The topology of number line.

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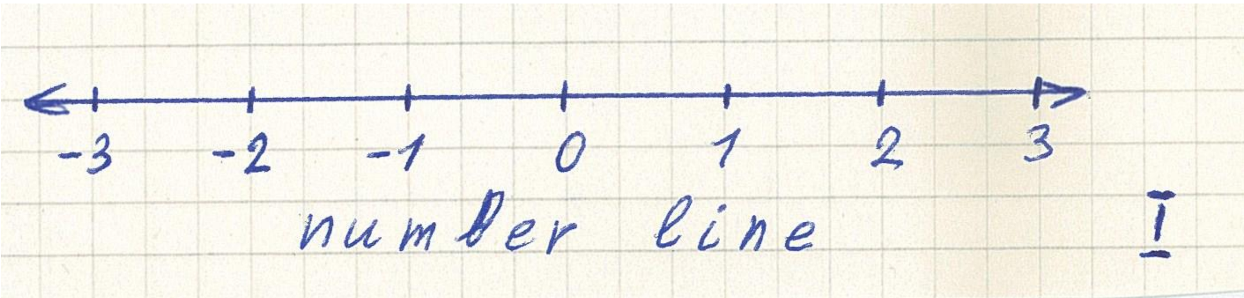
Abstract

This paper deals with the number line. Usually it is considered as one-dimensional object. But if to take into account the infinite large and the infinite little numbers, then this line turns out to be more complex object with infinite self-crossings in some many-dimensional space.

Content

1 Big and little numbers.

The number line for medium numbers looks like:



Let us consider the big numbers. The equation for the first big number is:

$$X + 1 = X \qquad (1)$$

Let us designate the solution of equation (1) as N1. N1 is very similar with the number of natural numbers.

And let us form the second big number – N2- so:

$$N2 = 2^{N1} \qquad (2)$$

$$N2 > N1 \qquad (3)$$

And N2 is very similar with the number of real numbers.

The third big number we build in the same manner:

$$N3 = 2^{N2} \qquad (4)$$

$$N3 > N2 \qquad (5)$$

The fourth big number:

$$N4 = 2^{N3} \qquad (6)$$

$$N4 > N3 \qquad (7)$$

And so on …

Let us consider the little numbers. The equation for the first little number is:

$$x + 1 = 1 \qquad (8)$$

Let us designate the solution of equation (8) as n1. From (1) we have:

$$N1 - N1 = 1 \qquad (9)$$

$$N1 * (1 - 1) = 1 \quad (10)$$

From (8) we have: $n1 = 1 - 1 \qquad (11)$

(10) + (11) = (12) $n1 = 1/N1 \qquad (12)$

Let us form the second little number n2 so: $n2 = 1/N2 \quad (13)$

(3) + (12) + (13) = (14) $n2 < n1 \qquad (14)$

And $n3 = 1/N3$ (15)

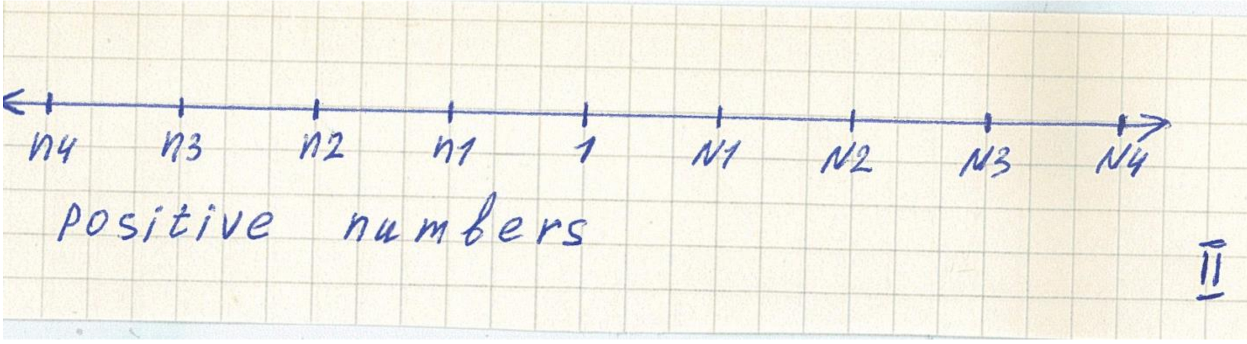
$n3 < n2$ (16)

