Abstract
The objects that occur in nature can be categorized in several levels. In this collection every level except the first level is built from lower level objects. This collection represents a simple model of nature. The model exploits the possibilities that mathematical concepts provide. Also typical physical ingredients will be used.
The paper splits the hierarchy of objects in a logic model and a geometric model. These two hierarchies partly overlap.
The underlying Hilbert Book Model is a simple self-consistent model of physics that is strictly based on quantum logic. This paper refines quantum logic to Hilbert logic such that it more directly resembles its lattice isomorphic companion, which is a separable Hilbert space. The HBM extends these static sub-models into a dynamic model that consists of an ordered sequence of the static sub-models.
The paper is founded on three starting points: • A sub-model in the form of traditional quantum logic that represents a static status quo. • A correlation vehicle that establishes cohesion between subsequent members of a sequence of such sub-models. • The cosmological principle.
Further it uses a small set of hypotheses. It turns out that the cosmological principle is already a corollary of the first two points.
The paper explains or indicates the explanation of all features of fundamental physics that are encountered in the discussed hierarchy which ranges from propositions about physical objects until composites of elementary particles. Amongst them are the cosmological principle, the existence of quantum physics, the existence of a maximum speed of information transfer, the existence of physical fields, the origin of curvature, the origin of inertia, the dynamics of gravity, the existence of elementary particles, the existence of generations of elementary particles and the existence of the Pauli principle.
No model of physics can change physical reality.
Any view on physical reality involves a model
Drastically different models can still be consistent in themselves.
The Hilbert Book Model is a simple self-consistent model of physics.
This model steps with universe-wide progression steps from one sub-model to the next one. Each of these sub-models represents a static status quo of the universe. The sub-models are strictly based on traditional quantum logic.
The HBM is a pure quaternion based model. Conventional physics is spacetime based. When both models are compared, then the progression quantity (which represents the page number in the Hilbert Book model) corresponds to proper time in conventional physics.
The length of a smallest quaternionic space-progression step in the HBM corresponds to an "infinitesimal" coordinate time step in conventional physics.
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The "secret" of physics is the way that it enumerates its countable sets, such that these approach a corresponding continuum.

1 Introduction

I present you my personal view on the hierarchy of objects that occur in nature. Only the lowest levels are extensively treated. Composite particle objects are treated in a more general way. Cosmology is touched. For the greater part, the model is deduced. For that reason the model is founded on a solid and well accepted foundation. That foundation is traditional quantum logic. The model does not aim at experimental verification of its results, but it uses experimentally verified results of physics as a guidance. The model uses mathematical tools for extending the foundation. In some cases “new” mathematics is applied.

The paper is founded on three starting points:

- A sub-model in the form of traditional quantum logic that represents a static status quo.
- A correlation vehicle that establishes cohesion between subsequent members of a sequence of such static sub-models.
- The cosmological principle.

Further it uses a small set of hypotheses. It turns out that the cosmological principle is already a corollary of the first two points.

This hierarchy model is in concordance with the main principles of the Hilbert Book Model[^1]. Since the HBM is strictly based on the axioms of traditional quantum logic, the same will be the case for the logic part of the object hierarchy model. The Hilbert Book Model gets its name from the fact that traditional quantum logic can only represent a static status quo and for that reason dynamics must be represented by an ordered sequence of these static models. The similarity with a sequence of pages and a book is obvious.

The object hierarchy model adds two fundamental starting points. First, a correlation vehicle must provide sufficient cohesion between the subsequent members of the sequence. Second, the model must obey the cosmological principle. The cohesion must not be too stiff otherwise no dynamics will take place.

The cosmological principle means that at large scales, universe looks the same for whomever and wherever you are. One of the consequences is that at larger scales universe possesses no preferred directions. It is quasi-isotropic (on average isotropic).

This paper is part of the ongoing HBM project. Many of the mathematical concepts that are touched here are treated in greater depth in Q-Formule: [http://vixra.org/abs/1210.0111](http://vixra.org/abs/1210.0111)


The paper explains[^2] all features of fundamental physics that are encountered in the discussed hierarchy which ranges from propositions about physical objects until elementary particles and their composites. Amongst them are the cosmological principle, the existence of quantum physics, the existence of a maximum speed of information transfer, the existence of physical fields, the origin of curvature, the origin of inertia, the dynamics of gravity, the existence of

[^1]: [http://vixra.org/abs/1209.0047](http://vixra.org/abs/1209.0047)
[^2]: Or it indicates a possible explanation
elementary particles, the existence of generations of elementary particles, the existence of the Pauli principle and the history of the universe.
On the other hand the current HBM does not explore further than composites that are constructed from elementary particles. It only touches some aspects of cosmology.
New mathematics is involved in the dynamic generation of potential functions.
2 General notes

2.1 Measurability and perceptibility

The first priority of the HBM is to understand how this model works and it is not her primary task to verify whether nature behaves that way. This is compensated by pursuing a strong degree of self-consistence of the model. At the same time the knowledge of how nature works is a guide in the development of the model. This makes HBM a deduced model rather than a model that is based on observed or experimentally verifiable facts.

For example the HBM uses proper time instead of coordinate time. Proper time is a Lorentz invariant measure of time. The corresponding clock ticks at the location of the observed item. Our common notion of time is coordinate time. The coordinate time clock ticks at the location of the observer. The HBM does not bother about the fact that in general proper time cannot practicably be measured. The HBM adds to this fact that all proper time clocks are synchronized. Further, the model includes lower level objects that cannot be observed as individuals. Only as groups these objects become noticeable.

The result of this target specification is that the HBM introduces its own methodology that often deviates considerably from the methodology of contemporary physics. The advantage is that this approach enables the researcher to dive deeper into the undercrofts of physics than is possible with conventional methodology.

As a consequence the HBM must be reluctant in comparing these methodologies and in using similar names. Confusions in discussion groups about these items have shown that great care is necessary. Otherwise, the author can easily be accused from stealing ideas from other theories that are not meant to be included in the HBM model.

This again will make it difficult to design measurements. Measuring methods are designed for measuring physical phenomena that are common in contemporary physics. This is best assured when is sought for phenomena that are similar between the model and contemporary physics. This action contradicts the caution not to use similar terms and concepts. This is the main reason why the HBM does not make experimental verification to its first priority.

On the other hand, also contemporary physics contains items that cannot be measured. For example color charge is an item that cannot (yet) be measured. As indicated above, proper time is a concept that also exists in contemporary physics, but in general it cannot be measured. Contemporary physics uses the field concept, but except for the cases that the fields are raised by properties of separate particles contemporary physics does not bother what causes the field.

2.2 Generators, spread and descriptors.

In the model, generators produce coherent groups of discrete objects that are spread over an embedding continuum. The density distribution and the current density distribution of these coherent groups are continuous functions that describe and categorize these groups. Depending on a suitable Green’s function, the distributions of discrete objects also correspond to potential functions. Due to the way in which the potential is generated, the potential functions correspond to a local curvature of the embedding space. This can be comprehended when the groups are generated dynamically in a rate of one element per progression step. During its very short existence the element transmits a spherical wave\(^3\) that slightly folds and thus curves the embedding space. The wave keeps proceeding. It represents a trace of the existence of the element that survives the element when it is long gone. The elements act as step stones and

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\(^3\) For anisotropic elements the message is transmitted by an anisotropic wave.
together they form a micro-path for the corresponding group. This micro-movement can be considered as a combination of a quasi-oscillation and a quasi-rotation. Indirectly, the generator influences space curvature. The descriptors can only describe the influence of the potentials on the local space curvature. The resulting potential is an integral and therefore a rather static effect. The generator can be described by the convolution of a sharp continuous function and a low scale spread function. In this way, the spreading part can be seen as the activator of local space curvature, while the derivative of the sharp part defines a local metric that can be considered as the descriptor of the local curvature. The two parts must be in concordance. In this way two kinds of descriptors of local curvature exist. The first is the density distribution that describes the spread of the discrete objects. It corresponds to a potential function. The second descriptor is the local metric. Since these functions act on different scales they can usually be treated separately. The origin of the local curvature is the dynamic stochastic process that produces the low scale spread of the discrete objects. As described above these objects transmit waves that curve the local space. The HBM suggests the combination of a Poisson process that is coupled to a binomial process, where the attenuation of the binomial process is implemented by a 3D spread function. For each coherent group, the elements are generated at a rate of one element per progression step. The stochastic generator process will generate according to a standard plan. Thus at each location where it is active it produces locally in principle the same kind of patterns. However, these patterns cause space curvature. The local curvature is generated by the considered group and by neighboring groups. Due to the variance in space curvature, the center location of the pattern may move. Both effects disturb the natal state of the distributions that are generated by the generating process. Since the patterns are generated with a single element per progression step, the generation poses a large chance to not generate the target natal shape but instead a distorted shape that in addition is spread over the path that the center location decides to follow. The produced distribution can still be described by a continuous function, but that function will differ from the continuous function that describes the undisturbed natal state. So the generation process is characterized by two functions. The first one represents the characteristics of the local generation process. It describes the natal state of the intended distribution. It is more a prospector than a descriptor. The second one describes the actually produced distribution that is distorted by the local space curvature and spread out by the movement of the center location. Further the generation of the distribution may not be completely finished, because not enough elements were generated since the generation of the pattern was started. The generated element only lives during the current progression step. In the next step a newly generated element replaces the previous object. At any instant the generated distribution consists of only one element. Thus for its most part the distribution can be considered as a set of virtual elements that lived in the past or will live in the future. The virtual distribution together with its current non-virtual element represents a pattern. The local curvature is partly caused by the pattern itself, but for another part it is caused by neighbor patterns. The previous description of the natal generation can be imagined visually. At a rate of one element per progression instant the generator produces step stones that are used by the generated building block. The step stones are located randomly in a coherent region of 3D space. The building block walks along these step stones. As a consequence even at rest the building block follows a stochastic micro-path. Any movement of the building block as a whole, will be superposed on the micro-path. At every arrival at a step stone, the building block transmits its

See: The enumeration process.
presence via a wave that slightly folds and thus curves the embedding continuum. These waves and the transmitted content constitute the potentials of the building block. Nobody said that the undercroft of physics behaves in a simple way!

2.3 Coupling and events
The HBM introduces the notion of \textit{coupling} of fields. It also means that a state of not being coupled exists. Coupling is described by a \textit{coupling equation}, which is a special kind of \textit{differential continuity equation}\footnote{See: Coupling}.

Coupling takes place between \textit{stochastic fields}. Stochastic fields describe \textit{density distributions} and \textit{current density distributions of lower order objects}. The distributions are generated by a local generation process that in each progression step produces ONE lower order object per stochastic field.

Coupling is implemented by messages that are transmitted in the embedding continuum by the active elements of the distribution via spherical waves\footnote{For anisotropic elements the message is transmitted by an anisotropic wave.} that slightly fold and thus curve this continuum. Together these waves constitute the potentials that are raised by the distribution. It is sensible to presume that the element generator reacts on the potentials that are active in that location.

When the particle is annihilated, the coupling stops. This also means that no further spherical potential waves are generated. However, the existing waves keep flowing away from their original source. They keep extending their reach with light speed.

In order to keep the considered group coherent, an inbound or outbound micro-move must on average be followed by a move in a reverse direction. This must hold separately in each spatial dimension. Thus in each spatial dimension a kind of quasi oscillation takes place. The synchronization of this quasi oscillation may differ per dimension. In a similar way a quasi-rotation can exist. A certain kind of coupling of fields may be based on induced synchronization of these quasi oscillations and quasi-rotations.

Coupling becomes complicated when it involves coupling dependencies that live in different dimensions. Such cases can no longer be solved by separating the problem per dimension. It also means that the problem is inherently quaternionic and cannot be solved by simple complex number based technology. This occurs in the coupling equation of elementary particles where two quaternionic functions are coupled that belong to different discrete symmetry sets. Dirac has solved this problem by applying spinors and Dirac matrices. The HBM solves this with quaternionic methodology.

2.4 Wave particle duality
A point-like object can hop along a stochastically distributed set of step stones that together form a micro-path. The step stones form a coherent distribution that can be described by a continuous object density distribution. Via a properly selected Green’s function the step stone distribution can also be converted into a potential function. Each suitable Green’s function corresponds to a corresponding potential function. A direct conversion from density distribution to a potential function is also possible and uses a dedicated Green’s function.

These higher level objects are different views of the same thing. Let us call it a particle.

Both the density distribution and the potential function have a Fourier transform and can be considered as a wave package. Problem with this view is the fact that the step stones only are used in a single progression instant. So most of the time the step stones are virtual. This becomes less relevant when the step stone distribution is generated according to a given plan. In that case the plan represents the
particle. Now we have a higher level object that at the same time is a point-like particle and will act as a wave package. This idea is exploited by the Hilbert Book Model.
3 The logic model
In this chapter the basic phenomena of physics will be deduced from its logical foundation. The HBM choses traditional quantum logic as its most basic foundation. In 1936, this foundation was suggested by Garret Birkhoff and John von Neumann.\(^7\)

3.1 Static status quo

3.1.1 Quantum logic
The most basic level of objects in nature is formed by the propositions that can be made about the objects that occur in nature. The relations between these propositions are restricted by the axioms of traditional quantum logic. This set of related propositions can only describe a static status quo. In mathematical terminology the propositions whose relations are described by traditional quantum logic form a lattice. More particular, they form an orthomodular lattice that contains a countable infinite set of atomic (=mutually independent) propositions. Within the same quantum logic system multiple versions of sets of these mutually independent atoms exist. In this phase of the model the content of the propositions is totally unimportant. As a consequence these atoms form principally an unordered set.\(^8\) Only the interrelations between the propositions count. Traditional quantum logic shows narrow similarity with classical logic, however the modular law, which is one of the about 25 axioms that define the classical logic, is weakened in quantum logic. This is the cause of the fact that the structure of quantum logic is significantly more complicated than the structure of classical logic.

