«The Quantum Mechanics» in the Model of 4D Matter

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In the frame of the model of 4D matter («4D medium» or «4D ether») it is given the description of some basic notions of the quantum mechanics such as the uncertainty principle of Heisenberg, the atomic spectrum, the wave function, the de Broyle's wave and some other.

The quantum mechanics takes the significant place in the contemporary physics. It has many achievements in the development of our knowledge in the field of microworld. There is the trial to penetrate it on macroworld [1]. With that in many ways such knowledge has the formal character that lowers the cognitive value of this theory that is so much different as compared with the classical physics. Below the endeavour to give the alternative description of the notions of quantum mechanics is undertaken.

In [1] it was made some suggestion on the construction of the atom in the model of 4D medium. Here we are considering the atom in details taking into account the idea about the 4D whirls as the representatives of the elementary particles in the model.

The main feature of this idea is as follows. The Universe is the close four-dimensional region of space filled by the particular matter, 4D medium. While because of the surface tension the Universe has the spherical form in the whole, its border locally, due to its enormous size, can be presented as the 3D plane

\[ x_4 = 0 \]  \hspace{1cm} (1)

We are perceiving this border as our usual 3D World because the light and other electromagnetic waves supposed to be some special waves on the surface of the Universe.

The particle supposed to have the spatial longitudes in the additional dimension. Its beginning is on the border, in the World, and the end is lost in the «depth» of the Universe. For the brevity we call such presentation of the particle as 4D whirls or vortex.

As it was shown [2], the tilt of the whirl with respect to the border is corresponded to the movement of the particle. In this case the velocity of the particle is determined by the simple expression which lead to the natural treatment of the results of the special theory of relativity, but without real contraction of the longitudinal size and time delay [3]

\[ V = c \sin \alpha \]  \hspace{1cm} (2)

where \( c \) is the light speed and \( \alpha \) is the angle between the axis of the whirl and the normal to the border at the point of the cross-section of the whirl and the border. For the border (1) this normal coincides with the \( x_4 \) axis.
The uncertainty principle

There was obtained [4] the next expression for the impulse of the particle that is compatible with that being in the special theory for the free particle

\[ p = m_0 c \tan \alpha \]  

where \( m_0 \) is the rest mass of the particle biased with the so called relativistic mass, or mass in motion, \( m \) by the relation

\[ m = \frac{m_0}{\cos \alpha} \]  

Then we can put down the usual expression for the impulse \( p = mV \) taking into account Eq.(1).

When the considering of the atom, which for a certainty will be taken the hydrogen atom, we will assume that the nucleus of the atom is at rest. It means that the whirl corresponding by us to the nucleus is placed normally to the border.

Then if the electron is taken as the whirl too, its form in the atom can be determined under consideration of the process of the possible generation of the atom. To be near at the nucleus it must move to it or, by other words, have the tilt to the nucleus side. Of cause, the approaching is came about due to the electrostatic interaction with the nucleus because electron and nucleus has different charges. Let us imagine such sloped vortex approaching to the nucleus. Its “lower” part will come by the nucleus because the interaction of the currents existing along the vortices is weak as it depents on the distance as the reverse third power. So it needs very small adjusting distance for electron to be catched by nucleus. But when the “upper” part of the electron, the part linked to the border, is approaching to nucleus, the electrostatic interaction depending from the distance as the reverse square will capture it and the whole vortex representing the electron will “coil” around the vortex representing the nucleus. As a result the structure of the atom in the form of the spiral can be presented by the system of following equations

\[
\begin{align*}
x_1 &= a \cos (k_1 x_4 - \omega_1 t) \\
x_2 &= a \sin (k_1 x_4 - \omega_1 t) \\
x_3 &= 0 \\
x_4 &= u_4 t
\end{align*}
\]  

where \( a \) is the radius of the spiral, \( k_1 \) is the wave vector, \( \omega_1 \) is the angular velocity of the electron rotation around the nucleus. The time \( t \) is the parameter the changing of which gets all spiral points. The picture proposed is the spiral rotating in 4D space which also can be considered as the unrotating spiral moving with velocity \( \omega_1/k_1 \) along \( x_4 \) axis. Below it will be shown that this phase velocity is equal \( c \).