$n4 = 1/N4$ (17)

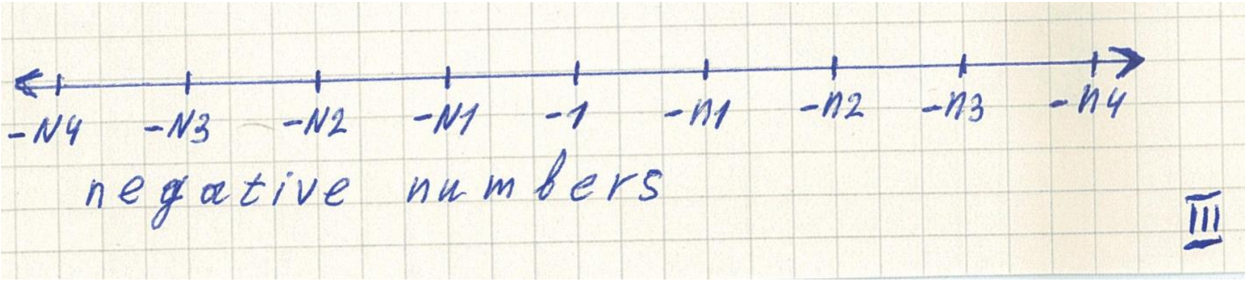
$n4 < n3$ (18)

And so on … .

So we have now two number lines:



