Scientia Magna
Vol. 2 (2006), No. 3, 81-87

Sequences of pentagonal numbers[1]

Arun S. Muktibodh
Mohota Science College,
Umred Rd. Nagpur-440009, India.

Abstract Shyam Sunder Gupta [3] has defined Smarandache consecutive and reversed Smarandache sequences of triangular numbers. Delfim F.M.Torres and Viorica Teca [1] have further investigated these sequences and defined mirror and symmetric Smarandache sequences of triangular numbers making use of Maple system. Working on the same lines we have defined and investigated consecutive, reversed, mirror and symmetric Smarandache sequences of pentagonal numbers of dimension 2 using the Maple system .

Keywords Figurate number, Smarandache consecutive sequence, Smarandache mirror sequence.

§1. Introduction

Figurate number is a number which can be represented by a regular geometrical arrangement of equally spaced points. If the arrangement forms a regular polygon the number is called a polygonal number. Different figurate sequences are formed depending upon the dimension we consider. Each dimension gives rise to a system of figurate sequences which are infinite in number.

In this paper we consider a figurate sequence of pentagonal numbers of dimension 2. Henceforth, unless and otherwise stated, "pentagonal numbers" will mean pentagonal numbers of dimension 2.

The n-th pentagonal number $t_n, n \in N$ of dimension 2 is defined by $t_n = 2n + \frac{5}{2}n(n-1) - n^2$.

We can obtain the first k terms of Pentagonal numbers in Maple as:

```
> t:= n->2*n + (5\2)*n*(n-1)-n^{2}:

> first := k -> seq(t(n), n=1...20);
```

For example first 20 terms are:

```
> first(20);

1, 5, 12, 22, 35, 51, 70, 92, 117, 145, 176, 210, 247, 287, 330,
     376, 425, 477, 532, 590
```

For constructing Smarandache sequence of pentagonal numbers we require the operation of concatenation on the terms of above sequence which is defined in Maple as;

[1]This work is supported by UGC under Minor project F. No. 23-245/06.

```
> conc :=(n,m) -> n*10^length(m)+m:
```

We define Smarandache consecutive sequence $\{scs_n\}$ for pentagonal numbers recursively as;
$scs_1 = u_1$,
$scs_n = conc(scs_{n-1}, u_n)$
Using Maple We have obtained first 20 terms of Smarandache consecutive sequence of pentagonal numbers;

```
> conc :=(n,m)-> n*10^length(m)+m:
> scs_n := (u,n)-> if n = 1 then u(1)else conc(scs_n(u,n-1),u(n)) fi:
> scs := (u,n)-> seq (scs_n(u,i),i=1...n):
> scs(t,20);
```

```
1, 15, 1512, 151222, 15122235, 1512223551, 151222355170,
      15122235517092, 15122235517092117, 15122235517092117145,
      15122235517092117145176, 15122235517092117145176210,
      15122235517092117145176210247,
      15122235517092117145176210247287,
      15122235517092117145176210247287330,
      15122235517092117145176210247287330376,
      15122235517092117145176210247287330376425,
      15122235517092117145176210247287330376425477,
      15122235517092117145176210247287330376425477532,
      15122235517092117145176210247287330376425477532590
```

Same sequence can be displayed in "triangular form" as;

```
> show := L -> map(i ->print(i),L):
> show([scs(t,20)]);
```

```
1
15
1512
151222
15122235
1512223551
151222355170
15122235517092
15122235517092117
15122235517092117145
15122235517092117145176
15122235517092117145176210
15122235517092117145176210247
15122235517092117145176210247287
```

```
15122235517092117145176210247287330
15122235517092117145176210247287330376
15122235517092117145176210247287330376425
15122235517092117145176210247287330376425477
15122235517092117145176210247287330376425477532
15122235517092117145176210247287330376425477532590
```