3.1.2 Hilbert logic
The set of propositions of traditional quantum logic is lattice isomorphic with the set of closed subspaces of a separable Hilbert space. However there exist still significant differences between this logic system and the Hilbert space. This gap can be closed by a small refinement of the quantum logic system.
Step 1: Define linear propositions (also called Hilbert propositions) as quantum logical propositions that are characterized by a number valued strength or relevance. This number is taken from a division ring.
Step 2: Require that linear combinations of atomic propositions also belong to the logic system. Call such propositions linear propositions. Another name for these propositions is Hilbert propositions.
Step 3: introduce the notion of relational coupling between two linear propositions. This measure has properties that are similar to the inner product of Hilbert space vectors.
Step 4: Close the subsets of the new logic system with respect to this relational coupling measure. The relational coupling measure can have values that are taken from a suitable division ring. The resulting logic system will be called Hilbert logic.
The Hilbert logic is lattice isomorphic as well topological isomorphic with the corresponding Hilbert space.
In this correspondence, Hilbert propositions are the equivalents of Hilbert vectors. General quantum logic propositions are the equivalents of (closed) subspaces of a Hilbert space. The measure of the relational coupling between two Hilbert propositions is the equivalent of the inner product between two Hilbert vectors.

\(^8\) This fact will prove to be the underpinning of the cosmologic principle.
Due to this similarity the Hilbert logic will also feature operators\(^9\).

In a Hilbert logic, linear operators can be defined that have atomic Hilbert propositions as their eigen-propositions. Their eigenspace is countable.

In a Hilbert logic system the **superposition principle** holds. A linear combination of Hilbert proposition is again a Hilbert proposition.


### 3.2 Dynamic model

A dynamic model can be constructed from an ordered sequence of the above static sub-models. Care must be taken to keep sufficient coherence between subsequent static models. Otherwise, the model just represents dynamical chaos. However, some deviation must be tolerated, because otherwise, nothing dynamical will happen in this new dynamic model. The cohesion is established by a suitable correlation vehicle.

#### 3.2.1 Correlation vehicle

The correlation vehicle uses a toolkit consisting of an enumerator generator, a reference continuum and a continuous function that maps the enumerators onto the continuum. The function is a continuous function of both the sequence number of the sub-models and the enumerators that are attached to a member of the selected set of atomic propositions. The enumeration is artificial and is not allowed to structurally add extra characteristics or functionality to the attached proposition. For example, if the enumeration takes the form of a coordinate system, then this coordinate system cannot have a unique origin and it is not allowed to structurally introduce preferred directions. These restrictions lead to an affine space. The avoidance of preferred directions produces problems in multidimensional coordinate systems. As a consequence, in case of a multidimensional coordinate system the correlation vehicle must use a smooth touch. This means, that at very small scales the coordinate system must get blurred. This means that the guarantee for coherence between subsequent sub-models cannot be made super hard. Instead coherence is reached with an acceptable tolerance. In any case a super hard coherence is unwanted.

#### 3.2.2 Isomorphic model

The natural form of the enumeration system can be derived from the lattice isomorphic companion of the quantum logic sub-model. Or it can be derived via a corresponding Hilbert logic system. Here we follow the historical development that was initialized by Birkhoff and von Neumann.

In the third decade of the twentieth century Garret Birkhoff and John von Neumann\(^10\) were able to prove that for the set of propositions in the traditional quantum logic model a mathematical lattice isomorphic model exists in the form of the set of the closed subspaces of an infinite dimensional separable Hilbert space. The Hilbert space is a linear vector space that features an inner vector product. It offers a mathematical environment that is far better suited for the formulation of physical laws than what the purely logic model can provide.

Some decades later Constantin Piron\(^11\) proved that the only number systems that can be used to construct the inner products of the Hilbert vectors must be division rings. Later Solèr’s theorem

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\(^9\) The Hilbert logic does not feature dynamic operators.


\(^11\) C. Piron 1964; _Axiomatique quantique_
formulated this discovery more precisely. The only suitable division rings are the real numbers, the complex numbers and the quaternions\textsuperscript{12}. Quaternions can be seen as combinations of a real scalar and a 3D (real) vector. The number system of the quaternions represent a 1+3D coordinate system. It can be shown that the eigenvalues of normal operators must also be taken from the same division ring.

Since the set of real numbers is multiple times contained in the set of complex numbers and the set of complex numbers is multiple times contained in the set of quaternions, the most extensive isomorphic model is contained in an infinite dimensional quaternionic separable Hilbert space. For our final model we will choose the quaternionic Hilbert space, but first we study what the real Hilbert space model and the complex Hilbert space model provide. What can be done by using a quaternionic Hilbert space can also be done in a real or complex Hilbert space by adding extra structure\textsuperscript{13}.

It appears that a cross product of two quaternionic Hilbert spaces no longer equals a quaternionic Hilbert space\textsuperscript{14}. The HBM does not use such cross products.

The set of closed subspaces of the Hilbert space represents the set of propositions that forms the static quantum logic system. Like the sets of mutually independent atoms in the quantum logic system, multiple sets of orthonormal base vectors exist in the Hilbert space. The base vectors do not form an ordered set. However, a so called normal operator will have a set of eigenvectors that form a complete orthonormal base. The corresponding eigenvalues may provide a means for enumeration and thus for ordering these base vectors. An arbitrary normal operator will in general not fit the purpose of providing an affine eigenspace Usually the eigenvalues of a normal operator introduce a unique n origin and in the case of a multidimensional eigenspace the eigenspace may structurally contain preferred directions. Still, suitable enumeration operators exist. Several things can already be said about the eigenspace of the wanted enumeration operator. Its eigenspace is countable. It has no unique origin. It does not show preferred directions. Its eigenvalues can be embedded in an appropriate reference continuum.

As part of its corresponding Gelfand triple\textsuperscript{15} a selected separable Hilbert space forms a sandwich that features uncountable orthonormal bases and (compact) normal operators with eigenspaces that form a continuum. A reference continuum can be taken as the eigenspace of the corresponding enumeration operator that resides in the Gelfand triple of this reference Hilbert space.

Together with the pure quantum logic model, we now have a dual model that is significantly better suited for use with calculable mathematics. Both models represent a static status quo. The Hilbert space model suits as part of the toolkit that is used by the correlation vehicle.

As a consequence, an ordered sequence of infinite dimensional quaternionic separable Hilbert spaces forms the isomorphic model of the dynamic logical model.

3.2.2.1 Hierarchy

The refinement of quantum logic to Hilbert logic also can deliver an enumeration system. However, the fact that the selected separable Hilbert space offers a reference continuum via its Gelfand triple make the Hilbert space more suitable for implementing the Hilbert Book Model.

\textsuperscript{12} Bi-quaternions have complex coordinate values and do not form a division ring.

\textsuperscript{13} \url{http://math.ucr.edu/home/baez/rch.pdf}

\textsuperscript{14} The result is an abstraction to a real Hilbert space.

\textsuperscript{15} See \url{http://vixra.org/abs/1210.0111} for more details on the Hilbert space and the Gelfand triple. See the paragraph on the Gelfand triple.
The two logic systems feature a hierarchy that is replicated in the Hilbert space. Quantum logic propositions can be represented by closed sub-spaces of the Hilbert space. Atomic Hilbert propositions can be represented by base vectors of the Hilbert space. The base vectors that span a closed sub-space belong to that sub-space. This situation becomes interesting when the base vectors are eigenvectors. In that case the corresponding eigenvalues can be used to enumerate the eigenvectors of the Hilbert space operator and the corresponding eigen atoms of the Hilbert logic operator.

A similar hierarchy can be found when a coherent set of lower order objects forms a building block. Here the lower order objects correspond to atomic Hilbert propositions and to corresponding Hilbert base vectors. The building block corresponds to the quantum logical proposition and to the corresponding closed Hilbert subspace.
3.2.2.2 Correspondences
Several correspondences exist between the sub models:

<table>
<thead>
<tr>
<th>Quantum logic</th>
<th>Hilbert space</th>
<th>Hilbert logic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Propositions:</td>
<td>Subspaces:</td>
<td>Vectors:</td>
</tr>
<tr>
<td>$a$, $b$</td>
<td>$a, b$</td>
<td>$</td>
</tr>
<tr>
<td>atoms</td>
<td></td>
<td>atoms</td>
</tr>
<tr>
<td>$c$, $d$</td>
<td></td>
<td>$</td>
</tr>
<tr>
<td>Relational complexity: $C_{\text{complexity}}(a \cap b)$</td>
<td>Relational complexity: $C_{\text{complexity}}(a \cap b)$</td>
<td>Inner product: $\langle a</td>
</tr>
<tr>
<td>Inclusion:</td>
<td>Inclusion:</td>
<td>Linear combination: $\alpha a + \beta b$</td>
</tr>
<tr>
<td>$(a \cup b)$</td>
<td>$(a \cup b)$</td>
<td>$\alpha</td>
</tr>
<tr>
<td>For atoms $c_i$: $\bigcup_i c_i$</td>
<td>Subspace $\left{ \sum_i \alpha_i</td>
<td>c_i\rangle \right}_{\forall \alpha_i}$</td>
</tr>
</tbody>
</table>

The distribution

$$a(i) \equiv \{(a|c_i\rangle)_{\forall \alpha_i}$$

has no proper definition in quantum logic. It can be interpreted via the Hilbert logic and Hilbert space sub-models.

3.3 Affine space
The set of mutually independent atomic propositions is represented by an orthonormal set of base vectors in Hilbert space. Both sets span the whole of the corresponding structure. An arbitrary orthonormal base is not an ordered set. It has no start and no end. It is comparable to an affine space. However, all or a part of these base vectors can be enumerated for example with rational quaternions. Enumeration introduces an artificial origin and may introduce artificially preferred directions. Thus, in general enumeration will apply to a part of the affine space. As is shown in the last paragraph this enumeration process defines a corresponding normal operator.

The installation of the correlation vehicle requests the introduction of enumerators. The enumeration may introduce an ordering. In that case the attachment of the numerical values of the enumerators to the Hilbert base vectors defines a corresponding operator. It must be remembered that the selection of the enumerators and therefore the corresponding ordering is kind of artificial. The eigenspace of the enumeration operator has no unique origin and is has no natural preferred directions. Thus it has no natural axes. It can only indicate the distance between two or more locations. It will be shown that for multidimensional rational enumerators the distance is not precise. In that case the enumeration represents a blurred coordinate system. Both in the Hilbert space and in its Gelfand triple, the enumeration can be represented by a normal enumeration operator.

3.4 Continuity
3.4.1 Arranging dynamics
Embedding the enumerators in a continuum highlights the interspacing between the enumerators. Having a sequence of static sub-models is no guarantee that anything happens in the dynamic
model. A fixed (everywhere equal) interspacing will effectively lame any dynamics. A more effective dynamics can be arranged by playing with the sizes of the interspacing in a stochastic way. This is the task of an enumerator generator.

3.4.2 Establishing coherence

The coherence between subsequent static models can be established by embedding each of the countable sets in an appropriate continuum. For example the Hilbert space can be embedded in its Gelfand triple. The enumerators of the base vectors of the separable Hilbert space or of a subspace can also be embedded in a corresponding continuum. In the reference Hilbert space that continuum is formed by the values of the enumerators that enumerate a corresponding orthonormal base of the Gelfand triple\(^\text{16}\). For subsequent Hilbert spaces a new appropriate embedding continuum will be used, but that continuum is curved. Next a correlation vehicle is established by introducing a continuous allocation function that controls the coherence between subsequent members of the sequence of static models. It does that by creating a moderate relocation in the countable set of the enumerators that act in the separable Hilbert space by mapping them to the embedding continuum. The relocation is controlled by a stochastic process. In fact the differential of the allocation function is used to specify the small scale working space for this stochastic process\(^\text{17}\). The allocation function also takes care of the persistence of the embedding continuum.

The allocation function uses a combination of progression and the enumerator id as its parameter value. The value of the progression might be included in the value of the id. Apart from their relation via the allocation function, the enumerators and the embedding continuum are mutually independent\(^\text{18}\). For the selected correlation vehicle it is useful to use numbers as the value of the enumerators. The type of the numbers will be taken equal to the number type that is used for specifying the inner product of the corresponding Hilbert space and Gelfand triple.

The danger is then that in general a direct relation between the value of the enumerator of the Hilbert base vectors and the embedding continuum is suggested. An exception is formed by the selected reference Hilbert space. So, for later Hilbert spaces a warning is at its place. Without the allocation function there is no relation between the value of the enumerators and corresponding values in the embedding continuum that is formed by the Gelfand triple. However, there is a well-defined relation between the images\(^\text{19}\) produced by the allocation function and the selected embedding continuum\(^\text{20}\). The relation between the members of a countable set and the members of a continuum raises a serious one-to-many problem. That problem can easily be resolved for real Hilbert spaces and complex Hilbert spaces, but it requires a special solution for quaternionic Hilbert spaces. That solution is treated below.

Together with the selected embedding continuum and the Hilbert base enumeration set the allocation function defines the \textit{evolution} of the model.

3.5 Hilbert spaces

Hilbert spaces represent quantum logical systems and associated Hilbert logic systems.