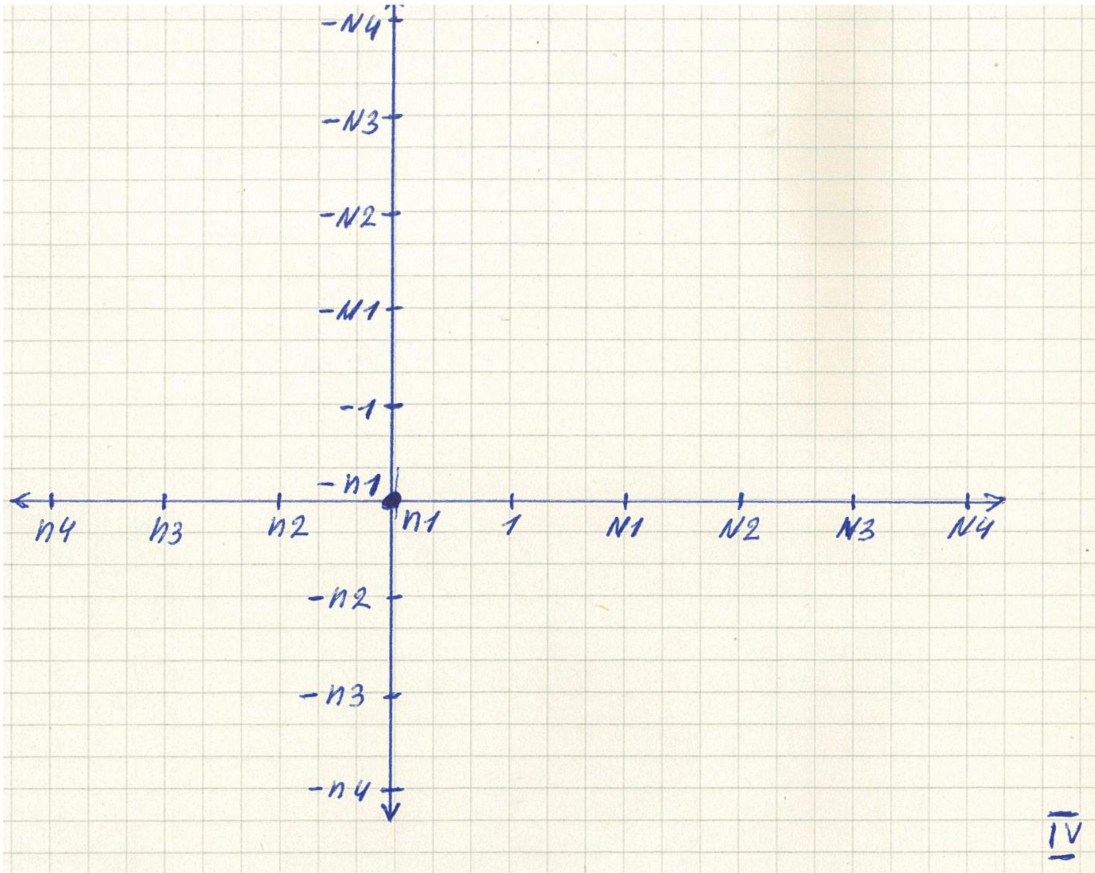
and



2 Crossings and self-crossings of positive and negative number lines.

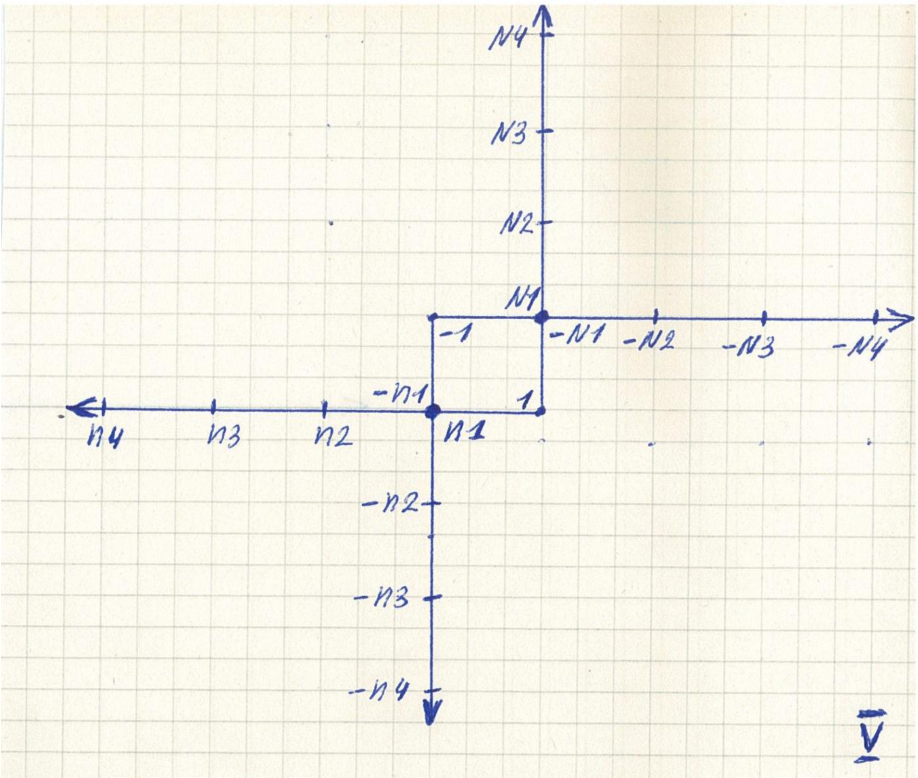
The definition (11) gives us the equation (19):

-n1=-(1-1)=-1+1=1-1=n1 -n1=n1 (19) So we can draw this:



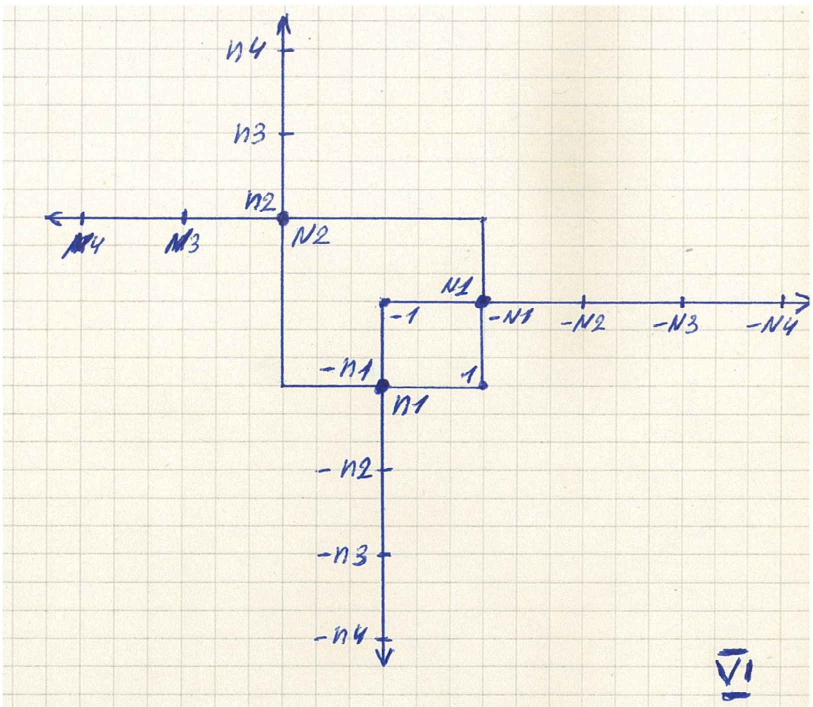
The definition (1) gives us the equation (20):

N1 + 1 = N1; -N1 - 1 = -N1; -N1 = -N1+1; -N1 = N1 (20) So we can draw:



(2)+(13)+(20): n2=N2 (21)

Now we can draw:



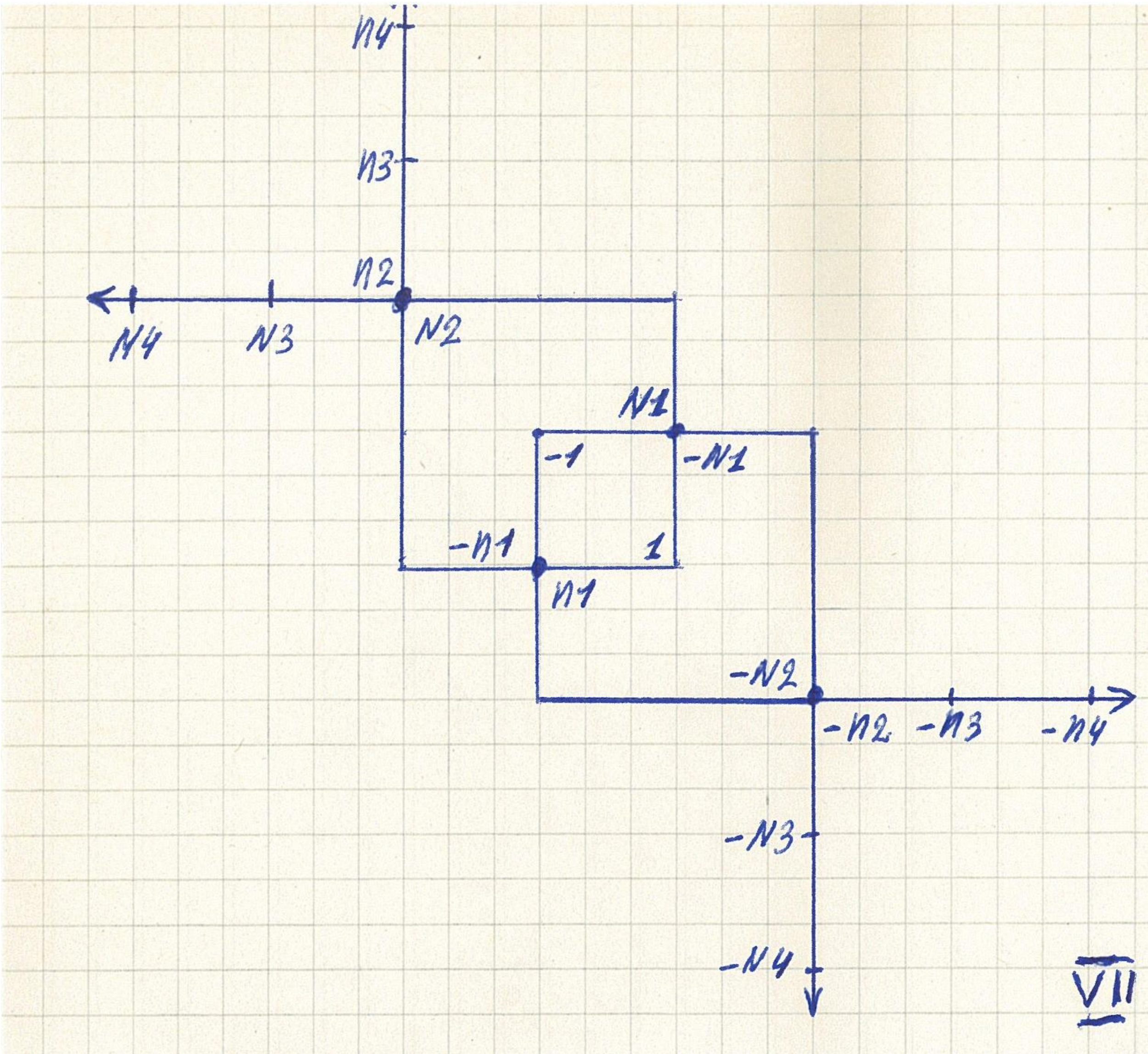
We have from (21):