The meaning \( x_4 = 0 \) corresponds to the instantaneous position of the electron in the World. It should be noted that when we speak about the axis of the 4D vortex we mean the \( x_4 \) axis, but when we speak about rotation (5) we understand that it occurs around the plane created by the axes \( x_3 \) and \( x_4 \). It means that under the conserving of the vortex axis direction the values of the coordinates \( x_1, x_2 \) and \( x_3 \) can interchang with each other arbitrarily or by any law. Moreover, these values can be changed with \( x'_1, x'_2 \) and \( x'_3 \), which are obtained from the previous set of coordinates by any orthogonal transformation.

Thus one can say that the rotation (5) is “degenerated” rotation around \( x_4 \) axis. The
points of the spiral may be situated in any points of the sphere surface with radius $a$ under projection on the plane (1). Therefore the three-dimensional momentum impulse of the whole spiral, or the momentum impulse averaged with respect to coordinate $x_4$, or the orbital momentum $l$, is vanished for the state $1s$, which is considered here.

The product $a\omega_1$ is the velocity of the electron rotation around the nucleus. It can be expressed by Eq.(2) through the angle and seed of light. It let to present the moment impulse of the electron without taking into account the above-mentioned degeneration as

$$l = pa$$

This value is constant that means the constant size and pace of the spiral and the constant rotation velocity of the spiral around its axis. The angle $\alpha$ is also taken some constant value which we denote as $\alpha_0$.

One can say in the terms of quantum mechanics that the values $p$ and $a$ are the «uncertainties» of the values of impulse of the electron and its distance from the nucleus. Really, «observing» for the rotating spiral in three-dimensional World, we can't determine the exact place where is the electron, the whole spiral, located and which the velocity it has. So we can put together the momentum impulse $l$ and the Planck constant $\hbar$. So in contrast with the approximate uncertainty relation of Heisenberg we can put down the precise equality

$$pa = \hbar$$

Of course, the motion of the vortex is not the single motion of the medium even in the case of the consideration of single vortex. The points of the medium neighbouring to those determined by Eq.(5) are also somewhat moving. Therefore the vortex in the essence is not the one-dimensional line in space, the string, but the rather complicated four-dimensional object in the medium. Then one is to consider the expression (7) as approximate that means the uncertainties in the values of the coordinates and impulses of the points belonging to the vortex. But when we are talking about the positions of the points on the vortex axis, we are still to think of Eq.(7) as the exact relation.

**The fine structure constant**

Using Eq.(7), one can express the tangent of the angle $\alpha_0$ in Eq.(3) in the next view

$$tg \alpha_0 = \frac{\hbar}{am_e c}$$

If we put here the Bohr radius $a_0 = \hbar^2 / m_e e^2$ instead of $a$, where $e$ is the charge of electron and $m_e$ is its rest mass, we get so called the fine structure constant, for which the accepted notation is also the letter $\alpha$,

$$tg \alpha_0 = \frac{e^2}{\hbar c} \approx \frac{1}{137}$$

In this content it has the geometrical meaning of the tangent of the tilt angle. For hydrogen it consists just 25 angular minutes that correspond to the electron rotation velocity of 2187633 mps by the Eq.(2).
**The electromagnetic waves**

The step of the spiral $\lambda_1$, with which an electron in the atom is associated by us, is determined as $2\pi a \tan \alpha_0$. It is equal $4.5563 \times 10^{-8}$ m and the half of its reverse value is called as the Rydberg constant

$$R_\infty = \frac{1}{2\lambda_1} = \frac{m_e e^4}{4\pi \hbar^2 c}$$ (10)

We used here the Bohr radius as the radius of the spiral and Eq.(9) for $\tan \alpha_0$. This observation lets us to suppose that the lengths of the electromagnetic waves absorbing and radiating by atom are commensurate with $\lambda_1$.

