Universal Gravitational Constant Via Proton

**Abstract**: Using a formula including the proton mass and Compton’s wavelength, it is obtained the value of the universal gravitational constant by two orders of magnitude more accurate than the recommended CODATA value [1].

**Introduction**

The dimension of the universal gravitational constant $G$ is $M^{-1}L^3T^{-2}$. If it is expressed in natural units [2], it has value by definition (in Planck units equals 1). The exact value of the constant is also possible in any other system in which $G$, or the values from which it could be directly derived, would by definition have exact values. That is not possible in the International System of Units [3] because in that system only the speed of light with dimensions $L^2T^{-2}$ has exact value and can be used for determining $G$. For example, if in that system Planck mass and length would have the value by definition, then by using formula: $G=c^2l_{pl}/m_{pl}$ ($c$ – speed of light, $l_{pl}$ – Planck length, $m_{pl}$ – Planck mass), $G$ would also have exact value. The same result could be obtained by applying some other combinations of the exactly defined values.

There is a large number of formulas which feature $G$, and still its value is known for its low accuracy in the SI. The reason for that is that the values which are included in the calculation of $G$ are difficult to determine experimentally or cannot be determined at all. It is more common for those values to even be determined via the known $G$. Hence, in the following formulas at least one of the Planck values is always present:

$$G = c^2l_{pl}/m_{pl}$$

$$G = l_{pl}^3/m_{pl}t_{pl}^2$$

$$G = hc/\pi'm_{pl}^2$$

Taken from [1]:
- Planck length: $1.616\ 199\ e^{-35}$ m
- Planck mass: $2.176\ 51\ e^{-8}$ kg
- Planck time: $5.391\ 06\ e^{-44}$ s
- Planck constant: $6.626\ 069\ 57\ e^{-34}$ Js

Therefore we have:

Newtonian constant of gravitation: $6.673\ 84\ e^{-11}$ m$^3$ kg$^{-1}$ s$^{-2}$

in the similar range of accuracy. On the right is the value of uncertainty expressed by $1\sigma$, standard deviations. In the text below, the uncertainty will be shown in brackets, after the value of the physical quantity. Therefore, for the accurate determination of $G$ it is necessary to express this constant via the physical constants whose values can be determined experimentally with great accuracy.

**Formula for $G$**

Starting from the statement "**Parts are dependent on the whole (Universe) and are also an integral part of the whole; therefore, the whole is also dependent on the parts!**" I developed a methodology which produced results in the articles published at [4]. Especially the article [5] shows the accuracy of determining the mass of tau particles by using the original formula.

Let's define the mathematical constants:
t=log(2π,2)=2.651496..., Cycle, cy=e^{2π}=535.49165..., Half cycle, z=e^{2π/2}=267.74582776...

The masses of the universe and proton are as follows:

\[ M_u=1.73944912\times10^{53} \text{ kg} \] [6], \[ m_p=1.672621777\times10^{-27} \text{ kg} \] [1]

From [7], \( p \) – the constant related to the proton is:

\[ p = \log(m_u / m_p, 2) \] (1)

And also:

\[ z = e^{2\pi} / 2 = \log(m_u / m_z, 2) \] (2)

Then we can define, let call it the proton shift, \( z_p \):

\[ z_p = z - p = \log(m_p / m_z, 2) = 1.9350609435 \] (3)

We will also use physical constants \( \mu \) – proton-to-electron mass ratio and \( \alpha \) – inverse fine-structure constant from [1]. They can also be used to determine the proton shift:

\[ z_p = (\mu/\alpha' + 1)/(\mu/\alpha' + 2) + 1 = 1.9350609435 \] (4)

Or:

\[ z_p = 2 - \frac{1}{\mu/\alpha' + 2} = 1.9350609435 \] (5)

Also, from (3) and (5):

\[ p = e^{2\pi / 2} - z_p = z - z_p = 265.8107668 \] (6)

If \( m_p \) is the proton mass and \( \lambda_p \) stands for the proton Compton wavelength, we obtain the following formula:

\[ G = c^2 m_p^{-1} * \lambda_p * 2^{(-cy/4 + 3zp/2 + t/2)} \] (7)

Or:

\[ G = c^2 m_p^{-1} * \lambda_p * 2^{(z-3p/2 + t/2)} \] (8)

Or:

\[ G = c^2 m_p^{-1} * \lambda_p * \sqrt{2\pi} * 2^{(cy-3p)} \] (9)
All the physical quantities in (8) are related to the proton and are accurately determined experimentally.