\(^{16}\) See Gelfand triple
\(^{17}\) The differential defines a local metric.
\(^{18}\) This is not the case for the reference Hilbert space in the sequence. There a direct (close) relation exists.
\(^{19}\) Later these images will be called Qpatches
\(^{20}\) Later the nature of this embedding continuum will be revealed. In later Hilbert spaces the embedding continuum is constituted by potentials.
3.5.1 Real Hilbert space model
When a real separable Hilbert space is used to represent the static quantum logic, then it is sensible to use a countable set of real numbers for the enumeration. A possible selection is formed by the natural numbers. Within the real numbers the natural numbers have a fixed interspacing. Since the rational number system has the same cardinality as the natural number system, the rational numbers can also be used as enumerators. In that case it is sensible to specify a (fixed) smallest rational number as the enumeration step size. In this way the notion of interspacing is preserved and can the allocation function do its scaling task. In the realm of the real Hilbert space model, the continuum that embeds the enumerators is formed by the real numbers. The values of the enumerators of the Hilbert base vectors are used as parameters for the allocation function. The value that is produced by the allocation function determines the target location for the corresponding enumerator in the embedding continuum. The interspacing freedom is used in order to introduce dynamics in which something happens. In fact what we do is defining an enumeration operator that has the enumeration numbers as its eigenvalues. The corresponding eigenvectors of this operator are the target of the enumerator. With respect to the logic model, what we do is enumerate a previously unordered set of atomic propositions that together span the quantum logic system and next we embed the numerators in a continuum. The correlation vehicle takes care of the cohesion between subsequent quantum logical systems. While the progression step is kept fixed, the (otherwise fixed) space step might scale with progression. Instead of using a fixed smallest rational number as the enumeration step size and a map into a reference continuum we could also have chosen for a model in which the rational numbered step size varies with the index of the enumerator.

3.5.2 Gelfand triple
The Gelfand triple of a real separable Hilbert space can be understood via the enumeration model of the real separable Hilbert space. This enumeration is obtained by taking the set of eigenvectors of a normal operator that has rational numbers as its eigenvalues. Let the smallest enumeration value of the rational enumerators approach zero. Even when zero is reached, then still the set of enumerators is countable. Now add all limits of converging rows of rational enumerators to the enumeration set. After this operation the enumeration set has become a continuum and has the same cardinality as the set of the real numbers. This operation converts the Hilbert space into its Gelfand triple and it converts the normal operator in a new operator that has the real numbers as its eigenspace. It means that the orthonormal base of the Gelfand triple that is formed by the eigenvectors of the new normal operator has the cardinality of the real numbers. It also means that linear operators in this Gelfand triple have eigenspaces that are continuaums and have the cardinality of the real numbers. The same reasoning holds for complex number based Hilbert spaces and quaternionic Hilbert spaces and their respective Gelfand triples.

3.5.3 Complex Hilbert space model
When a complex separable Hilbert space is used to represent quantum logic, then it is sensible to use rational complex numbers for the enumeration. Again a smallest enumeration step size is introduced. However, the imaginary fixed enumeration step size may differ from the real fixed enumeration step size. The otherwise fixed imaginary enumeration step may be scaled as a

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21 Later, in the quaternionic Hilbert space model, this freedom is used to introduce space curvature and it is used for resolving the one to many problem.

22 This story also applies to the complex and the quaternionic Hilbert spaces and their Gelfand triples.
function of progression. In the complex Hilbert space model, the continuum that embeds the enumerators of the Hilbert base vectors is formed by the system of the complex numbers. This continuum belongs as eigenspace to the enumerator operator that resides in the Gelfand triple. It is sensible to let the real part of the Hilbert base enumerators represent progression. The same will happen to the real axis of the embedding continuum. On the real axis of the embedding continuum the interspacing can be kept fixed. Instead, it is possible to let the allocation function control the interspacing in the imaginary axis of the embedding continuum. The values of the rational complex enumerators are used as parameters for the allocation function. The complex value of the allocation function determines the target location for the corresponding enumerator in the continuum. The allocation function establishes the necessary coherence between the subsequent Hilbert spaces in the sequence. The difference with the real Hilbert space model is, that now the progression is included into the values of the enumerators. The result of these choices is that the whole model steps with (very small, say practically infinitesimal) fixed progression steps.

In the model that uses complex Hilbert spaces, the enumeration operator has rational complex numbers as its eigenvalues. In the complex Hilbert space model, the fixed enumeration real step size and the fixed enumeration imaginary step size define a maximum speed. The fixed imaginary step size may scale as a function of progression. The same will then happen with the maximum speed, defined as space step divided by progression step. However, if information steps one step per progression step, then the information transfer speed will be constant. Progression plays the role of proper time. Now define a new concept that takes the length of the complex path step as the step value. Call this concept the coordinate time step. Define a new speed as the space step divided by the coordinate time step. This new maximum speed is a model constant. Proper time is the time that ticks in the reference frame of the observed item. Coordinate time is the time that ticks in the reference frame of the observer. Coordinate time is our conventional notion of time.

Again the eigenvectors of the (complex enumeration) operator are the targets of the enumerator whose value corresponds to the complex eigenvalue.

In the complex Hilbert space model the squared modulus of the quantum state function represents the probability of finding the location of the corresponding particle at the position that is defined by the parameter of this function.

If we ignore the case of negative progression, then the complex Hilbert model exist in two forms, one in which the interspacing appears to expand and one in which the interspacing decreases with progression.

### 3.5.4 Quaternionic Hilbert space model

When a quaternionic separable Hilbert space is used to model the static quantum logic, then it is sensible to use rational quaternions for the enumeration. Again the fixed enumeration step sizes are applied for the real part of the enumerators and again the real parts of the enumerators represent progression. The continuum that embeds the enumerators is formed by the number system of the quaternions. The scaling allocation function of the complex Hilbert space translates into an isotropic scaling function in the quaternionic Hilbert space. However, we may instead use a full 3D allocation function that incorporates the isotropic scaling function. This new allocation

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23 In fact coordinate time is a mixture of progression and space. See paragraph on spacetime metric.

24 The situation that expands from the point of view of the countable enumeration set, will contract from the point of view of the embedding continuum of enumerators.
function may act differently in different spatial dimensions. However, when this happens at very large scales, then it conflicts with the cosmological principle. At those scales the allocation function must be quasi isotropic. The allocation function is not allowed to create preferred directions. Now the enumeration operator of the Hilbert space has rational quaternions as its eigenvalues. The relation between eigenvalues, eigenvectors and enumerators is the same as in the case of the complex Hilbert space. Again the whole model steps with fixed progression steps. In the quaternionic Hilbert space model the real part of the quantum state function represents the probability of finding the location of the corresponding particle at the position that is defined by the parameter of this function.

### 3.5.4.1 Curvature and fundamental fuzziness

The spatially fixed interspacing that is used with complex Hilbert spaces poses problems with quaternionic Hilbert spaces. Any regular spatial interspacing pattern will introduce preferred directions. Preferred directions are not observed in nature and the model must not create them. A solution is formed by the **randomization of the interspacing**. Thus instead of a fixed imaginary interspacing we get an average interspacing. This problem does not play on the real axis. On the real axis we can still use a fixed interspacing. The result is an **average maximum speed**. This speed is measured as space step per coordinate time step, where the coordinate time step is given by the length of the 1+3D quaternionic path step. Further, the actual location of the enumerators in the embedding continuum will be determined by the combination of a sharp continuous allocation function and a Quaternionic Probability Amplitude Distribution (QPAD) that specifies the local blur. The form factor of the blur may differ in each direction and is set by the differential of the sharp allocation function. The total effect is given by the convolution of the sharp allocation function and a non-deformed QPAD. The result is a blurred allocation function. A QPAD is a descriptor. It describes the distribution of a set of discrete objects. **The requirement that the cosmological principle must be obeyed is the cause of a fundamental fuzziness of the quaternionic Hilbert model. It is the reason of existence of quantum physics.**

An important observation is that the blur mainly occurs locally. The blur has a very limited extent. The blur corresponds to a potential function that has an unlimited extent, but its influence decreases with distance.

At larger distances the freedom that is tolerated by the allocation function causes **curvature of observed space**. However, as explained before, at very large scales the allocation function must be quasi isotropic. The local curvature is described by the differential of the sharp part of the allocation function.

The continuous part of the allocation function defines the current embedding continuum. In fact it determines the eigenspace of a corresponding operator that resides in the Gelfand triple. Apart from the exceptional case of the reference Hilbert space, the selection of this operator poses a problem. The HBM selects the superposition of all gravitational potentials as the proper choice for subsequent Hilbert spaces. This picture only tells that space curvature might exist. It does not describe the origin of space curvature. For more detailed explanation, please see the paragraph on the enumeration process.

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25 Preferred directions are in conflict with the cosmological principle.
26 Quasi-isotropic = on average isotropic.
3.5.4.2 Discrete symmetry sets
Quaternionic number systems exist in 16 versions (sign flavors\textsuperscript{27}) that differ in their discrete symmetry sets. The same holds for sets of rational quaternionic enumerators and for continuous quaternionic functions. Four members of the set represent isotropic expansion or isotropic contraction of the imaginary interspacing. At large scales two of them are symmetric functions of progression. The other two are at large scales anti-symmetric functions of progression. We will take the symmetrical member that expands with positive progression as the reference rational quaternionic enumerator set. Each member of the set corresponds with a quaternionic Hilbert space model. Thus apart from a reference continuum we now have a reference rational quaternionic enumerator set. Both reference sets meet at the reference Hilbert space. Even at the instance of the reference Hilbert space, the allocation function must be a continuous function of progression.

When the real parts are ignored, then eight sign flavors result. These eight flavors are discerned by their “color” and their handedness.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{sign_flavors.png}
\caption{Sign flavors (discrete symmetries) Colors N, R, G, B, \(\overline{R}\), \(\overline{G}\), \(\overline{B}\), W Right or Left handedness R,L}
\end{figure}

A similar split in quaternionic sign flavors occurs with continuous quaternionic functions. In the picture they are listed as \(\psi^0\)…\(\psi^7\). For each discrete symmetry set of their parameter space, the function values of the continuous quaternionic distribution exist in 16 versions that differ in their discrete symmetry set. Within the target domain of the continuous quaternionic distribution the symmetry set will stay constant.

3.5.4.3 Generations and Qpatterns
The local generator of enumerators can depending on its characteristics generate a certain distribution of randomized enumerators. A Poisson generator combined by a binomial process that is implemented by a suitable 3D isotropic spread function can implement a suitable distribution. That distribution is described by a QPAD. A single QPAD can correspond to

\textsuperscript{27} See paragraph on Qpattern coupling
multiple distributions. The QPAD corresponds to the characteristics of the generator, but depending on its starting condition the generator can generate different distributions.

If generators with different characteristics exist, then several generations of local QPAD’s exist.

HYPOTHESIS 1: For a selected generation the following holds:

Apart from the adaptation of the form factor that is determined by the local curvature and apart from the discrete symmetry set of the QPAD, the natal QPAD’s are everywhere in the model the same.

Therefore we will call the distribution of objects that is described by this basic form of the selected QPAD generation a Qpattern. For each generation, QPAD’s exist in 16 versions that differ in their discrete symmetry set. Each Qpattern has a weighted center location, which is called Qpatch.

At each progression step, all generators produce only a single element of the distribution. This means that each Hilbert space contains only one element of the Qpattern. That element is called Qtarget.

3.5.4.4 Microstate
A Qpattern corresponds with the statistic mechanical notion of a microstate. A microstate of a gas is defined as a set of numbers which specify in which cell each atom is located, that is, a number labeling the atom, an index for the cell in which atom s is located and a label for the microstate.

3.6 Optimal ordering
In the Hilbert space it is possible to select a base that has optimal ordering for the eigenvalues of a normal operator. Optimally ordered means that these sections are uniformly distributed and that stochastic properties of these sections are the same. In the Hilbert logic system a similar selection is possible for the set of mutually independent atomic propositions. There the atoms are enumerated by the same set of rational quaternionic values.

For the Hilbert spaces it means that in the Gelfand triple a corresponding operator exist whose eigen space maps onto the well-ordered eigenspace of the operator that resides in the Hilbert space. We will call these operators “reference operators”.

3.7 The reference Hilbert space
The reference Hilbert space is taken as the member of the sequence of Hilbert spaces at the progression instance where the allocation function is a symmetric function of progression that expands in directions that depart from the progression value of the reference Hilbert space. At large and medium scales the reference member of the sequence of quaternionic Hilbert spaces is supposed to have a quasi-uniform distribution of the enumerators in the embedding continuum. This is realized by requiring that the eigenspace of the enumeration operator that acts in the Gelfand triple of the zero progression value Hilbert space represents the reference embedding continuum.

With other words, at this instance of progression, the rational quaternionic enumeration space is flat. For the reference Hilbert space the isotropic scaling function is symmetric at zero

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28 See paragraph on generations
30 quasi-uniform = on average uniform.
progression value. Thus for the reference Hilbert space at the reference progression instance the
distribution of the enumerators will realize a densest packaging\textsuperscript{31}.

\textit{For all subsequent Hilbert spaces the embedding continuum will be taken from potentials that}
\textit{are generated in earlier Hilbert spaces.}

\textsuperscript{31} The densest packaging will also be realized locally when the geometry generates black regions.
3.8 The embedding continuum
For the reference Hilbert space the embedding continuum is taken from a flat normal location operator that resides in its Gelfand triple of the Hilbert space. That continuum is the virginal reference continuum.
For subsequent Hilbert spaces the embedding continuum for fermions is formed by the superposition of all potentials that are generated by objects that lived in previous Hilbert spaces. The result is a curved version of the virginal reference continuum.
Bosons use an embedding continuum that is formed by the potentials that are emitted locally in previous Hilbert spaces.

