Universal Gravitational Constant Via Proton

Abstract: Using a formula including the proton mass and Compton’s wavelength for the proton, I obtained the value of the universal gravitational constant by two orders of magnitude more accurate than the recommended CODATA value [1].

Introduction

The dimension of the universal gravitational constant $G$ is $M^{-1}L^3T^{-2}$. If it is expressed in natural units [2], it has value by definition (in Planck units equals 1). The exact value of the constant is also possible in any other system in which $G$, or the values from which it could be directly derived, would by definition have exact values. That is not possible in the International System of Units [3] because in that system only the speed of light with dimensions $L^2T^{-2}$ has exact value and can be used for determining $G$. For example, if in that system Planck mass and length would have the value by definition, then by using formula: $G=c^2l_{pl}/m_{pl}$ (c – speed of light, $l_{pl}$ – Planck length, $m_{pl}$ – Planck mass), $G$ would also have exact value. The same result could be obtained by applying some other combinations of the exactly defined values.

There is a large number of formulas which feature $G$, and still its value is known for its low accuracy in the SI. The reason for that is that the values which are included in the calculation of $G$ are difficult to determine experimentally or cannot be determined at all. It is more common for those values to even be determined via the known $G$. Hence, in the following formulas at least one of the Planck values is always present:

\[ G = c^2l_{pl}/m_{pl} \]
\[ G = l_{pl}^3/m_{pl}t_{pl}^2 \]
\[ G = \hbar c/\pi'm_{pl}^2 \]

Taken from [1]:

- Planck length: $1.616\,199\,e^{-35}$ m, $0.000\,097\,e^{-35}$ m
- Planck mass: $2.176\,51\,e^{-8}$ kg, $0.000\,13\,e^{-8}$ kg
- Planck time: $5.391\,06\,e^{-44}$ s, $0.000\,32\,e^{-44}$ s
- Planck constant: $6.626\,069\,57\,e^{-34}$ Js, $0.000\,000\,29\,e^{-34}$ Js

Therefore we have:

- Newtonian constant of gravitation: $6.673\,84\,e^{-11}$ m$^3$ kg$^{-1}$ s$^{-2}$

in the similar range of accuracy. On the right is the value of uncertainty expressed by $1\sigma$, standard deviations. In the text below, the uncertainty will be shown in brackets, after the value of the physical quantity. Therefore, for the accurate determination of $G$ it is necessary to express this constant via the physical constants whose values can be determined experimentally with great accuracy.

Formula for $G$

Starting from the statement "Parts are dependent on the whole (Universe) and are also an integral part of the whole; therefore, the whole is also dependent on the parts!" I developed a methodology which produced results in the articles published on [4]. Especially the article [5] shows the accuracy of determining the mass of tau particles by using the original formula.

Let's define the mathematical constants:
The masses of the universe and proton are as follows:

$$M_u=1.73944912E+53 \text{ kg} \ [6], \ m_p=1.672621777E-27 \text{ kg} \ [1]$$

From [7], $p$ – the constant related to the proton is:

$$p = \log\left(\frac{m_u}{m_p}, 2\right) \quad (1)$$

And also:

$$z = e^{2\pi/2} = \log\left(\frac{m_u}{m_z}, 2\right) \quad (2)$$

Then we can define, let call it the proton shift, $z_p$:

$$z_p = z - p = \log\left(\frac{m_p}{m_z}, 2\right) = 1.9350609435 \quad (3)$$

We will also use physical constants $\mu$ – proton-to-electron mass ratio and $\alpha$ – inverse fine-structure constant from [1]. They can also be used to determine the proton shift:

$$z_p = (\mu/\alpha'+1)/(\mu/\alpha'+2) + 1 = 1.9350609435 \quad (4)$$

Or:

$$z_p = 2 - \frac{1}{\mu/\alpha'+2} = 1.9350609435 \quad (5)$$

Also, from (3) and (5):

$$p = e^{2\pi} - z_p = 265.8107668 \quad (6)$$

If $m_p$ is the proton mass and $\lambda_p$ stands for the proton Compton wavelength, we obtain the following formula:

$$G = c^2 m_p^{-1} \cdot \lambda_p^{-1} \cdot 2^{\left(-cy/4+3zp/2+t/2\right)} \quad (7)$$

Or:

$$G = c^2 m_p^{-1} \cdot \lambda_p^{-1} \cdot 2^{\left(-zp/2+t/2\right)} \quad (8)$$

Or:

$$G = c^2 m_p^{-1} \cdot \lambda_p^{-1} \cdot \sqrt{2\pi} \cdot 2^{(cy-3p)} \quad (9)$$
All the physical quantities in (8) are related to the proton and are accurately determined experimentally.