To endorse it we will find the kinetic energy $T_1$ of the rotating spiral in the basic state, that is in the state when the atom is not able to radiate. For small angle $\alpha_0$ we can determine it as the difference between the full energy in this state $E_1 = mc^2$ and the rest energy $E_0 = m_0 c^2$:

$$T_1 = mc^2 \left( 1 - \frac{1}{\cos \alpha_0} - 1 \right) \approx \frac{p^2}{2m_e} = \frac{m_e c^2 \tan \alpha_0}{2 \lambda_1} = \frac{m_e e^4}{2 \hbar^2 \lambda_1} = \frac{2\pi \hbar c}{\lambda_1} = \hbar \omega_1$$ (11)

The so-called excited state, if being able to describe in the spiral form, is to “inscribe” into basic state. By the other words the wave length of the first excited state must be two times greater then $\lambda_1$, the second three times and so on. The frequency of the n-th excited state will become the n times less. But when the radius of the spiral is left constant the correspondent expression of the kinetic energy of this mode

$$\frac{1}{2} m_0 a_0 \left( \frac{\omega_{mn}^2}{n^2} \right) = \frac{\hbar \omega_1}{n^2}$$

will approach to zero while $n \to \infty$.

Therefore the full energy at this limit must be the value to exceed $T_1$. Only in the case of putting an additional energy equal to $T_1$ to the atom we can “uncoil” the spiral and to enter by this action the electron in the free state with energy $E_0$. Thus we can write down the following expression for common energy in the n-th state

$$E_n = E_1 + T_1 - \frac{T_1}{n^2} = E_1 + \left( 1 - \frac{1}{n^2} \right) \hbar \omega_1$$ (12)

and the excited states can be treated as the foundations created under the absorbing the light quanta with energies $E_2 - E_1 = \hbar \omega_{12} = 3/4 \hbar \omega_1$, $E_3 - E_1 = \hbar \omega_{13} = 8/9 \hbar \omega_1$ and so on for the transitions from the first excited state, the second one and so on to the basic state. Thus the Lyman series of the radiation spectrum is produced. Also the transitions between any other excited states are possible.

From the experiments it is known that the whole spectrum of these energies can be presented by the well-known Balmer formular that can be get from Eq.(12)

$$\hbar \omega_{mn} = \hbar \omega_1 \left( \frac{1}{n^2} - \frac{1}{m^2} \right)$$ (13)

where n and m are any whole numbers. The lengths of the light wave can be expressed as

$$\frac{1}{\lambda_{nm}} = \frac{1}{2\lambda_1} \left( \frac{1}{n^2} - \frac{1}{m^2} \right)$$ (14)

For the other chemical elements having nucleus charge Z these two equations should be
multiply to $Z^2$. Moreover due to the shielding the nucleus by the inner electrons the values of the angle $\alpha$ for the spirals of the outer electrons are changing and the Rydberg constant will be different from the (10).

The following picture of the absorption and emission of the light by the atom can be presented. Let us the wave moving forwards the atom along the border hypersurface has such length and frequency that transit the atom with energy $E_1$ to the state with energy $E_2$. The action that the wave endows to the spiral of the electron bears the impulsive character and puts on the sole wave, the soliton, being formed. The local region on the “upper” part of the spiral with the steps gone together is appeared. Then the soliton begins to move along the spiral axis, reflects approaching some limit in the “depth” of the medium and returns. Reaching the medium border it runs out from it in the same form of the light wave or the electromagnetic radiation as was absorbed earlier. Thus the delay between the moments of the light absorbing and radiating is explained.

The soliton can be described by the displacements $\Delta x_4$ that the spiral (5) gives under the portion of light is absorbed. If the action was weakly nonlinear and the medium was weakly dispersive, the equation of motion for soliton may be the Korteweg-de Vries equation

$$u_t + uu_x + u_{x^4} = 0$$  \hspace{1cm} (15)

where $u = \frac{d(\Delta x_4)}{dt}$. It has one-soliton solution

$$u = 3V \text{sech}^2\left[\frac{\sqrt{V}}{2}(x_4 - Vt)\right]$$  \hspace{1cm} (16)

where V is the velocity of soliton. It is clear from this solution that the more velocity the more displacement field. So the soliton energy is dependent from the velocity. Therefore one can suppose that for our case the energies can take only strictly determined discrete values which correspond those given by Eq.(13). This supposition also means that the time delay between the moments of absorption and irradiation depends from the frequency. It correspondent to the double time interval for passing soliton along the vortex.