**Testing the formula for G**

Here we will test the formula (8) by using the historical CODATA values. The CODATA values for \( \hat{\alpha}, \mu, \lambda_p, m_p \) are shown in Table 1, columns 1, 2, 4 and 5. There, for example, we can see that each of the four physical constants in 2010 [1] have at least two significant digits more than \( G \), while the value of the speed of light \( c \) is exact by definition.

The seventh column of Table 1 shows the value of \( G \) determined by the formula (8), so that once the upper value \( G' \) is determined based on the CODATA values \( (\hat{\alpha}, \mu, \lambda_p, m_p) \) for the corresponding year, and once the lower value \( G \). The upper and lower values determine the uncertainty \(+/-1\sigma\), shown in brackets. Value \((G'-G)/2\) is adopted to represent \(1\sigma\).

**Table 1**

<table>
<thead>
<tr>
<th>Year</th>
<th>CODATA</th>
<th>(\hat{\alpha} = 1/\alpha)</th>
<th>(\mu = m_p/c_e)</th>
<th>(c)</th>
<th>Compton (\lambda_p)</th>
<th>(m_p)</th>
<th>(G)</th>
<th>(G')</th>
</tr>
</thead>
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<tr>
<td>1969</td>
<td>137.03602(21)</td>
<td>1836.1090(110)</td>
<td>299792500</td>
<td>1.3214409(90)</td>
<td>1.672614(11)</td>
<td>6.6732(31)</td>
<td>6.67402(92)</td>
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<td>1973</td>
<td>137.036040(110)</td>
<td>1836.15152(70)</td>
<td>299792458</td>
<td>1.3214099(22)</td>
<td>1.6726485(86)</td>
<td>6.6720(41)</td>
<td>6.67382(46)</td>
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<tr>
<td>1986</td>
<td>137.0359895(61)</td>
<td>1836.152701(37)</td>
<td>299792458</td>
<td>1.32141002(12)</td>
<td>1.6726231(10)</td>
<td>6.67259(85)</td>
<td>6.673836(16)</td>
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<td>6.6742(10)</td>
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<td>2006</td>
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<td>6.67384(80)</td>
<td>6.673836(30)</td>
<td></td>
</tr>
</tbody>
</table>

Table 1 shows that the value of \( G \) determined by the formula in year 1973 achieved the accuracy from year 2010 in [1]. The value of \( G \) determined by the formula for year 2010 has two significant digits more than the CODATA value.

**Figure 1** Universal gravitational constant – \( G \) in the 1969–2010 period
CODATA values [1] and values achieved by formula (8)
Figure 1 visually presents the advantage of determining the value of \( G \) by applying the formula in relation to the CODATA method.

**Conclusion**

The article shows the predictive power of the formula (8) for determining the value of the universal gravitational constant \( G \) by applying physical constants whose experimental determination gives the values much more accurate than the experimentally obtained \( G \).

In the formula (9), the values are:

\[
R_u = \lambda_p \sqrt{2\pi} \times 2^{(\epsilon y - p)} = 1.2916530 \times 10^2 \text{ m}
\]  
(10)

\[
M_u = m_p \times 2^p = 1.73944912 \times 10^2 \text{ kg}
\]  
(11)

\( R_u \) is radius of universe and \( M_u \) is mass of universe.

Then, from (9), (10) and (11):

\[
G = c^2 M_u^{-1} * R_u = M_u^{-1} * R_u^3 * T_u^{-2}
\]  
(12)

which is the basic and simple formula presenting the essence of the universal gravitational constant. There is also a possibility to determine \( G \) even more accurately through other constants or even exactly by redefining the International System of Units.

**Novi Sad, October 2013**

**References:**

4. [viXra.org open e-Print archive](http://viXra.org)