**Testing the formula for G**

Here we will test the formula (8) by using the historical CODATA values. The CODATA values for \( \dot{a}, \mu, \lambda_p, m_p \) are shown in Table 1, columns 1, 2, 4 and 5. There, for example, we can see that each of the four physical constants in 2010 [1] have at least two significant digits more than \( G \), while the value of the speed of light \( c \) is exact by definition.

The seventh column of Table 1 shows the value of \( G \) determined by the formula (8), so that once the upper value \( G' \) is determined based on the CODATA values (\( \dot{a}, \mu, \lambda_p, m_p \) – for the corresponding year), and once the lower value \( G \). The upper and lower values determine the uncertainty +/-1σ, shown in brackets. Value \((G' - G)/2\) is adopted to represent 1σ.

**Table 1**

<table>
<thead>
<tr>
<th>Year</th>
<th>CODATA Values [1]:</th>
<th>c</th>
<th>Compton ( \lambda_p )</th>
<th>( m_p )</th>
<th>( G )</th>
<th>Formula value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \dot{a}=1/\alpha )</td>
<td>( \mu=m_p/m_e )</td>
<td>( \times 10^{-15} ) m</td>
<td>( \times 10^{-27} ) kg</td>
<td>( \times 10^{-11} ) kg ( m^3 ) s (^{-2} )</td>
<td></td>
</tr>
<tr>
<td>1969</td>
<td>137.03602(21)</td>
<td>1836.1090(110)</td>
<td>299792500</td>
<td>1.3214409(90)</td>
<td>1.672614(11)</td>
<td>6.6732(31)</td>
</tr>
<tr>
<td>1973</td>
<td>137.036040(110)</td>
<td>1836.15152(70)</td>
<td>299792458</td>
<td>1.3214099(22)</td>
<td>1.6726485(86)</td>
<td>6.6720(41)</td>
</tr>
<tr>
<td>1986</td>
<td>137.0359895(61)</td>
<td>1836.152701(37)</td>
<td>299792458</td>
<td>1.32141002(12)</td>
<td>1.6726231(10)</td>
<td>6.67259(85)</td>
</tr>
<tr>
<td>1998</td>
<td>137.03599976(50)</td>
<td>1836.1526675(39)</td>
<td>299792458</td>
<td>1.321409847(10)</td>
<td>1.67262158(13)</td>
<td>6.673(10)</td>
</tr>
<tr>
<td>2002</td>
<td>137.03599911(46)</td>
<td>1836.15267261(85)</td>
<td>299792458</td>
<td>1.3214098555(88)</td>
<td>1.67262171(29)</td>
<td>6.6742(10)</td>
</tr>
<tr>
<td>2006</td>
<td>137.035999679(94)</td>
<td>1836.15267247(80)</td>
<td>299792458</td>
<td>1.3214098446(19)</td>
<td>1.672621637(83)</td>
<td>6.67428(67)</td>
</tr>
<tr>
<td>2010</td>
<td>137.035999074(45)</td>
<td>1836.152672457(75)</td>
<td>299792458</td>
<td>1.32140985623(94)</td>
<td>1.672621777(74)</td>
<td>6.67384(80)</td>
</tr>
</tbody>
</table>

Table 1 shows that the value of \( G \) determined by the formula in year 1973 achieved the accuracy from year 2010 in [1]. The value of \( G \) determined by the formula for year 2010 has two significant digits more than the CODATA value.

**Figure 1** Universal gravitational constant – \( G \) in the 1969–2010 period

CODATA values [1] and values achieved by formula (8)
Figure 1 visually presents the advantage of determining the value of G by applying the formula in relation to the CODATA method.

**Conclusion**

The article shows the predictive power of the formula (8) for determining the value of the universal gravitational constant G by applying physical constants whose experimental determination gives the values much more accurate than the experimentally obtained G.

In the formula (9), the values are:

\[
R_u = \lambda_p \sqrt{2\pi \cdot 2^{(c y - p)}} = 1.2916530E + 26 \text{ m} \tag{10}
\]

\[
M_u = m_p \cdot 2^p = 1.73944912E + 53 \text{ kg} \tag{11}
\]

\(R_u\) is radius of universe and \(M_u\) is mass of universe.

Then, from (9), (10) and (11):

\[
G = c^2 M_u^{-1} \cdot R_u = M_u^{-1} \cdot R_u^{3} \cdot T_u^{-2} \tag{12}
\]

which is the basic and simple formula presenting the essence of the universal gravitational constant. There is also a possibility to determine G even more accurately through other constants or even exactly by redefining the International System of Units.

Novi Sad, October 2013

**References:**

4. [viXra.org open e-Print archive](http://viXra.org)