This fact seems to be observed by experiment and let to evaluate how much Eq.(15) corresponds to the real equation of motion for “inner” soliton that is created after the absorbing of light by the hypothesis.

It is interesting to note that the frequency equal to $\frac{3}{4}$ from the basic one is characteristic for the overtone firstly found by Pythagore. It calls quarta in musics.

Thus the quantum of light or photon is the run-out part of the electron in the excited state, the part of the spiral, It is presented schematically in the following two pictures.

Fig.1. Photon as the vortex tube with its start and end at the hypersurface is moving in the direction of $x_1$. 
The stream in the center of such vortex line along the $x_1$-axis keeps the motion of the photon with the uppermost velocity $c$. Such picture has some resemblance with the one obtained by smoker who can bring forth the ring of smoke.

Earlier the similar model with the closed vortex ring was proposed for the neutrino [6]. The particle is invisible in this case for its motion is into the bulk of the medium.

**The wave function**

By differentiating Eqs. (5) we get the set of velocities of the points of the spiral-electron in atom

\begin{align}
    u_1 &= -a \omega \sin (k_1 x_4 - \omega_1 t) \\
    u_2 &= a \omega \cos (k_1 x_4 - \omega_1 t) \\
    u_3 &= 0 \\
    u_4 &\neq 0
\end{align}

We can take into consideration the complex velocity determined the real part as $a \omega$ and the imaginary one as $u_4$. Then one can call the next expression as the wave function

$$
\psi = a \omega + i u_4
$$

(18)

if adopt the normalization used in the quantum mechanics that means in the multiplication of $\psi$ on the following constant coefficient

$$
\int |\psi|^2 d^4 x = 1
$$

(19)

Then one can say that taking into account above-told for the uncertainty relation the square of the wave function determines the density of the kinetic energy of the medium points. To some expend it reflects the probability of the being the particle, the electron in our case, in the given point of three-dimensional space. Our point of view is closer to the interpretation of de Broyle who tied it with the real wave of matter. In common case the wave function can be determined as $\Psi = |\psi| + i u_4$, where the module is talen from the usual three-dimensional velocity of the medium points. It becomes clear that the quantum-mechanical consideration based on the using of the wave function can gine only quite short discription of the behavior of the particle even in three-dimensional space rather then in four-dimesional one. It can't give the full description of the processes that can take place in 4D medium — of course, if such madium really exists.

The cause of such situation from our point of view is that the demand of the superposition priciple superimposed on the wave function is rather hard. The equations for the 4D medium velocity field is essentially nonlinear ones in distinct to the equations of motion for the wave function given under this principle.
The unit system

To simplify the accounting we can present the following «quantum» system of units biased
to atom. It puts the rest mass and the charge of electron together with the light speed to one unit.
Then the next row of sizes is obtained
\[
\begin{align*}
\lambda_1 &= 2\pi\hbar^3, \quad a_0 = \hbar^2, \quad \lambda_c = 2\pi\hbar, \quad r_0 = 1,
\end{align*}
\]
where \(\lambda_c\) is the Compton length of wave for electron, \(r_0\) is the classic radius of electron that
are equal in usual units \(2\pi\hbar/m_ec\) and \(e^2/m_ec^2\), respectively, and the Plank constant
\(\hbar = 1/\tan\alpha_0\), that is equal to the reverse of the fine structure constant in this units, gives the scale
on which the parameters of electron in atom changes. Namely, the “radius of the orbit” of electron
in \(a_0\) in \(2\pi\hbar\) times less the length of the period of one rotation of spiral around the vortex axis
in the additional dimension. The classic radius and the Compton length \(r_0\) in \(\lambda_c\), that are in the
same relation with respect to each other, obviously related to the parameters characteristic to the
inner construction of the vortex. This fact affords ground for the supposition that the electron is
“built” in a somewhat similar manner that was proposed above for hydrogen atom,