3.9 The cosmological principle revisited
The enumeration process attaches an artificial content to the each of the members in the unordered set of atomic propositions. The unrestricted enumeration with rational quaternions generates an artificial origin and it generates artificial preferred directions that are not present in the original set of atomic propositions. The correlation vehicle is not allowed to attach this extra functionality to the original propositions. However, the vehicle must still perform its task to establish cohesion between subsequent sub-models. One measure is to turn the enumeration space into an affine space. An affine space has no origin. The next measure is to randomize the enumeration process sufficiently such that an acceptable degree of cohesion is reached and at the same time a quasi-isotropy of this affine space is established. This measure requires the freedom of some interspacing, which is obtained by assigning a lowest rational number. In principle, a lowest rational number can be chosen for the real part and a different smallest base number can be chosen for the imaginary part. This choice defines a basic notion of speed. The resulting (imaginary) space is on average isotropic. The randomization results in a local blur of the continuous function that regulates the enumeration process.
The result of these measures is that roughly the cosmologic principle is installed. Thus, in fact the cosmological principle is a corollary of the other two starting points.
However, according to this model, apart from the low scale randomization, the universe would be quite well ordered. After a myriad of progression steps this medium to large scale ordering is significantly disturbed.
Looking away from any point in universe is in fact looking back in proper time. Looking as far as is physically possible will open the view at a reference member of the Hilbert Book Model. This reference member represents a densest and well-ordered packaging. This will result in a uniform background at the horizon of the universe.
The well-known microwave background radiation is not fully uniform and is expelled by members that are close to the densest packaged member.
4 The HBM picture
In the advance of quantum physics two views on quantum physics existed.

4.1 The Schrödinger picture
The Schrödinger picture describes a dynamic implementation in Hilbert space in which the quantum states carry the time dependence. The operators are static.

4.2 The Heisenberg picture
The Heisenberg picture describes a dynamic implementation in Hilbert space in which the operators (represented by matrices) carry the time dependence. The quantum states are static.

4.3 The Hilbert Book Model picture
In the HBM picture an ordered sequence of Hilbert spaces and their corresponding Gelfand triples are used. Each of these spaces represent a static status quo.
In the HBM the whole Hilbert space carries the proper time dependence. Both the enumeration operator and the patterns that represent the quantum state functions depend on the progression parameter. For each Qpattern the Hilbert space contains only the actual element. Thus if only a single Hilbert space is considered, then the Qpatterns cannot be recognized. The virtual elements are not actually present in any member of the sequence of Hilbert spaces. The virtual elements can only exist as place holders. However, the potentials of Qpatterns act as traces of the existing and passed Qpatterns. They affect the embedding continuum that is formed by the potentials of particles that existed in the past.
The correlation vehicle ensures the cohesion between subsequent Hilbert spaces and takes care of the persistence of the emitted potentials.
The potentials survive the extinction of the sources that created them. If they do not compensate each other, then they exist forever. Gravitation potentials do not compensate each other. This fact renders the HBM into a never ending story.

4.4 The operational picture
In the operational picture only a single Hilbert space and its Gelfand triple are used. An operator that resides in the Hilbert space acts as the reference operator. It has an equivalent in the Gelfand triple and the eigenspaces of these operators map onto each other in an orderly fashion. Together with the Hilbert space and Gelfand triple these reference operators represent the static part of the model.
The eigenvalues of the reference operators represent the progression value in their real part.
In the Hilbert space and in its Gelfand triple the correlation vehicle supports the existence of progression dependent operators. This concerns a stochastically operating operator in the Hilbert space and for each potential type a compact normal operator that installs the temporal behavior of these potentials.
The correlation vehicle uses the eigenspaces of the reference operators as its parameter spaces. It uses eigenspaces of other operators as its target space. As a consequence these operators depend on progression.
This picture comes close to the Heisenberg picture, but it does not keep states static.

4.5 Discussion
Obviously the Hilbert Book Model selects the HBM picture. According to the feel of the author this picture offers the cleanest view.
The Hilbert space and Gelfand triple hulls together with the reference operators form the static part of both the HBM picture and the operational picture. In the HBM picture this static part is represented by the reference Hilbert space, its Gelfand triple and the reference operators. There is one small exception to this static behavior: the eigenvalues of the reference operators represent the progression value in their real parts. Not all of the eigenvectors of the Hilbert space reference operator are constantly in use. Annihilation and (re)creation events regulate this usage. The models only use a huge subspace of the Hilbert space(s). Enumeration is considered to be an artificial action and the enumerators must be seen as to be embedded in an affine space.
5 Fields
Field theory exists independent of what it describes. It describes fields varying from fluid dynamics, via electromagnetism to gravitation. You can describe scalar fields and vector fields separately or combined in a quaternionic field. Apart from that tensor fields exist. Fields can be seen as variations of a continuum such as photons and gluons. Other types of fields can be seen as representing the distribution of the density of discrete objects and the corresponding current densities. Fields can also represent the potentials of these distributions of discrete objects. Examples of this last category are gravitation fields and electrostatic fields. The type of the potential is set by its Green’s function. All these fields have many similarities and some differences. Only in case of density distributions and corresponding potentials the fields describe the same objects, which form the discrete distribution that underlies these fields. The elements of the distributions are treated as anonymous objects. However, it is also possible to enumerate them and allow each individual object to possess a series of properties. The elements can also share properties. These properties will characterize the distribution and the corresponding fields.
6 The enumeration process

It is not yet clear how Qpatterns will be shaped. This information can be derived from the requirements that are set for the correlation vehicle. We will start with a suggestion for the enumeration process that for this vehicle will lead to the wanted functionality.

HYPOTHESIS 2: At small scales the enumeration process is governed by a Poisson process. The lateral spread that goes together with the low scale randomization of the interspacing plays the role of a binomial process. The combination of a Poisson process and a binomial process is again a Poisson process, but locally it has a lower efficiency than the original Poisson process. The binomial distribution is implemented by a continuous 3D spread function.

As an example, we consider the special situation that this combination produces a 3D normal distribution. For a large number of enumerator generations the resulting Poisson distribution resembles a Gaussian distribution\(^{32}\). If the generated enumerators are considered as charge carriers, then the corresponding potential has the shape of an Error function divided by \(r\). Already at a short distance from its center location the potential function starts decreasing with distance \(r\) as a \(1/r\) function\(^{33}\).

6.1 Gravity and electrostatics

Potentials depend on the Green’s function that is used to convert the corresponding density distribution into a potential function. Apart from their Green’s function, gravity and electrostatics can be treated by similar equations.

<table>
<thead>
<tr>
<th>Description</th>
<th>Gravity</th>
<th>Electrostatics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Field (g = -\nabla \varphi)</td>
<td>(E = -\nabla \varphi)</td>
<td></td>
</tr>
<tr>
<td>Force (F = mg)</td>
<td>(F = QE)</td>
<td></td>
</tr>
<tr>
<td>Gauss law (\langle \nabla, g \rangle = -4\pi G \rho)</td>
<td>Gauss law (\langle \nabla, E \rangle = \frac{Q}{\varepsilon})</td>
<td></td>
</tr>
<tr>
<td>Poisson law (\Delta \varphi = \langle \nabla, \nabla \varphi \rangle) (\Delta \varphi = 4\pi G \rho)</td>
<td>Poisson law (\Delta \varphi = -\frac{Q}{\varepsilon})</td>
<td></td>
</tr>
<tr>
<td>Greens function (-\frac{\rho(r')}{</td>
<td>r - r'</td>
<td>})</td>
</tr>
<tr>
<td>Single charge potential (\varphi = -\frac{4\pi G m}{</td>
<td>r</td>
<td>})</td>
</tr>
<tr>
<td>Single charge field (g = -\frac{4\pi G m}{</td>
<td>r</td>
<td>^2} r)</td>
</tr>
<tr>
<td>Two charge force (F = -\frac{4\pi G m_1 m_2}{</td>
<td>r</td>
<td>^3} r)</td>
</tr>
<tr>
<td>Mode attracting</td>
<td>Mode repelling</td>
<td></td>
</tr>
</tbody>
</table>

The table shows that the Greens functions of both fields differ in sign. For the gravitation potential the Green’s function is charged with the local “charge” density \(\rho(r')\). For the electrostatic potential the Green’s function is charged with a (constant) electric charge \(Q\). The Yukawa potential\(^{34}\) uses a short range Green’s function:

\(^{32}\) http://en.wikipedia.org/wiki/Poisson’s_equation#Potential_of_a_Gaussian_charge_density

\(^{33}\) http://farside.ph.utexas.edu/teaching/em/lectures/node28.html through node31

\(^{34}\) http://en.wikipedia.org/wiki/Yukawa_potential
In this example we use the gravitational Green’s function. Since the items are carriers with charge $\rho_i$, the density distribution $\rho_i(r)$ correspond to a potential $\varphi(r)$. Every item contributes a term $\varphi_i(r - r_i) = \frac{-\rho_i}{|r - r_i|}$

$$\varphi(r) = \sum_i \varphi_i(r - r_i) = \sum_i \frac{-\rho_i}{|r - r_i|}$$

Example: If there is a static spherically symmetric Gaussian charge density

$$\rho_g(r) = \frac{\rho_c}{\sigma^3 \sqrt{2\pi}} \exp\left(\frac{-r^2}{2\sigma^2}\right)$$

where $\rho_c$ is the total charge, then the solution $\varphi(r)$ of Poisson’s equation, $\nabla^2 \varphi = \rho_g$ is given by

$$\varphi(r) = \frac{\rho_c}{4\pi \varepsilon} \text{erf}\left(\frac{r}{\sqrt{2} \sigma}\right) = \frac{-1}{4\pi \varepsilon} \int \frac{\rho_g(r')}{|r - r'|} d^3r'$$

where $\text{erf}(x)$ is the error function.

Note that, for $r$ much greater than $\sigma$, the erf function approaches unity and the potential $\varphi(r)$ approaches the point charge potential

$$\varphi(r) \approx \frac{-\rho_c}{4\pi \varepsilon r}$$

as one would expect. Furthermore the $\text{erf}$ function approaches 1 extremely quickly as its argument increases; in practice for $r > 3\sigma$ the relative error is smaller than one part in a thousand.

$$\frac{-\rho(r')}{|r - r'|} \exp(-\mu|r - r'|)$$

### 6.1.1 Bertrand’s theorem

Now we remember Bertrand’s theorem.\(^ {35} \):

Bertrand’s theorem states that only two types of central force potentials produce stable, closed orbits:

1. an inverse-square central force such as the gravitational or electrostatic potential

$$V(r) = \frac{-k}{r} \quad (1)$$

and

2. the radial harmonic oscillator potential

\(^ {35} \)http://en.wikipedia.org/wiki/Bertrand's_theorem.
\[ V(r) = \frac{1}{2} k r^2 \]  

According to this investigation it becomes acceptable to assume that the undisturbed shape of the Qpatterns can be characterized as 3D Gaussian distributions\(^{36}\). Since this distribution produces the correct shape of the gravitation potential, \textit{it would explain the origin of curvature.}\(^{36}\)

6.2 The internal dynamics of Qpatterns

A Qpattern is generated in a pace of one element per progression step. A corresponding allocation operator that resides in the Hilbert space will reflect these Qtags in its eigenspace. During each progression step an increment is added to the static potential function. This is performed by transmitting a message to the environment of the Qtag. The Qtag is the element, which is currently active. For isotropic Qtags\(^ {37}\) the message is sent in the form of a 3D tsunami-like spherical wave. Depending on the discrete symmetric difference with the embedding continuum to which the building block couples, the wave is either spherical or anisotropic. The wave folds the embedding continuum. This is the mechanism, which is used in order to transport the message. By repeating that message for every Qtag a constant stream of messages is produced that together form a wave pattern that oscillates with ultra-high frequency\(^ {38}\). If the Qpattern does not move, then at some distance the situation looks as if a “sine” wave is transmitted from a single source. If the Qpattern oscillates, then that ultra-high frequency wave gets a lower frequency amplitude and a phase modulation.

\textit{For anisotropic Qtags the message is transmitted by an anisotropic wave.}

The waves curve the embedded continuum. The effect on local curvature diminishes with distance from the Qtags. This effect is described by the corresponding potential function\(^ {39}\). The sharp continuous part of the allocation registers the effect on the embedding continuum and stores this data for the creation of the next version of the embedding continuum. A corresponding operator that resides in the Gelfand triple will reflect the embedding continuum in its eigen space.

\(^{36}\) It might be clear that in this way an explanation is given for the effect of a Qpattern on local curvature.

\(^{37}\) See Discrete symmetry sets.

\(^{38}\) That frequency is determined by the progression step size.

\(^{39}\) See: Waves that spread information.
6.3 Qpatterns

Qpatterns of a given generation have a fixed natal shape. The Qpattern is a dynamic building block. Qpatterns extend over many progression steps. A Qtarget lasts only during a single progression step.

A Qpattern is a coherent collection of objects that are distributed in space. This coherent distribution can be described by two density distributions. The first one is a scalar function that describes the distribution of the density of the spatial locations. The second one describes the corresponding current density distribution. It administers the displacement since the last element generation. The distribution of discrete objects correspond to potential functions. For each suitable Green’s function a corresponding potential function exists. In this way the scalar density distribution correspond to a set of scalar potential functions and the current density distribution corresponds to a set of 3D vector potential functions.

A direct conversion from density distribution to a potential function uses a dedicated Green’s function. Each suitable Green’s function gives a corresponding potential function. Each natal Qpattern corresponds to a plan. Not all enumerations that are required for generating the planned Qpattern must be used during the life of the actual Qpattern. Per progression step the generator creates only a single member of the Qpattern and that member is replaced in the next step by another member. Qpatterns contain one actual member and for the rest it consists of virtual members. The actual member is a location where an event can happen. This actual element is called Qtarget. That event may be the annihilation of the Qpattern. After that the generation of new elements stops.

