (16) is written to define the final iso-radius iso-state of $T^2_\hat{r}(m)$ as
\[ T^2_\hat{r}(m_1) : \hat{r}_1 \equiv \hat{r}(m_1) \equiv 2m_1 \equiv \frac{1}{\hat{\kappa}(m_1)} > 0, \] (22)
to identify the final iso-2-sphere IHR iso-state $T^2_\hat{r}(m_1)$, where $\hat{r}(m_1) > 0$ is the final iso-radius, $\hat{\kappa}(m_1) > 0$ is the final iso-curvature, and $m_1 > 0$ is the final quantity, such that $m_1 \in M$, regardless of whether the positive-definite $M$ is continuous or discrete. Therefore, for this final case we assign $m = m_1$ for eq. (8) to establish
\[ \bar{y}_{\hat{r}(m_1)} \equiv \bar{y} \times \hat{r}(m_1), \forall \bar{y} \in T^2 \to \forall \bar{y}_{\hat{r}(m_1)} \in T^2_\hat{r}(m_1) \] (23)
so eq. (18) becomes
\[ f(\hat{r}(m_1), T^2) : T^2 \to T^2_\hat{r}(m_1) \]
\[ f^{-1}(\hat{r}(m_1), T^2_\hat{r}(m_1)) : T^2_\hat{r}(m_1) \to T^2. \] (24)

9. Ninth, given the impact of eqs. (19–24), the initial iso-2-sphere IHR iso-state $T^2_\hat{r}(m_0)$ (of initial iso-radius $\hat{r}(m_0)$) is iso-topically lifted to the final iso-2-sphere IHR iso-state $T^2_\hat{r}(m_1)$ (of final iso-radius $\hat{r}(m_1)$) via
\[ \bar{y}_{\hat{r}(m_1)} \equiv \bar{y}_{\hat{r}(m_0)} \times \frac{\hat{r}(m_1)}{\hat{r}(m_0)}, \forall \bar{y}_{\hat{r}(m_0)} \in T^2_\hat{r}(m_0) \to \forall \bar{y}_{\hat{r}(m_1)} \in T^2_\hat{r}(m_1) \] (25)
for the iso-transition and its inverse

\[
 f(\hat{r}_{\hat{m}_1}, T^2_{\hat{r}(m_0)}) : T^2_{\hat{r}(m_0)} \rightarrow T^2_{\hat{r}(m_1)}
\]

\[
 f^{-1}(\hat{r}_{\hat{m}_1}, T^2_{\hat{r}(m_1)}) : T^2_{\hat{r}(m_1)} \rightarrow T^2_{\hat{r}(m_0)}
\]

with the iso-radius ratio \( \hat{r}_{\hat{m}_1} / \hat{r}_{\hat{m}_0} \) and the corresponding iso-curvature ratio \( \hat{c}_{\hat{m}_0} / \hat{c}_{\hat{m}_1} \) characterize the iso-transition to establish that \( T^2_{\hat{r}(m_0)} \), \( T^2_{\hat{r}(m_1)} \), and \( T^2_{\hat{r}(m_1)} \) are indeed locally iso-morphic.

10. Tenth, we note that the iso-transition between \( T^2_{\hat{r}(m_0)} \) and \( T^2_{\hat{r}(m_1)} \) depends on \( \Delta_m \) and complies with the trichotomy:

- **Case** \( \Delta_m < 0 \): \( T^2_{\hat{r}(m_0)} \) is de-magnified to become \( T^2_{\hat{r}(m_1)} \) via the iso-topic lifting \( T^2_{\hat{r}(m_0)} \rightarrow T^2_{\hat{r}(m_1)} \) because \( m_1 < m_0 \) so \( \hat{r}(m_1) < \hat{r}(m_0) \).

- **Case** \( \Delta_m = 0 \): \( T^2_{\hat{r}(m_0)} \) is equivalent to \( T^2_{\hat{r}(m_1)} \) via the iso-topic lifting \( T^2_{\hat{r}(m_0)} \rightarrow T^2_{\hat{r}(m_1)} \) because \( m_1 = m_0 \) so \( \hat{r}(m_1) = \hat{r}(m_0) \).