The electron in such supposition is presented in the form of the vortex tube with its radius
\(r_0\) in \(2\pi\hbar\) times less the length of the period \(\lambda_c\) of one rotation of the stream of the medium
around the tube axis. The direction of the stream points out on the charge and therefore is reverse to
that the positron has. Its vorticity is the spin. It can take as the rotation of the electron spiral on atom
the different direction in three-dimensional space. Only an action of the outer magnetic field can
give the orientation for the spin. The full moment \(j\) that the atom has due to electron system is the sum
\[
\hat{s} = l + s
\]

The de Broyle wave

De Broyle shown that for any particle moving with the impulse \(p\) one can juxtapose the
wave with the wave vector \(k\) :
\[
p = \hbar k
\]
This fact that found its approwvement in the experiments on the diffraction particles on clysals one
can give the next tratement in the frame of the model proposed.

Let us consider the moving electron. We know the movement of the material particle
corresponds the tilt of the particle vortex in the movement side. Let us it will be the slope in the
\(x_1\) direction. The velocity field for this vortex can be obtained changing the coordinates in (7) by
the following matrix
\[
M = \begin{bmatrix}
\cos\alpha & 0 & 0 & -\sin\alpha \\
0 & 1 & 0 & 0 \\
\sin\alpha & 0 & 0 & \cos\alpha
\end{bmatrix}
\]
As a result the picture shown on Fig.3 is took place. One can imagine that due to the slope the
stream makes the splashes on the hypersurface (1) every time when the inner vorticity meets with
the hypersurface in the same phase. The inner wave corresponded to the Compton wave length
spreads to the outer de Broyle wave in the three-dimensional World.
Fig.3. It is shown the vortex tube position in the time moments \( t=0 \) and \( t=T \), the period of the stream rotation around the vortex axis. One stream line into the vortex tube is outlined.

For the particle with rest mass \( m_0 \) the Compton wavelength is equal in the usual units\[ \lambda_C = \frac{m_e}{m_0} \] . The de Broyle wave length put together with it is equal\[ \lambda = \frac{\lambda_C m_e}{m_0 \sin \alpha} \] (24)

and the correspondent wave vector is equal\[ k = \frac{m_0 v}{\hbar} = \frac{p}{\hbar \sqrt{1 + p^2 / (m_0 c)^2}} \] (25)

Therefore we get\[ p = \frac{\hbar k}{\sqrt{1 - (k/k_C)^2}} \] (26)

where \( k_C \) is the Compton wave vector of the electron. One can see that the length of de Broyle’s wave can not be less \( \lambda_C \). Under \( k \ll k_C \) we get (22) for the modules of the vectors \( p \) and \( k \).

**Conclusion**

Thus some basic notions of quantum mechanics find their more natural representation in the model of four-dimensional medium. Of course, the main one among them is the Plank constant or the fine structure constant.

On the example of the interrelation of \( \lambda_1 \) and \( a_0 \) one can conclude that in quantum mechanics multiplying any physical value, or its representation, on the Plank constant we are made the transition from the imaginary part of this value that is in the additional dimension to the real part. For example, the real part of the expression \( i \hbar \frac{\partial \psi}{\partial t} \) is corresponded to the multiplied on the “action quant”, on the Plank constant, acceleration or more strictly the time derivative of the normalized velocity belonging to the fourth dimension. According to the Schroedinger equation it is equal to the real part of the product of Hamiltonian \( H \) and the normalized 4-velocity \( H \psi \). For the velocity is the complex value in quantum mechanical representation the Hamiltonian must be complex too and, moreover, hermitian. In the view of the model proposed it become obvious that the Schroedinger equation by no means can not be the equation of motion for the points of 4D
matter. The acceleration for them looks different, as for example the equation of Navier-Stokes one.

So the basic presentation of quantum mechanics can be proposed in the more simplest terms approaching it to the notions of the classical physics.