n2 = N2

Let us multiply this equation on -1:

-n2 = -N2 (22)

And we can draw:



From (4), (21) we have: $N3 = 2^{N2} = e^{n2*\ell n(2)} = 1 + n2 * \ell n(2) + \cdots$

Let us define M1 so: $M1 = 1 + n2 * \ell n(2) + \cdots$ (23) Then N3 = M1 (24)

From (15), (23), (24) we have: $n3 = 1/N3 = 1 - n2 * \ell n(2) + \cdots$

Let us define m1 so: $m1 = 1 - n2 * \ell n(2) + \cdots$ (25)

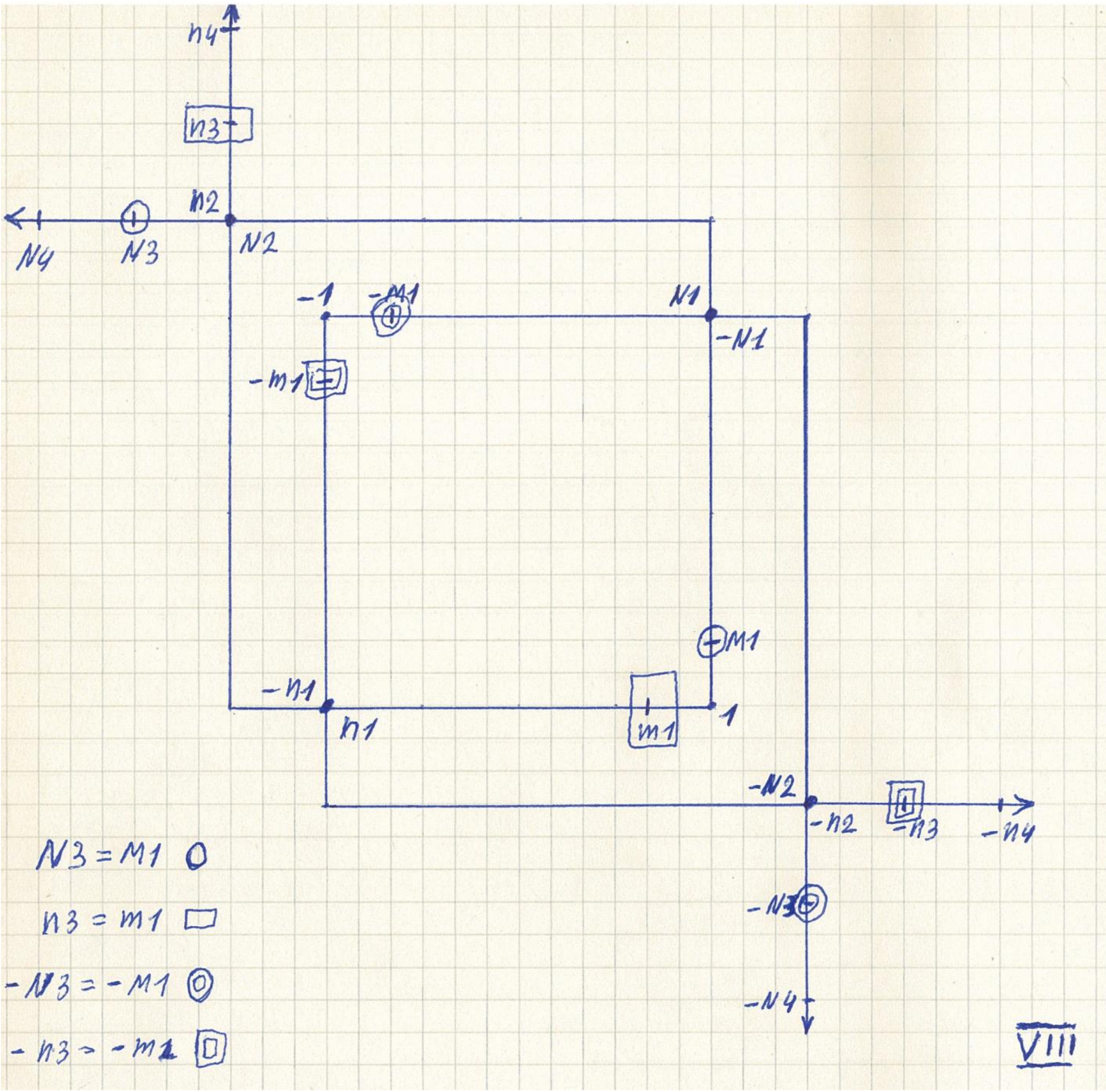
Then n3 = m1 (26)

And from (24) and (26) we have :

-N3 = -M1 (27)

-n3 = -m1 (28)

Then we can draw:



It is necessary to raise the arrow from N3 into the third dimension and fly it over two lines and then descent to the M1 and transfix it. And N4 with arrow will be on the other side of the plane of drawing. And N3 will coincide with M1. It will be not only with (N3 and M1), but also with other 3 pairs of points: (n3 and m1), (-N3 and –M1), (-n3 and –m1).

From (6), (24), (23) : $N4 = 2^{N3} = 2^{M1} = 2^{1+n2*\ell n(2)+\cdots} = 2 + 2 * n2 * \ell n(2) + \cdots$

Let us define M2 so: $M2 = 2^{M1} = 2 + 2 * n2 * \ell n(2) + \cdots$ (29)

Then N4 = M2 (30)

From (17), (30), (29) :