- **Case** \( \Delta_m > 0 \): \( T^2_{\hat{r}(m_0)} \) is magnified to become \( T^2_{\hat{r}(m_1)} \) via the iso-topic lifting \( T^2_{\hat{r}(m_0)} \rightarrow T^2_{\hat{r}(m_1)} \) because \( m_1 > m_0 \) so \( \hat{r}(m_1) > \hat{r}(m_0) \).

11. Finally, given the new and developing framework of [13, 14, 15, 16] that characterizes the effective physical state of black holes for an emission or absorption transition, we are motivated to define the effective iso-mathematical state of dynamic iso-2-sphere IHRs (which are also spherically-symmetric objects) for a transition from \( T^2_{\hat{r}(m_0)} \) to \( T^2_{\hat{r}(m_1)} \). Therefore, given the physical black hole effective radius definition from eq. (5) of [16], we implement the dynamic iso-topic lifting of [11, 12] and define the iso-mathematical effective iso-2-sphere IHR iso-radius as

\[
 T^2_{\hat{r}(m_0)} \rightarrow T^2_{\hat{r}(m_1)} : \hat{r}_E \equiv \hat{r}_E(m_0, m_1) \equiv 2m_E(m_0, m_1) \equiv \frac{1}{\kappa_E(m_0, m_1)} > 0,
\]
where $\hat{\kappa}_E(m_0, m_1)$ is the effective iso-2-sphere IHR iso-curvature and inverse of the iso-unit, such that the effective iso-2-sphere IHR quantity is defined as

$$m_E(m_0, m_1) \equiv \frac{m_0 + m_1}{2},$$

which is simply the average of $T_{\hat{r}(m_0)}^2$'s initial quantity $m_0$ and $T_{\hat{r}(m_1)}^2$'s final quantity $m_1$.

At this point, we’ve assembled the effective iso-radius encoding platform for representing dynamic iso-sphere IHR [12] iso-states and iso-transitions in the iso-dual 4D space-time topology [8].

3 Conclusion

In this work, we successfully assembled the effective iso-radius for a dynamic iso-sphere IHR [12] in the iso-dual 4D space-time IHR topology [8] and introduced the corresponding notion of effective iso-state to begin encoding the iso-transition between two distinct iso-states. For this, the procedure and step-by-step developing results were presented in Section 2, and applies to both continuous and discrete dynamic iso-sphere IHRs. Also, we demonstrated that all of these outcomes comply with the exterior and interior IHR iso-duality [12]. To recapitulate the final results more precisely, we defined—for the dynamic iso-sphere IHR $T_{\hat{r}(m)}^2$—the effective iso-radius $\hat{r}_E(m_0, m_1)$ as the average of the initial iso-radius $\hat{r}(m_0)$ and the final iso-radius $\hat{r}(m_1)$ in eqs. (27–28), which correspond to the initial dynamic iso-sphere IHR $T_{\hat{r}(m_0)}^2$ and the final dynamic iso-sphere IHR $T_{\hat{r}(m_1)}^2$, respectively.

The results, constructions, and implications of this preliminary investigation are significant because they exemplify alternative modes of cutting-edge iso-mathematics research that facilitate a heightened quantifiable characterization of dynamic iso-sphere IHRs [12] in terms of effective iso-states for iso-transitions with iso-duality. Hence, given that iso-sphere IHRs are equipped with topological deformation order parameters [6, 7, 8], the next logical step of this analysis should be to implement iso-topic liftings [1, 2, 3, 4, 5] for the order parameters and then topologically incorporate these “iso-deformations” into the existing effective iso-state definition. From there, we may build on this platform and continue to develop the framework by
exploring and assessing the frontiers of iso-, geno-, and hyper- mathematics [1, 2, 3, 4, 5]. Thus, this developing class of dynamic iso-sphere IHRs warrants further development, scrutiny, collaboration, and hard work in order to advance it for future application in the discipline of science.

References
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