The reversed Smarandache sequence (rss)associated with a given sequence $\{u_n\}, n \in N$ is defined recursively as;

$$rss_1 = u_1,$$
$$rss_n = conc(u_n, rss_{n-1}).$$

In Maple we use the following program;

```
> rss_n :=(u,n) -> if n=1 then u(1) else conc(u(n),rss_n(u,n-1)) fi:
> rss := (u,n) -> seq(rss_n(u,i),i=1..n):
```

We get the first 20 terms of reversed Smarandache sequence of pentagonal numbers;

```
> rss(t,20);

  1, 51, 1251, 221251, 35221251, 5135221251, 705135221251,
        92705135221251, 11792705135221251, 14511792705135221251,
        17614511792705135221251, 21017614511792705135221251,
        24721017614511792705135221251,
        28724721017614511792705135221251,
        33028724721017614511792705135221251,
        37633028724721017614511792705135221251,
        42537633028724721017614511792705135221251,
        47742537633028724721017614511792705135221251,
        53247742537633028724721017614511792705135221251,
        59053247742537633028724721017614511792705135221251
```

Smarandache Mirror Sequence (sms)is defined as follows:

$$sms_1 = u_1,$$
$$sms_n = conc(conc(u_n, sms_{n-1}), u_n).$$

The following program gives first 20 terms of Smarandache Mirror sequence of pentagonal numbers.

```
> sms_n := (u,n) -> if n=1 then
```

```
> u(1)
> else
> conc(conc(u(n),sms_n(u,n-1)),u(n))
> fi:
> sms :=(u,n) ->seq(sms_n(u,i), i=1..n):
> sms(t,20);

 1, 515, 1251512, 22125151222, 352212515122235, 5135221251512223551,
       70513522125151222355170, 927051352212515122235517092,
       117927051352212515122235517092117,
       145117927051352212515122235517092117145,
       176145117927051352212515122235517092117145176,
       210176145117927051352212515122235517092117145176210,
       247210176145117927051352212515122235517092117145176210247,
       287247210176145117927051352212515122235517092117145176210\
       247287, 3302872472101761451179270513522125151222355170921171\
       45176210247287330, 3763302872472101761451179270513522125151\
       22235517092117145176210247287330376, 4253763302872472101761\
       4511792705135221251512223551709211714517621024728733037642\
       5, 47742537633028724721017614511792705135221251512223551709\
       2117145176210247287330376425477, 53247742537633028724721017\
       6145117927051352212515122235517092117145176210247287330376\
       425477532, 590532477425376330287247210176145117927051352212\
       515122235517092117145176210247287330376425477532590.
```

Finally Smarandache Symmetric sequence (sss) is defined as:

$$sss_{2n-1} = conc(bld(scs_{2n-1}), rss_{2n-1}),$$

$$sss_{2n} = conc(scs_{2n}, rss_{2n}), n \in N.$$

where the function "bld" (But Last Digit) is defined in Maple as:

```
> bld := n->iquo(n,10):
```

First 20 terms of Smarandache Symmetric sequence are obtained in Maple as:

```
> bld :=n-> iquo(n,10):
> conc := (n,m)-> n*10^length(m)+m:
> sss_n := (u,n) -> if type(n,odd) then
> conc(bld(scs_n(u,(n+1)/2)),rss_n(u,(n+1)/2))
> else
> conc(scs_n(u,n/2),rss_n(u,n/2))
> fi:
> sss := (u,n) -> seq(sss_n(u,i), i=1..n):
> sss(t,20);
```

```
1, 11, 151, 1551, 1511251, 15121251, 15122221251, 151222221251,
      151222335221251, 1512223535221251, 1512223555135221251,
      15122235515135221251, 151222355177051352212 51,
      1512223551707051352212 51, 15122235517099270513522125 1,
      151222355170929270513522125 1,
      1512223551709211117927051352212 51,
      15122235517092117111792705135221251,
      151222355170921171414511792705135221251,
      15122235517092117145145117927051352212 51.
```