Each realization of a Qpattern corresponds to a micro-path that runs along step stones. The Qpatch may move and/or oscillate. The actual distribution of Qtargets spreads along the actual path of the building block. This actual path differs from the planned micro-path. The contributions to the potentials are transmitted by Qtargets at the halts along the actual path. The natal Qpattern can be described by a temporal function that produces a stochastic spatial location at every subsequent progression interval.

Since the collection is generated in a rate of one element per progression step, the contributions to the potential functions are also generated in that rate and at the locations of the actual elements, which are the Qtargets. It is shown above that the potential functions are generated with the help of spherical waves\(^{40}\) that with light speed move away from the locations of the elements that generated them.

These waves are emitted with a fixed ultra-high frequency. In the HBM no higher frequency exists.

Only if the Qpattern stays fixed at a single location in an non-curved part of the embedding continuum, then that location will see the generation of a virtual Qpattern that takes a shape that approaches the planned target distribution. It will take a huge number of progression steps to reach that condition.

A moving Qpattern will be spread along the path of the corresponding building block. A move of the building block may affect the life of the realizable part of the Qpattern\(^{41}\).

6.3.1 Micro-paths

Qpatterns are representatives of nature’s building blocks. They are coherent collections of lower order objects that each can be considered as a location where the building block can be. These

\(^{40}\) For anisotropic Qpatterns the message is transmitted by an anisotropic wave.

\(^{41}\) http://en.wikipedia.org/wiki/Particle_decay
objects are generated in a rate of one element per progression step. The situation can be interpreted as if the building block hops from step stone to step stone. These micro-movements form a micro-path in the form of a random string. At each arrival at a step stone the building block emits a message. That emission contributes to the potentials of the building block. The emission does not affect the natal Qpattern. However, it may affect the actual Qpattern. In order to stay at the same position, a step in a given direction will on average be followed by a step in the reverse direction. Otherwise the average location will move away or the pattern will implode or explode. This means that the particle moves along a micro-path and this path is characterized by quasi-oscillations. Similarly the path may show quasi-rotations.

6.3.2 Qpattern history
A Qpattern can be created and it can be annihilated. If a Qpattern is annihilated, then the generator stops producing new elements. Thus, also the generation of new potential waves will stop. However, existing potential waves will keep proceeding. The last generated wave closes a train of previous waves. This edge moves away with light speed. A previously “static” potential will be replaced by a dynamic phenomenon. The annihilation frees the identifier of the Qpattern and makes it available for reuse. In this way the identifiers of the Qpatterns refer to their virgin equivalents that were born in the reference Hilbert space.

Looking away is looking back in proper time. Looking back as far as is possible is looking back at the virginal state of the historic Qpattern. Looking as far away as is possible is looking at the virginal state. In this way a Qpattern can be coupled both to its past and to its distant background. On the other side this means that the transmitted potential waves from this virgin state reach the current local Qpattern.

The superposition of all transmitted potentials that contributed in the past to the local potential results in huge background potential that acts as a (curved) embedding continuum (for fermions).

6.3.3 Fourier transform
A QPAD that has the form of a QPAD of a Gaussian distribution has a Fourier transform that also has the form of a QPAD of a Gaussian distribution. However, the characteristics of the distributions will differ.

The QPAD of a coupled Qpattern is compact in configuration space and wide spread in canonical conjugated space.

The Fourier transform of a QPAD is its characteristic function\(^\text{42}\). It is again a quaternionic function. It is the characteristic function of the underlying distribution of discrete objects.

6.4 Qtargets
In fact the actual elements, called Qtargets, are represented by three different rational quaternions. These rational quaternions define locations or displacements relative to an embedding continuum. That continuum might be curved.

1. The real part of the first quaternion represents progression. Its imaginary part acts as the identifier of the element. For each Qtarget, it plays the role of the corresponding parameter. It equals the imaginary part of the parameter at zero progression value. The Qtargets walk through a path as a function of progression.

\(^{42}\) http://en.wikipedia.org/wiki/Characteristic_function_(probability_theory)
2. The imaginary part of the second quaternion defines the location of the Qtarget. Its real part specifies the local density. It also acts as the relevance factor of the corresponding Hilbert proposition.

3. The imaginary part of the third quaternion defines the displacement. The discrete symmetry set of this quaternion constitutes the charge of the Qtarget. Apart from the discrete symmetry set this third quaternion contains no new information. It contains the displacement of the last Qtarget to the current Qtarget.

The planned and the actual distribution can be described by a charged carrier density distribution and a corresponding current density distribution. Via appropriate Green’s functions these density distributions correspond to a scalar potential and a corresponding vector potential. The potentials reflect the transmittance of the existence and the discrete properties of the Qtargets.

Since Qtargets are elements of Qpatterns and their identifier is also Qtarget of a Qpattern that existed at zero progression value, the two patterns are connected as well.

6.5 New mathematics

The idea that spherical waves implement the contribution that Green’s functions add to the potential functions, represents new mathematics. This is quite clear for the gravitational potential. The emitted wave folds and thus curves the embedding continuum. In this way curvature can be explained.

It is less clear for other potentials. Especially the encoding of charge information in the emitted information is not yet properly established. This encoding uses the difference in discrete symmetry between the Qtarget and the embedding continuum.

6.5.1 Waves that spread information

A Qtarget exists during a single progression step. Even when they belong to the same Qpattern will subsequent Qtargets be generated at different locations. If the Qtarget is generated, then in the embedding continuum the Qtarget corresponds to a 3D tsunami-like wave that has its source at the location of the Qtarget. After the disappearance of the Qtarget the wave keeps spreading out. The waves that belong to preceding Qtargets and the waves that belong to other Qpatterns will interfere with that wave. If the Qpatch is stationary, then at sufficient distance it will look as if the waves are generated by a single source. The train of emitted waves will resemble an ultra-high frequency oscillating wave. The amplitude of this oscillating wave decreases with distance from the source. For isotropic spherical waves, this is the reason of the contribution of the term $\frac{Q}{|r-r'|}$ to the static potential integral.

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43 For anisotropic Qpatterns the message is transmitted by an anisotropic wave.
44 See elementary particle properties
Example: Generation process with one element per progression instant. Here we use the electrostatic Green’s function.

- Poisson process coupled to a binomial process
  - Binomial process implemented by a 3D spread function
  - Produces a 3D distribution
- Which approaches a 3D Gaussian distribution
  - \( \rho_f(r) = \frac{Q}{\sigma^3\sqrt{2\pi}} \exp \left( \frac{-r^2}{2\sigma^2} \right) \)
- This corresponds to a scalar potential of the form
  \[
  \phi(r) = \frac{Q}{4\pi\varepsilon} \text{erf} \left( \frac{r}{\sqrt{2}\sigma} \right) = \frac{1}{4\pi\varepsilon} \int \frac{\rho_f(r')}{|r - r'|} d^3r' \approx \frac{Q}{4\pi\varepsilon r} (r \gg \sigma)
  \]
- And a vector potential of the form
  \[
  \mathbf{A} = \frac{Q}{4\pi\varepsilon r} (r \gg \sigma)
  \]
  - Charge \( Q \) is represented by the discrete symmetry set of the carrier

If an event occurs, then the generator stops generating Qtargets for this Qpattern in the configuration space. However the waves that have been started will proceed spreading over the embedding configuration space.

When the local generator stops generating then no new waves will be formed. The last wave and foregoing waves proceed spreading with light speed.

The fact that the waves keep spreading is a consequence of the characteristics of the correlation vehicle, which is implemented by the enumerator generating mechanism. That mechanism also regenerates the embedding continuum for use in the next progression step.

The scalar potential functions and vector potential functions that correspond to the charge and current density distributions reflect the transmission of the information that is transmitted by the Qtargets.

The potential functions reveal the existence of the Qpattern. The potentials can be observed without affecting the Qpattern.

### 6.5.2 Waves that shrink space

The 3D tsunami-like spherical waves appear to shrink space. The local shrinkage diminishes when the distance from the source increases. As a consequence the influence diminishes as \( 1/r \).

Also this fact is a consequence of the actions of the enumerator generating mechanism.

All quaternionic quantum state functions are fields (they are quaternionic probability amplitude distributions) that extend over a limited region of the embedding space. Their potentials extend over a part of universe that falls within the information horizon of the corresponding particles.
When a particle annihilates, then the information about its existence keeps spreading. However, no new information is generated. The potential functions act as traces of Qpatterns. The 3D tsunami-like wave that spreads this information appears to shrink the space where it passes. However its influence diminishes with distance. For spherical waves the influence diminishes with distance $r$ as $1/r$.

As long as a particle lives, it keeps sending these tsunami-like waves. This might be the way that gravitation/space curvature is implemented.
6.5.3 Spreading electric charge information
The Qtarget also contains information about the electric charge of the corresponding particle. The process corresponds to the way that gravitational information is transmitted. In this case not the existence and local density, but the charge is transmitted. The charge is determined by the discrete symmetry of the Qtarget in comparison to the discrete symmetry of the embedding continuum. Only the symmetries of the imaginary parts that encode displacement are relevant.

6.5.4 Huygens principle
The correlation vehicle applies the Huygens principle. It means that in every progression step, every location on a wave front can be seen as a source of a new wave. The Huygens principle acts differently for waves that operate in different numbers of dimensions. The Green’s function differs accordingly. For odd dimensions the mechanism works in the commonly understood way.

6.6 Quasi oscillations and quasi rotations
In order to keep the distribution coherent on average in each dimension any step in positive direction must be followed by a step in negative direction. With other words a kind of quasi oscillation takes place. This oscillation can be synchronous to a reference or it can be asynchronous. This (a)synchrony may differ per dimension. In a similar way a quasi-rotation can exist.
A special kind of coupling/interaction between fields can be the result of these induced quasi oscillations and or quasi rotations, where distant sources of oscillating potentials induce this coupling with local oscillations.

6.7 Distant Qtargets
The Qtargets of distant Qpatterns also send messages that encode their presence in 3D tsunami-like waves. These waves contribute to a huge local potential. This effect represents the origin of inertia. Together the potentials of all Qpatterns constitute a local potential that can act as an embedding continuum.
It is a bit strange that electrostatic potential plays no role in this effect. Probably the electric potential is shielded in a similar way as the Yukama potential, but then on a cosmic rather than on an nuclear scale.
In this respect http://en.wikipedia.org/wiki/Common_integrals_in_quantum_field_theory may show interesting.

6.8 Spurious elements
The distributions need not be generated in coherent distributions as is the case with Qpatterns. Coherent distributions correspond to potential functions that are constructed dynamically in a large series of steps. In extreme cases the distribution consists of a single element that pops up and disappears in a single progression step. During its existence the element still produces a tsunami-like signal in the form of a spherical wave that travels in the embedding continuum. Again this wave causes a local curvature. In large numbers these spurious elements may cause a noticeable effect.

45 An interesting discussion is given at: http://www.mathpages.com/home/kmath242/kmath242.htm
46 See Inertia
47 For anisotropic elements the message is transmitted by an anisotropic wave.
6.9 The tasks of the generator

The primary task of the generator is the generation of Qtargets that are part of Qpatterns. After the generation and vanishing of the Qtarget the generator takes care of the transmission of the information about the generation incident over the space in which the Qtarget was produced. This is done in the form of the described 3D tsunami-like waves. This is the second task of the generator. When the generator stops generating Qtargets for the current Qpattern, then it does not transmit new information but the correlation mechanism keeps supporting the existing flow of information. This means that a third task of the correlation mechanism is the care for the survival of the embedding continuum when the Qtargets vanish.

The transmission of incident information causes space curvature. The sharp part of the generator function describes the influence of space curvature. It does this via its differential which specifies a local metric.

Apart from describing the curvature the correlation mechanism also recreates at every progression step the corresponding embedding continuum.
7 Geometric model
The geometric model applies the quaternionic Hilbert space model. From now on the complex Hilbert space model and the real Hilbert space model are considered to be abstractions of the quaternionic model. It means that the special features of the quaternionic model bubble down to the complex and real models. For example both lower dimensional enumeration spaces will show blur at small enumeration scales. Further, both models will show a simulation of the discrete symmetry sets that quaternionic systems and functions possess. This can be achieved with spinors and Dirac matrices or with the combination of Clifford algebras, Grassmann algebras and Jordan algebras

The real and complex models suit in situations where phenomena can be decoupled from the dimensions in which they appear.
At large scales the model can properly be described by the complex Hilbert space model. After a sufficient number of progression steps, at very large scales the quaternionic model is quasi isotropic.
We will place the reference Hilbert space at zero progression value. This reference Hilbert space can be a subspace of a much larger Hilbert space. However, in the reference Hilbert subspace a state of densest packaging must reside.
Quaternionic numbers exist in 16 discrete symmetry sets. When used as enumerators, half of this set corresponds with negative progression and will not be used in this geometric model. As a consequence we will call the Hilbert space at zero progression value the start of the model. This model does not start with a Big Bang. Instead it starts in a state that is characterized by densest packaging of the Qpatches. This reference sub-model is well-ordered.

7.1 Quaternionic distributions
Quaternionic distributions consist of a real scalar distribution and an imaginary 3D vector distribution.
It is the sum of a symmetric distribution and an asymmetric distribution.
The complex Fourier transform of a symmetric (complex) function is a cosine transform. It is a real function.
The complex Fourier transform of an anti-symmetric (complex) function is a sine transform. It is an imaginary function.
This cannot directly be translated to quaternionic functions. The simplest solution is to consider the symmetric parts and asymmetric parts separately. An asymmetric quaternionic function is always anisotropic. A symmetric function can be isotropic.
As shown before the continuous quaternionic distributions can be interpreted as descriptors of the density distribution of a coherent distribution of discrete objects. However the potential functions that can be derived from coherent distributions of discrete objects are also quaternionic functions.
In the HBM these associated potentials can be considered to be generated dynamically.

