The relation of colour charge to electric charge

Dirac has shown how the Klein-Gordon equation can be factored into two linear parts using $4 \times 4$ Dirac gamma matrices.


\[
(\partial^2 - \nabla^2 - \partial_z^2 - \partial^2_z + m^2) I = (-i[sy^0 \partial_0 + ry^1 \partial_x + gy^2 \partial_y + by^3 \partial_z] - m I) (i[sy^0 \partial_0 + ry^1 \partial_x + gy^2 \partial_y + by^3 \partial_z] - m I)
\]

where r,g,b and s equal +1 or -1.

For leptons r,g,b all equal -1 and for quarks two of r,g,b are equal to +1 and the third equals -1. The signs are all negated for anti-particles as in the equation above.

When $s = +1$, count the number of plus signs (say) for r,g,b which is 0 for leptons and 2 for quarks.

When $s = -1$, count the number of minus signs (say) for r,g,b which is 3 for leptons and 1 for quarks.

For material particles as in the equation above.

A charged particle moving in an electromagnetic field will have $\partial_0, \partial_1, \partial_2, \partial_3$ modified to $\gamma_0 \partial_0, \gamma_1 \partial_1, \gamma_2 \partial_2, \gamma_3 \partial_3$ by the scalar and vector potentials of the field, where $\gamma_0, \gamma_1, \gamma_2, \gamma_3, \gamma_4$ do not commute with each other. Thus:

\[
(-i[sy^0 \partial_0 + ry^1 \partial_x + gy^2 \partial_y + by^3 \partial_z] - m I) (i[sy^0 \partial_0 + ry^1 \partial_x + gy^2 \partial_y + by^3 \partial_z] - m I)
\]

\[
= (\gamma_0^2 - \gamma_i^2 - \gamma_j^2 - \gamma_k^2 + m^2) I
\]

\[
+ sy^0 [r y^1 (\gamma_0^\dagger \gamma_0 \partial_1 - \gamma_0 \gamma_0^\dagger \partial_1) + g y^2 (\gamma_0^\dagger \gamma_0 \partial_2 - \gamma_0 \gamma_0^\dagger \partial_2) + b y^3 (\gamma_0^\dagger \gamma_0 \partial_3 - \gamma_0 \gamma_0^\dagger \partial_3)] + g b y^2 y^3 (\gamma_0^\dagger \gamma_0 \partial_3 - \gamma_0 \gamma_0^\dagger \partial_3) + r g y^1 y^2 y^3 (\gamma_0^\dagger \gamma_0 \partial_2 - \gamma_0 \gamma_0^\dagger \partial_2) + r g y^1 y^2 y^3 (\gamma_0^\dagger \gamma_0 \partial_3 - \gamma_0 \gamma_0^\dagger \partial_3)
\]

\[
= (\gamma_0^2 - \gamma_i^2 - \gamma_j^2 - \gamma_k^2 + m^2) I
\]

\[
+ sy^0 [r y^1 (\gamma_0^\dagger \gamma_0 \partial_1 - \gamma_0 \gamma_0^\dagger \partial_1) + g y^2 (\gamma_0^\dagger \gamma_0 \partial_2 - \gamma_0 \gamma_0^\dagger \partial_2) + b y^3 (\gamma_0^\dagger \gamma_0 \partial_3 - \gamma_0 \gamma_0^\dagger \partial_3)] + g b y^2 y^3 (\gamma_0^\dagger \gamma_0 \partial_3 - \gamma_0 \gamma_0^\dagger \partial_3) + r g y^1 y^2 y^3 (\gamma_0^\dagger \gamma_0 \partial_2 - \gamma_0 \gamma_0^\dagger \partial_2) + r g y^1 y^2 y^3 (\gamma_0^\dagger \gamma_0 \partial_3 - \gamma_0 \gamma_0^\dagger \partial_3)
\]