We find out primes from a large number (first 500 terms) of various Smarandache sequences defined so far. We have used Maple 6 on Pentium 3 with 256 Mb RAM. We first collect the lists of first 500 terms of the consecutive, reversed, mirror and symmetric sequences of Pentagonal numbers:

```
 > st :=time(): Lscs500:=[scs(t,500)]: printf("%a seconds",round(time()-st));
16 seconds
> st :=time(): Lrss500:=[rss(t,500)]: printf("%a seconds",round(time()-st));

18 seconds
> st :=time(): Lsms500:=[sms(t,500)]: printf("%a seconds",round(time()-st));
50 seconds
> st :=time(): Lsss500:=[sss(t,500)]: printf("%a seconds",round(time()-st));
11 seconds
```

Further we find the number of digits in the 500th term of each sequence:

```
> length(Lscs500[500]),length(Lrss500[500]);

                              2626, 2626

> length(Lsms500[500]),length(Lsss500[500]);

                              5251, 2268
```

There exist no prime in the first 500 terms of Smarandache consecutive sequence of pentagonal numbers.

```
>st:= time():select(isprime,Lscs500);
> printf("%a minutes",round((time()-st)/60));

                                  [ ]
```

```
11 minutes
```

There are only two primes in the first 500 terms of reversed Smarandache sequence of pentagonal numbers.

```
>st:= time():select(isprime,Lrss500);
[221251,
10049299717989459817697410966479588795130943769362592877921329139
09065189915891828845287725870018628085562848478413583426827208201
78131780620799267923578547778627718076501758257515274482738157315
172490718327117770525698766923068587679476731066676660456541764792
641706355162935623226171261105605015990059302587075811557526569405
635755777552005462654055534875292252360518015124550692501424959549
051485104797247437469054637645850453274480744290437764326542757422
524175041251407554026239772392853880138320378423736736895364263596
0354973503734580341263367533227327823234031901314653103230602301752
9751293302891228497280852767627270268672646726070256762528524897245
1224130237512337523002226322226521901215402118220827204752012619780
1943719097187601842618095177671744217120168011648516172158621555515
2511495014652143571406513776134901320712927126501237612105118371157
2113101105110795105421029210045980195609322908788558626840081777957
7740752673157107690267006501630561125922573555515370519250174845467
6451043474187403038763725357734323290315130152882275226252501238022
6221472035192618201717161715201426133512471162108010019258527827156
5159053247742537633028724721017614511792705135221251]
```

```
> printf("%a minutes",round((time()-st)/60));
45 minutes
```

There is no prime in the first 500 terms of Smarandache mirror sequence.

```
>st:= time():select(isprime,Lsms500);
> printf("%a minutes",round((time()-st)/60));
```

```
[ ]
```

```
312 minutes
```

There are 4 primes in first 500 terms of Smarandache symmetric sequence of pentagonal numbers.

```
> st:= time():
```

```
> select(isprime,Lsss500);
> printf("%a minutes",round((time()-st)/60));

  [11, 151, 1512223551709211714514511792705135221251, 1512223551709\
        2117145176210247287330376425477532590651715782852925100100\
        1925852782715651590532477425376330287247210176145117927051\
        35221251]

50 minutes
```

There are some results which can be obtained by the readers as; How many pentagonal numbers are there in Smarandache consecutive , mirror and symmetric sequences of pentagonal numbers ?

References

[1] Delfim F.M., Viorica Teca, Consective, Reversed, Mirror and Symmetric Smarandache Sequences of Triangular Numbers, Scientia Magna, **2**(2005), pp. 39-45.

[2] Muktibodh A.S., Figurate Systems, Bull. Marathwada Mathematical Soc., **2**(2005), pp. 12-20.

[3] Shyam Sunder Gupta, Smarandache Sequence of Triangular Numbers, Smarandache Notions Journal, **14**, pp. 366-368.