7.2 RQE's
In principle the base vectors of the Hilbert space can be enumerated by members of a countable affine space. Here we concentrate on a huge subspace in which the base vectors are enumerated by rational quaternions. The huge subspace is covered by a large number of small dedicated subspaces that all are identified by a Qpatch region.

48 See: http://math.ucr.edu/home/baez/rch.pdf
The ordering and the corresponding origin of space become relevant when an observer object considers one or more observed objects. The real parts of the enumerators define progression. In conventional physics progression conforms to proper time. As a consequence according to our model, the equivalent of proper time steps with a fixed step.

RQE stands for **Rational Quaternionic Enumerator**. This lowest geometrical level is formed by the enumerators of a selected base of a selected member of the sequence of Hilbert spaces. The selected base vectors represent the atoms of the Hilbert logic system. In this level, the embedding continuum plays a secondary role. The sequence number corresponds with the progression value in the real part of the value of the RQE. In principle the enumerators enumerate a previously unordered set.

The dedicated subspaces are spanned by eigenvectors whose eigenvalues form the elements of **Qpatterns**. Qpatterns are identified by a **Qpatch** and by a **Qtarget**, which is the currently actual element. All other elements of the Qpattern and all other vectors of the dedicated subspace are virtual. Virtual means: “reserved, but currently not in use”.

For the reference Hilbert space its Gelfand triple delivers the reference continuum. For later Hilbert spaces the role of the reference continuum is taken over by one of the potentials. That potential is the gravitation potential.

### 7.2.1 Reference Hilbert space

A zero value of the real part of an RQE indicates its role in the reference Hilbert space. In the reference Hilbert space the Qpatches are well ordered and embedded in a reference continuum that is taken from the eigenspace of a reference operator that resides in the Gelfand triple of that reference Hilbert space.

The considered huge subspace of the selected reference member of the sequence of Hilbert spaces represents a state of densest packaging of the RQE’s. This means that in this subspace of the selected Hilbert space a normal allocation operator exists whose discrete and countable eigenspace has eigenvalues that are RQE’s, while in the Gelfand triple of this Hilbert space an allocation operator exists whose continuous eigenspace embeds the values of these RQE’s in a well ordered and dense way.

Due to this restriction the RQE-space is not afflicted with splits and ramifications.

Thus, both the RQE’s and the reference continuum are taken from the eigenspace of a corresponding normal allocation operator. These operators will be called reference operators. The Qpatches are linear combinations of a coherent set of RQE’s that correspond to eigenvectors, which together span the dedicated subspace. This dedicated subspace corresponds to a building block.

In the reference Hilbert space, the Qpatch is the average value of all RQE’s that belong to the building block. The RQE’s that belong to a building block can also be taken relative to the Qpatch of the building block. In that case they will be called relative RQE’s.

All aspects of the reference Hilbert space also occur in subsequent Hilbert spaces. In this respect the reference Hilbert space represents the static part of all Hilbert spaces.

### 7.2.2 Later Hilbert spaces

In later Hilbert spaces the embedding continuum is no longer flat as it is in the reference Hilbert space. Here the embedding continuum is formed by superposed potentials and is represented by the eigenspace of a dedicated operator that resides in the Gelfand triple. The corresponding potential is a special type. It is the gravitation potential.

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[^49]: http://en.wikipedia.org/wiki/Quaternion_algebra#Quaternion_algebras_over_the_rational_numbers
Due to space curvature, the value of the corresponding Qtarget will no longer correspond to the original RQE value in the flat continuum. It will become a function of the combination of the imaginary part of the original RQE and the new progression value. Thus RQE’s whose real parts are zero reside in the reference Hilbert space. Other RQE’s act as parameters for Qtargets.

The Qpatch of the building block will become the expectation value of the Qtargets. Thus, it also no longer equals the average value of the original RQE’s.

Qtargets are locations in a curved space. Only in the reference Hilbert space, that space is flat.

HYPOTHESIS 3: At the start of the life of the considered huge subspace the HBM used only one discrete symmetry set for its lowest level of geometrical objects. This discrete symmetry set is the same set that characterizes the reference continuum. This situation stays throughout the history of the model. This set corresponds with the set of eigenvalues of an RQE allocation operator that resides in the reference quaternionic Hilbert space model.

For each building block, in the reference Hilbert space one of the RQE’s becomes the actual element and will be called Qtarget. In each subsequent Hilbert space another RQE will be selected whose image becomes the Qtarget. The RQE selection occurs via a random process. In subsequent Hilbert spaces a new eigenvalue of the reference allocation operator becomes the parameter of the new Qtarget of the building block. These Qtarget parameters will differ in a random way from the original RQE. Thus Qtargets are continuous functions ($\phi$) of the corresponding RQE’s and they are random functions ($\psi$) of the virgin Qtarget of the building block. The convolution ($\mathcal{P}$) of the continuous function and the random function determine the location of the current Qtarget.

\[ \mathcal{P} = \phi \circ \psi \]  

The assignment of the value of the random function ($\psi$) occurs according to a given plan. Difference exists between the planned building block and the actually realized building block.

### 7.3 Potentials

RQE’s are the identifiers of the elements of a Qpattern. They are parameters of Qtargets. Qpatterns exist during a series of subsequent Hilbert spaces. They represent nature’s building blocks. The RQE’s reside in the reference Hilbert space, which occurred in the past. They also reside in the eigenspace of the current reference operator. For that eigenspace the real part of the RQE’s reflect the current progression value. The Qpatches reside in each of the subsequent Hilbert spaces. Qpatches are linear combinations of the elements of a Qpattern. They represent the expectation values of the Qtargets. The elements of the Qpatterns correspond to base vectors of dedicated Hilbert subspaces. The Qtargets emit contributions to the potentials of the Qpatterns. Potentials depend on their Green’s function. Apart from that, two kinds of potentials exist: scalar potentials and vector potentials. Potentials of the same type superpose. The potentials that possess sufficient reach may together add up to huge local potentials\(^{50}\). Locally the superposition of scalar potentials constitute a curved continuum that can be used to embed localizable objects. This continuum installs inertia for the embedded Qpatterns.

For all continuous quaternionic functions and for each discrete symmetry set of its parameter space, the function exists in 16 different discrete symmetry sets for its function values. In the HBM the discrete symmetry set of the RQE’s is fixed. The quaternionic potentials are continuous functions. Their superpositions constitute embedding continuums. This means that for vector potentials also 16 different embedding continuums exist.

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\(^{50}\) See Inertia
Also the allocation function exists in 16 different discrete symmetry sets for its function values. The sharp continuous part of the allocation function describes an embedding continuum. The allocation function keeps its discrete symmetry set throughout its life. Discrete symmetry sets do not influence the scalar potentials that are connected to object density distributions. Thus the superposition of these scalar potentials constitutes a special embedding continuum. This continuum characterizes the **Palestra**. It is described by the gravitation potential field. This does not say that in the realm of the Palestra no other potentials play their role.

### 7.4 Palestra

The second geometric level is a curved space, called Palestra. As ingredients, it consists of an embedding continuum, the embedded Qtarget set and a sharp continuous quaternionic allocation function. The local curvature is defined via the differential of the continuous (sharp) quaternionic allocation function. The parameter space of the allocation function embeds the RQE-set. Thus since the RQE-set is countable, the Palestra contains a countable set of images of the sharp allocation function. We have called these images “Qpatches”. They represent the “center locations” of Qpatterns or better said, the expectation values of the corresponding Qtarget values. The allocation function exists in 16 versions. The version determines the discrete symmetry set of the Qpattern.

The allocation function may include an isotropic scaling function. The differential of the allocation function defines an infinitesimal quaternionic step. In physical terms the length of this step is the infinitesimal coordinate time interval. The differential is a linear combination of sixteen partial derivatives. It defines a quaternionic metric\(^51\). The enumeration process adds a coordinate system. The selection of the coordinate system is arbitrary. The origin and the axes of this coordinate system only become relevant when the distance between locations must be handled. The origin is taken at the location of the current observer. The underlying space is an affine space. It does not have a unique origin. We only consider a compartment of the affine space.

### 7.5 Qpatches

The third level of geometrical objects consists of a countable set of space patches that occupy the Palestra. We already called them Qpatch regions. Qpatches are expectation values of the Qtarget images of the RQE’s that house in the first geometric object level. The set of RQE’s is used as parameter set for the part of the allocation function that produces the Qpatches. Apart from the rational quaternionic value of the corresponding RQE, their charge is formed by the discrete symmetry set, which will be shared by all elements of the corresponding Qpattern. The curvature of the second level space relates to the density distribution of the Qpatches and to the total energy of the corresponding Qpatterns. The Qpatches represent the weighted centers of the locations of the **regions**\(^52\) where next level objects can be detected. The name Qpatch stands for space patches with a quaternionic value. The charge of the Qpatches can be named Qsymm, Qsymm stands for discrete symmetry set of a quaternion. However, we already established that the value of the enumerator is also contained in the property set that forms the Qsymm charge.

The enumeration problems that come with the quaternionic Hilbert space model indicate that the Qpatches are in fact centers of a fuzzy environment that houses the potential locations where the actual RQE images (the Qtargets) can be found. The subsequent Qtargets form a micro-path.

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\(^{51}\) See the paragraph on the spacetime metric.

\(^{52}\) Not the exact locations.
7.6 QPAD’s and Qtargets
The fuzziness in the sampling of the enumerators and their images in the embedding continuum is described by a *quaternionic probability amplitude distribution* (QPAD). The squared modulus of the *complex probability amplitude distribution* (CPAD) represents the probability that an image of an RQE will be detected on the exact location that is specified by the value of the parameter of the blurred allocation function. In the QPAD this is represented by the real part of the QPAD.

A natal Qpattern is generated via a fixed statistical plan and is not disturbed by space curvature or a moving Qpatch. Since a Qpattern is generated by a stochastic process, the same natal QPAD can correspond to different natal Qpatterns. The **QPAD’s that describe natal Qpatterns have a flat parameter space in the form of a quaternionic continuum.** This natal QPAD adds blur to the sharp allocation function. The blurred allocation function is formed by the **convolution** of the sharp allocation function with the QPAD that describes the natal Qpattern. In this way the local form of the actually realized QPAD describes a deformed Qpattern. The adaptation concerns the form factor and the gradual displacement of the deformed QPAD. The form factor may differ in each direction. It is determined by the local differential of the sharp allocation function.

The image of an RQE that is produced by the **blurred allocation function** is a Qtarget. **Qtargets only live during a single progression step.** Qtargets mark the location where (higher level) objects may be detected. In this way QPAD’s exist in two types. The natal QPAD type describes the undisturbed Qpattern. It describes a fixed plan. The second QPAD type describes the potential Qtargets that at a rate of one element per progression step are **locally** generated by the blurred allocation function. That is why this second QPAD type is also called a local QPAD. The natal Qpattern can also be described by a temporal function that produces a stochastic spatial location at every subsequent progression interval. That natal Qpattern describes a natal micro-path.

The fact that Qtargets only exist during a single progression step means that on the instant of an event the generation of the Qpattern stops or proceeds in a different mode. Only if the Qpattern stays untouched, a rather complete Qpattern will be generated at that location. When the Qpatch moves, then the corresponding Qpattern smears out. With other words the natal QPAD is a plan rather than reality.

An event means that a Qpattern stops being generated or is generated in a different mode. Being generated means that it is coupled to an embedding continuum. The generator will create a relatively small pattern in that continuum. Coupling means that the generated Qpattern is coupled via its Qpatch to a mirror Qpattern that houses in the embedding continuum. This is reflected in the coupling equation\(^{53}\).

The parameter space of the blurred allocation function is a flat quaternionic continuum. The RQE’s form points in that continuum. Local QPAD’s are quaternionic distributions that contain a scalar potential in their real part that describes a density distribution of potential Qtargets. Further they contain a 3D vector potential in their imaginary part that describes the associated current density distribution of these potential Qtargets. Continuous quaternionic distributions exist in sixteen different discrete spatial symmetry sets. However, the QPAD’s inherit the discrete symmetry of their connected sharp allocation function. The QPAD’s superpose. Together they form a **global QPAD**. Depending on the Green’s functions, the local QPAD’s correspond to several types of quaternionic potential

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\(^{53}\) See coupling equation.
functions. These quaternionic potential functions combine a scalar potential and a vector potential. The QPAD’s are continuous functions. The objects that are described by these distributions form coherent countable discrete sets.

A Qtarget is an actually existing object. A Qpattern is a mostly virtual object. A natal Qpattern conforms to a plan. A QPAD may describe a Qpattern. In that case it describes a mostly virtual object. A natal QPAD describes a plan.

7.6.1 Inner products of QPAD’s and their Qpatches

Each Qpattern is a representative of a Hilbert subspace and indirectly the Qpattern represents a quantum logic proposition. The corresponding Qpatch is represented by a linear combination of Hilbert base vectors and is represented by a Hilbert proposition.