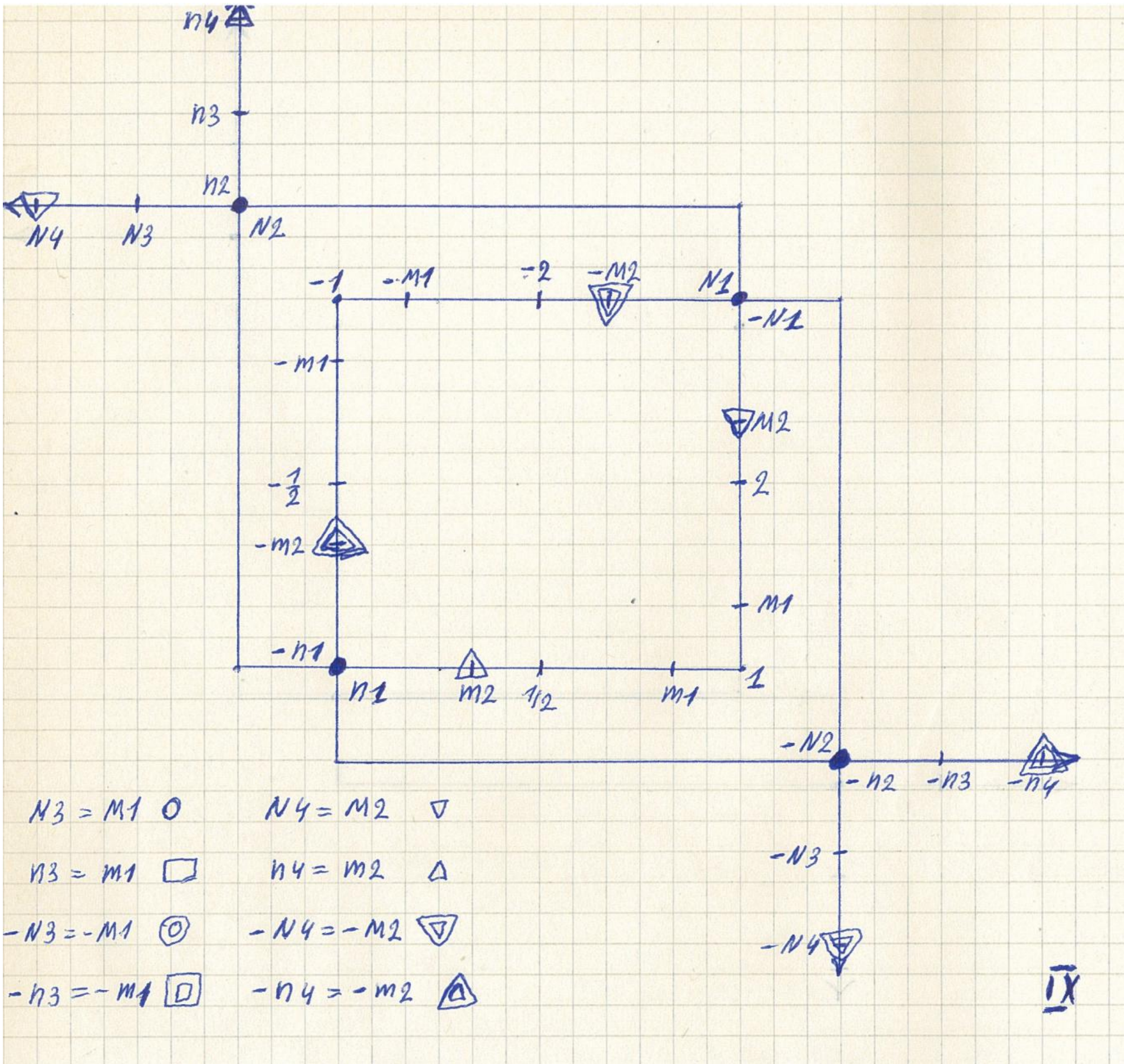
$$n4 = \frac{1}{N4} = 2^{-M1} = 2^{-1-n2*\ell n(2)-\cdots} = \frac{1}{2} - \frac{1}{2} * n2 * \ell n(2) + \cdots$$

Let us define m2 so: $m2 = 2^{-M1} = \frac{1}{2} - \frac{1}{2} * n2 * \ell n(2) + \cdots$ (31)

Then n4 = m2 (32)

And from (30), (32) we have: -N4 = -M2 (33) -n4 = -m2 (34)

Now we can draw:



And this process of forming new big, little, medium numbers and of discovery the coincidences between them is infinite.

We see now, that so simple number line has very complex topological structure that require the many-dimensional vector space to represent it fully.

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