Two QPAD’s a and b have an inner product defined by

\[ \langle a | b \rangle = \int_V a \cdot b \, dV \]  

Since the Fourier transform \( \mathcal{F} \) preserves inner products, the Parseval equation holds for the inner product:

\[ \langle a | b \rangle = \langle \mathcal{F}a | \mathcal{F}b \rangle = \langle \tilde{a} | \tilde{b} \rangle = \int_{\tilde{V}} \tilde{a} \cdot \tilde{b} \, d\tilde{V} \]  

QPAD’s have a norm

\[ |a| = \sqrt{\langle a | a \rangle} \]  

7.7 Blurred allocation functions

The blurred allocation function \( \mathcal{P} \) has a flat parameter space that is formed by rational quaternions. It is the convolution of the sharp allocation function \( \wp \) with a QPAD \( \psi \) that describes a planned natal Qpattern. \( \wp \) has a flat parameter space that is formed by real quaternions. \( \psi \) has a rational quaternionic parameter.

\[ \mathcal{P} = \wp \circ \psi \]  

\( \wp \) describes the long range variation and \( \psi \) describes the short range variation. Due to this separation it is possible to describe the effect of the convolution on the local QPAD as a deformed natal QPAD that on its turn describes a natal Qpattern, where the form factor is controlled by the differential \( d\wp \) of the sharp allocation function. The sharp part of the allocation function specifies the current embedding continuum. In fact this function defines the eigenspace of a corresponding operator that resides in the Gelfand triple of the current Hilbert space.

The planned Qpattern is the result of a Poisson process that is coupled to a binomial process, while the binomial process is implemented by a 3D spread function. This second part of the allocation function influences the local curvature. The differential of first part defines a quaternionic metric that describes the local spatial curvature. This means that the two parts must be in concordance with each other.

Fourier transforms cannot be defined properly for functions with a curved parameter space, however, the blurred allocation function \( \mathcal{P} \) has a well-defined Fourier transform \( \tilde{\mathcal{P}} \), which is the product of the Fourier transform \( \tilde{\wp} \) of the sharp allocation function and the Fourier transform \( \tilde{\psi} \) of the natal QPAD \( \psi \).

\[ \tilde{\mathcal{P}} = \tilde{\wp} \times \tilde{\psi} \]  

The Fourier transform pairs and the corresponding canonical conjugated parameter spaces form a double-hierarchy model.
The Fourier transform \( \tilde{P} \) of the blurred allocation function \( P \) equals the product of the Fourier transform \( \tilde{\varphi} \) of the sharp allocation function \( \varphi \) and the Fourier transform \( \tilde{\psi} \) of the natal Qpattern QPAD \( \psi \).

16 blurred allocation functions exist that together cover all Qpatches. One of the 16 blurred allocation functions acts as reference. The corresponding sharp allocation function and thus the corresponding actual QPAD have the same discrete symmetry set as the lowest level space. The fact that the blur \( \psi \) mainly has a local effect makes it possible to treat \( \varphi \) and \( \psi \) separately\(^{54}\).

### 7.8 Local and global QPAD’s

The model uses Qpatterns in order to implement the fuzziness of the local interspacing. After adaptation of the form factor to the differential of the sharp allocation function a local QPAD is generated. The non-deformed natal QPAD describes a natal Qpattern. Each Qpattern possess a private inertial reference frame\(^{55}\). The superposition of all deformed local QPAD’s, including the (deformed) descriptors of the higher generations of the Qpatterns, forms a global QPAD. Each of the 16 blurred allocation functions corresponds to a global QPAD. The global QPAD is the image of the corresponding allocation function.

Each of the natal Qpatterns extends over a restricted part of the embedding continuum. The probability amplitude of the elements of these Qpatterns diminishes with the distance from their center point\(^{56}\). The gravitation potential of a Qpattern extends over the whole embedding continuum.

### 7.9 Generations

Photons and gluons correspond to a special kind of fields. They differ in temporal frequency from the fields that constitute the potentials of particles. They can be interpreted as amplitude modulations of the potential generating fields. Two photon types and six\(^{57}\) gluon types exist\(^{58}\). For fermions, three generations of Qpatterns exist that have non-zero extension and that differ in their basic form factor. This paper does not explain these generations.

The generator of enumerators is for a part a random number generator. That part is implemented by a Poisson process and a subsequent binomial process. Generations correspond to different characteristics of the enumerator generator. Generations may differ in their quasi-oscillations and quasi-rotations.

### 7.10 Coupling

According to the coupling equation, coupling may occur because the two QPAD’s that constitute the coupling take the same location. Several reasons can be given for this coupling. The strongest reason is that the Qpattern generator produces two patterns that subsequently are coupled. Other reasons are:

- Coupling may occur between the local Qpattern and the potentials of very distant Qpatterns. This kind of coupling causes inertia. These coupling products appear to be fermions.

\(^{54}\) \( \psi \) concerns quantum physics. \( \varphi \) concerns general relativity.

\(^{55}\) See the paragraph on inertial reference frames.

\(^{56}\) See the paragraph on the enumeration process.

\(^{57}\) In the Standard Model gluons appear as eight superpositions of the six base gluons.

\(^{58}\) Bertrand’s theorem indicates that under some conditions, photons and gluons might be described as radial harmonic oscillators.
Coupling may occur between the local Qpattern and the potentials of locally situated Qpatterns. These coupling products appear to be bosons.

The fermion coupling uses the gravitation potential, which is a scalar potential. On itself this does not enforce a discrete symmetry. (Suggestion: That symmetry can be enforced/induced by involving the discrete symmetry of the parameter space and/or the discrete symmetry of the virgin Qpattern).

Coupling can also occur via induced quasi oscillations and or induced quasi rotations. These quasi-oscillations and quasi-rotations occur in the micro-paths of the Qpatterns. Because they differ in their discrete symmetry they may take part in a local oscillation where an outbound move is followed by an inbound move and vice versa\(^{59}\).

For fermions coupling also occurs with the RQE and with the historic Qpattern that belongs to this RQE.

### 7.11 Background potential

We use the ideas of Denis Sciama\(^ {60,61,62}\).

The superposition of all real parts of potentials of distant Qpatterns that emit potential contributions in the form of spherical waves produces a uniform background potential. At a somewhat larger distance \( r \) these individual scalar potentials diminish in their amplitude as \( 1/r \). However, the number of involved Qpatterns increases with the covered volume. Further, on average the distribution of the Qpatterns is isotropic and uniform. The result is a huge (real) local potential \( \Phi \)

\[
\Phi = -\int \frac{\tilde{\rho}_0}{r} dV = -\tilde{\rho}_0 \int \frac{dV}{r} = 2\pi R^2 \tilde{\rho}_0
\]

(1)

\[
\tilde{\rho} = \tilde{\rho}_0; \quad \tilde{\rho} = 0
\]

(2)

Apart from its dependence on the average value of \( \tilde{\rho}_0 \), \( \Phi \) is a huge constant. Sciama relates \( \Phi \) to the gravitational constant \( G \).

\[ G = \frac{-c^2}{\Phi} \]

(3)

If a local Qpattern moves relative to the universe with a uniform speed \( v \), then a vector potential \( A \) is generated.

\[
A = -\int \frac{v \tilde{\rho}_0}{c r} dV
\]

(4)

Both \( \tilde{\rho}_0 \) and \( v \) are independent of \( r \). The product \( v \tilde{\rho}_0 \) represents a current. Together with the constant \( c \) they can be taken out of the integral. Thus

\[
A = \Phi \frac{v}{c}
\]

(5)

Field theory learns:

\[
\mathbf{E} = -\nabla \Phi - \frac{1}{c} \cdot \dot{A}
\]

(6)

If we exclude the first term because it is negligible small, we get:

\[
\mathbf{E} = -\frac{\Phi}{c^2} \dot{v} = G \dot{v}
\]

(7)

The fields \( \Phi \) and \( A \) together form a quaternionic potential. However, this time the fields \( \Phi \) and \( A \) do not represent the potential of a Qpattern.

---

59 See: Coupling Qpatterns.
61 http://www.adsabs.harvard.edu/abs/1953MNRAS.113...34S
62 http://rmp.aps.org/abstract/RMP/v36/i1/p463_1
7.12 Interpretation
As soon as an acceleration of a local Qpattern occurs, an extra component \( \dot{A} \) of field \( \mathbf{E} \) appears that corresponds to acceleration \( \dot{v} \).\(^{63}\)
In our setting the component \( \nabla \Phi \) of the field \( \mathbf{E} \) is negligible. With respect to this component the items compensate each other’s influence. This means that if the influenced subject moves with uniform speed \( v \), then \( \mathbf{E} \approx 0 \). However, a vector potential \( A \) is present due to the movement of the considered local Qpattern. Any acceleration of the considered local item goes together with an extra non-zero \( \mathbf{E} \) field. In this way the universe of particles causes inertia in the form of a force that acts upon the scalar potential of the accelerating item.
The amplitude of \( \Phi \) says something about the number of coupled Qpatterns of the selected generation that exist in universe. If it is constant and the average interspacing grows with progression, then the universe dilutes with increasing progression. Also the volume of the reference continuum over which the integration must be done will increase with progression. The total energy of these coupled Qpatterns that is contained in universe equals:

\[
E_{\text{total}} = \sqrt{\int \frac{\rho_0}{r}^2 \ dV}
\]

The background potential \( \Phi \) is the superposition of the contributions of waves that are emitted by distant particles. The emission occurred with ultra-high frequency. This is the highest frequency that exists in the HBM. The background potential constitutes an embedding continuum.\( \text{The enumerator generator uses the background potential as the embedding continuum for its embedded products.} \)

The allocation function describes this embedding continuum and takes care of its permanence.
Fields that oscillate with a lower frequency, such as photons, are generated by oscillating sources and can be considered as amplitude modulations of the ultra-high frequency (potential) field.

7.13 Isotropic vector potential
The scalar background potential may be accompanied by a similar background vector potential that is caused by the fact that the considered volume that was investigated in order to calculate the scalar background potential is enveloped by a surface that delivers a non-zero surface integral. The isotropic background potential corresponds to an isotropic scaling factor. This factor was already introduced in the first phases of the model.

7.14 Quantum fluid dynamics
7.14.1 Quaternionic nabla
The quaternionic nabla stands for

\[
\nabla \equiv \left\{ \frac{\partial}{\partial \tau}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\}
\]

\[
\psi \equiv \psi_0 + \psi
\]

Here \( \tau \) stands for the progression parameter.

\[
\phi = \nabla \psi
\]

\[
\phi_0 = \nabla_0 \psi_0 - (\nabla, \psi)
\]

\[
\phi = \nabla_0 \psi + \nabla \psi_0 + \nabla \times \psi
\]

\(^{63}\) See: Inertia from the coupling equation.
Is the differential equation for continuous quaternionic distributions. Rearranging shows:
\[ \nabla \psi = \phi \]
This is the differential continuity equation. It holds for QPAD’s.

### 7.14.2 The differential and integral continuity equations

Let us approach the balance equation from the integral variety of the balance equation. Balance equation is another name for continuity equation.

We replace \( \psi \) by \( \rho \), \( \psi_0 \) by \( \rho_0 \) and \( \psi \) by \( \rho = \rho_0 v/c \).

\[ \rho \equiv \rho_0 + \rho \]  
(1)

When \( \rho_0 \) is interpreted as a charge density distribution, then the conservation of the corresponding charge\(^64\) is given by the continuity equation:

Total change within \( V = \) flow into \( V \) + production inside \( V \)

In formula this means:

\[ \frac{d}{d\tau} \int_V \rho_0 \, dV = \int_S \hat{n} \rho_0 \frac{v}{c} \, dS + \int_V s_0 \, dV \]  
(3)

\[ \int_V \nabla_0 \rho_0 \, dV = \int_V \langle \nabla, \rho \rangle \, dV + \int_V s_0 \, dV \]  
(4)

The conversion from formula (2) to formula (3) uses the Gauss theorem\(^65\). Here \( \hat{n} \) is the normal vector pointing outward the surrounding surface \( S \). \( v(\tau, q) \) is the velocity at which the charge density \( \rho_0(\tau, q) \) enters volume \( V \) and \( s_0 \) is the source density inside \( V \). In the above formula \( \rho \) stands for \( \rho = \rho_0 v/c \)

It is the flux (flow per unit area and unit time) of \( \rho_0 \).

The combination of \( \rho_0(q) \) and \( \rho(q) \) is a quaternionic skew field \( \rho(q) \) and can be seen as a probability amplitude distribution (QPAD). \( \rho \) is a function of \( q \).

\[ q \equiv q_0 + q; \ q_0 = \tau \]  
(5)

\( \rho(q) \rho^*(q) \) can be seen as an overall probability density distribution of the presence of the carrier of the charge. \( \rho_0(q) \) is a charge density distribution. \( \rho(q) \) is the current density distribution.

This results in the law of charge conservation:

\[ s_0(q) = \nabla_0 \rho_0(q) \mp \langle \nabla, (\rho_0(q)v(q) + \nabla \times a(q)) \rangle \]  
(6)

\[ = \nabla_0 \rho_0(q) \mp \langle \nabla, \rho(q) + A(q) \rangle \]  
\[ = \nabla_0 \rho_0(q) \mp \langle v(q), \nabla \rho_0(q) \rangle \mp \langle \nabla, v(q) \rangle \rho_0(q) \mp \langle \nabla, A(q) \rangle \)

The blue colored \( \pm \) indicates quaternionic sign selection through conjugation of the field \( \rho(q) \).

The field \( a(q) \) is an arbitrary differentiable vector function.

\[ \langle \nabla, \nabla \times a(q) \rangle = 0 \]  
(7)

\( A(q) \equiv \nabla \times a(q) \) is always divergence free. In the following we will neglect \( A(q) \).

Equation (6) represents a balance equation for charge density. What this charge actually is, will be left in the middle. It can be one of the properties of the carrier or it can represent the full ensemble of the properties of the carrier.

---

\(^64\) Also see Noether’s laws: [http://en.wikipedia.org/wiki/Noether%27s_theorem](http://en.wikipedia.org/wiki/Noether%27s_theorem)

Up to this point the investigation only treats the real part of the full equation. The full continuity equation runs:

\[ s(q) = \nabla \rho(q) = s_0(q) + s(q) \]  \hspace{1cm} (8)
\[ s_0(q) = 2\nabla_0 \rho_0(q) + (\nabla(q), \nabla \rho_0(q)) \nabla \rho_0(q) \]  \hspace{1cm} (9)
\[ s(q) = \pm \nabla_0 \rho(q) \pm \nabla \rho_0(q) \]  \hspace{1cm} (10)

The red sign selection indicates a change of handedness by changing the sign of one of the imaginary base vectors. Conjugation also causes a switch of handedness. It changes the sign of all three imaginary base vectors.

In its simplest form the full continuity equation runs:

\[ s(q) = \nabla \rho(q) \]  \hspace{1cm} (11)

Thus the full continuity equation specifies a quaternionic distribution \( s \) as a flat differential \( \nabla \rho \).

When we go back to the integral balance equation, then holds for the imaginary parts:

\[ \frac{d}{d\tau} \int_V \rho \, dV = -\oint_S \hat{n} \rho_0 \, dS - \oint_S \hat{n} \times \rho \, dS + \int_V s \, dV \]  \hspace{1cm} (12)
\[ \int_V \nabla \rho \, dV = -\int_V \nabla_0 \rho \, dV - \int_V \nabla \times \rho \, dV + \int_V s \, dV \]  \hspace{1cm} (13)

For the full integral equation holds:

\[ \frac{d}{d\tau} \int_V \rho \, dV + \oint_S \hat{n} \rho \, dS = \int_V s \, dV \]  \hspace{1cm} (14)
\[ \int_V \nabla \rho \, dV = \int_V s \, dV \]  \hspace{1cm} (15)

Here \( \hat{n} \) is the normal vector pointing outward the surrounding surface \( S \), \( v(q) \) is the velocity at which the charge density \( \rho_0(q) \) enters volume \( V \) and \( s_0 \) is the source density inside \( V \). In the above formula \( \rho \) stands for

\[ \rho = \rho_0 + \rho = \rho_0 + \rho_0 v \]  \hspace{1cm} (16)

It is the flux (flow per unit of area and per unit of progression) of \( \rho_0 \). \( \tau \) stands for progression (not coordinate time).
7.15 The coupling equation

The coupling equation is a special form of the continuity equation. $\psi$ is a normalized quaternionic distribution.

$$\langle \psi | \psi \rangle = \int_V |\psi|^2 \, dV = 1$$

$$\nabla \psi = \phi$$

We also normalize $\phi$ by dividing $a$ by a real factor $m$

$$\phi = m \varphi$$

$$\langle \varphi | \varphi \rangle = \int_V |\varphi|^2 \, dV = 1$$

This results in the coupling equation, which holds for coupled field pairs $\{\psi, \varphi\}$

$$\langle \phi | \phi \rangle = \int_V |\phi|^2 \, dV = m^2$$

$$\langle \nabla \psi | \nabla \psi \rangle = \int_V |\nabla \psi|^2 \, dV = m^2$$

This equation does not depend on $\varphi$, thus it also holds for composites. The coupling equation reads:

$$\nabla \psi = m \varphi$$

The quaternionic format of the Dirac equation for the electron is a special form of the coupling equation.

$$\nabla \psi = m \psi^*$$

The coupling equation appears to hold for elementary particles and simple composite particles. For anti-particles hold.

$$(\nabla \psi)^* = m \varphi^*$$

Due to the fact that the parameter space is not conjugated, equation (9) differs from equation (7).

The quaternionic format of the Dirac equation for the positron is a special form of the coupling equation for anti-particles.

$$(\nabla \psi)^* = m \psi$$

7.16 Energy

This makes $|\phi|$ to the distribution of the local energy and $m$ to the total energy of the quantum state function. The coupling equation can be split in a real equation and an imaginary equation.

$$\nabla_0 \psi_0 - \langle \nabla, \psi \rangle = m \varphi_0$$

$$\nabla_0 \psi + \nabla \psi_0 + \nabla \times \psi = m \varphi$$

Bold characters indicate imaginary quaternionic distributions and operators. Zero subscripts indicate real distributions and operators.

The quantum state function of a particle moving with uniform speed $\nu$ is given by

$$\psi = \chi + \chi_0 \nu$$

$$\chi_0 = \psi_0$$

Here $\chi$ stands for quantum state function of the particle at rest.

We introduce new symbols. In order to indicate the difference with Maxwell’s equations we use Gotic capitals:

$$\mathfrak{E} = \nabla_0 \psi + \nabla \psi_0$$

$$\mathfrak{B} = \nabla \times \psi$$

The local field energy $E$ is given by:
\[ E = |\phi| = \sqrt{\phi_0 \phi_0 + \langle \phi, \phi \rangle} = \sqrt{\phi_0 \phi_0 + \langle \mathbf{E}, \mathbf{E} \rangle + \langle \mathbf{B}, \mathbf{B} \rangle + 2 \langle \mathbf{E}, \mathbf{B} \rangle} \] (7)

The total energy is given by the volume integral

\[ E_{\text{total}} = \sqrt{\int_V |\phi|^2 \, dV} \] (8)

In a static situation the local energy \( E \) reduces to

\[ E_{\text{static}} = \sqrt{\langle \mathbf{V}, \psi \rangle^2 + \langle \mathbf{E}, \mathbf{E} \rangle + \langle \mathbf{B}, \mathbf{B} \rangle} \] (9)

### 7.16.1 Fourier transform

In a region of little or no space curvature the Fourier transform of the local QPAD can be taken.

\[ \forall \psi = \phi = m \varphi \] (1)
\[ \mathcal{M} \bar{\psi} = \bar{\phi} = m \bar{\varphi} \] (2)
\[ \langle \bar{\psi} | \mathcal{M} \bar{\psi} \rangle = m \langle \bar{\psi} | \bar{\varphi} \rangle \] (3)
\[ \mathcal{M} = \mathcal{M}_0 + \mathcal{M} \] (4)
\[ \mathcal{M}_0 \bar{\psi}_0 - \langle \mathcal{M}, \bar{\psi} \rangle = m \bar{\varphi}_0 \] (5)
\[ \mathcal{M}_0 \psi + \mathcal{M} \bar{\psi}_0 + \mathcal{M} \times \bar{\psi} = m \bar{\varphi} \] (6)
\[ \int_V \bar{\phi}^2 \, dV = \int_V (\mathcal{M} \bar{\psi})^2 \, dV = m^2 \] (7)

In general \( |\bar{\psi} \rangle \) is not an eigenfunction of operator \( \mathcal{M} \). That is only true when \( |\psi \rangle \) and \( |\varphi \rangle \) are equal. For elementary particles they are equal apart from their difference in discrete symmetry.

### 7.17 Elementary particles

Elementary particles are constituted by the coupling of two QPAD’s that belong to the same generation. One of the QPAD’s is the quantum state function of the particle. The other QPAD can be interpreted to implement inertia. Apart from their sign flavors these constituting QPAD’s form the same quaternionic distribution. However, the sign flavor may differ and their progression must have the same direction. It means that the density distribution is the same, but the signs of the flows of the concerned objects differ between the two distributions. The second QPAD only simulates a Qpattern. It represents the coupling of the quantum state function to the embedding continuum, which is used in constructing the potentials of the particle. Coupling of elementary particles is governed by a special coupling equation

The quantum state function is a mostly virtual distribution. Only one element is actual. The second QPAD is completely virtual.

The coupling uses pairs \( \{ \psi^x, \psi^y \} \) of two sign flavors of the same basic Qpattern and its corresponding QPAD, which is indicated by \( \psi^{\circ} \). The special coupling equation runs:

\[ \forall \psi^x = m \psi^y \] (1)

Corresponding anti-particles obey

\[ (\forall \psi^x)^* = m (\psi^y)^* \] (2)

As claimed above, coupling (also) occurs by embedding the message waves in the potential(s) of other particles.

In this specification the form of the quaternionic Dirac equations play a significant, but at the same time a very peculiar role. The fact that \( \psi^x \) and \( \psi^y \) must be equal apart from a discrete symmetry difference is very strange and it is highly improbable that this strong relation is constituted by accident. On the other hand it is known that the step stones couple to the embedding continuum. Two different types of this embedding continuum exists. The first
embedding continuum is formed by the superposition of the potentials of distant particles. This type of binding produces fermions. The second embedding continuum is formed by the superposition of the potentials of local particles. This type of binding produces bosons. It appears as if the correlation mechanism creates two rather than one distribution of step stones in which the descriptor of the first one plays the role of the quantum state function, while the descriptor of the second one plays the role of a mirror that has the sign flavor of the embedding continuum.

If the first Qpattern oscillates, then the second Qpattern oscillates asynchronous or partly in synchrony. This situation may differ per dimension. This results in 64 elementary particle types and 64 anti-particle types. Besides of that exist 8 oscillating potential types. The coupling has a small set of observable properties:

- coupling strength,
- electric charge,
- color charge and
- spin.

Due to the fact that the enumerator creation occurs in configuration space, the coupling affects the local curvature of the involved Palestras. Qpattern QPAD’s that belong to the same generation have the same shape. This is explained in the paragraph on the enumeration process. The difference between the coupling partners resides in the discrete symmetry sets. Thus, the properties of the coupled pair are completely determined by the sign flavors of the partners.

HYPOTHESIS 4: If the quaternionic quantum state function of an elementary particle couples to an embedding continuum that is formed by distant particles, then the particle is a fermion, otherwise it is a boson. The quantum state functions of anti-particles are coupled to canonical conjugates of the corresponding embedding continuums.

The fact that for fermions both the reference continuum and the reference enumerator set play a crucial role may indicate that the Pauli principle is based on this fact.

This paper does not give an explanation for the influence on the spin by the fact that the quantum state function is connected to an isotropic or an anisotropic Qpattern.

Photons and gluons are not coupled. They modulate the ultra-high frequency fields that constitute particle potentials.

In the standard model the eight gluons are constructed from superpositions of the six base gluons.

7.17.1 Reference frames

Each Qpattern possesses a reference frame that represents its current location, its orientation and its discrete symmetry. The reference frame corresponds with a Cartesian coordinate system that has a well-defined origin. Reference frames of different Qpatterns have a relative position. A Qpattern does not move with respect to its own reference frame. However, reference frames of different Qpatterns may move relative to each other. The reference frames reside in an affine space. Interaction can take place between reference frames that reside in different HBM pages and that are within the range of the interaction speed. Within the same HBM page no interaction is possible. Interaction runs from a reference frame to a frame that lays in the future of the sender. Coupling into elementary particles puts the origins of the reference frames of the coupled Qpatterns at the same location. At the same location reference frames are parallel. That does not mean that the axes have the same sign.
7.17.2 Coupling Qpatterns
This section uses the fact that coupling is caused by interfering with the embedding continuum. Fermions couple to the embedding continuum that is formed by the superposition of the potentials of distant particles. Bosons couple to the embedding continuum that is formed by the superposition of the potentials of local particles.

The coupling is represented by pairs \(\{\psi^x, \psi^y\}\) of two sign flavors of the same basic QPAD \(\psi^{\circ}\). Thus the corresponding coupling equation runs:

\[
\mathbf{\nabla} \psi^x = \psi^y 
\]  

(1)

The corresponding anti-particles obey

\[
(\mathbf{\nabla} \psi^x)^* = m (\psi^y)^* 
\]  

(2)

The partial anti-phase couplings must use different sign flavors. In the figure below \(\psi^{\circ}\) acts as the reference sign flavor.

The coupling and its effect on local curvature is treated in the section on the enumeration process.

\[ \begin{array}{cccc}
\psi^0 & N & R \\
\psi^1 & R & L \\
\psi^2 & G & L \\
\psi^3 & B & L \\
\psi^4 & \bar{B} & R \\
\psi^5 & \bar{G} & R \\
\psi^6 & \bar{R} & R \\
\psi^7 & \bar{N} & L \\
\end{array} \]

Figure 2: Sign flavors

Eight sign flavors
(discrete symmetries)
Colors N, R, G, B,\bar{R}, \bar{G}, \bar{B}, W
Right or Left handedness
R,L
7.17.3 Elementary particle properties
Elementary particles retain their discrete properties when they are contained in composite particles.

7.17.3.1 Spin
HYPOTHESIS 5: The size of the spin relates to the fact whether the coupled Qpattern is the reference Qpattern. The reference Qpattern QPAD has the reference sign flavor $\psi^{\circ}$. Each generation has its own reference Qpattern. Fermions couple to the reference Qpattern. Fermions have half integer spin. Bosons have integer spin. The spin of a composite equals the sum of the spins of its components.

7.17.3.2 Electric charge
HYPOTHESIS 6: Electric charge depends on the difference and direction of the imaginary base vectors for the Qpattern pair. Each sign difference stands for one third of a full electric charge. Further it depends on the fact whether the handedness differs. If the handedness differs then the sign of the count is changed as well.
The electric charge of a composite is the sum of the electric charge of its components.

7.17.3.3 Color charge
HYPOTHESIS 7: Color charge is related to the direction of the anisotropy of the considered Qpattern with respect to the reference Qpattern. The anisotropy lays in the discrete symmetry of the imaginary part. The color charge of the reference Qpattern is white. The corresponding anti-color is black. The color charge of the coupled pair is determined by the colors of its members. All composite particles are black or white. The neutral colors black and white correspond to Qpatterns that are isotropic with respect to the reference sign flavor.
Currently, color charge cannot be measured. In the Standard Model the existence of color charge is derived via the Pauli principle.

7.17.3.4 Mass
Mass is related to the internal energy of the Qpattern. More precisely stated, mass is related to the square root of the volume integral of the square of the local field energy $E^2 = |\nabla \psi|^2$. Any internal kinetic energy is included in $E$.

$$m^2 = \langle \nabla \psi | \nabla \psi \rangle = \int_V |\nabla \psi|^2 \, dV$$

The same mass rule holds for composite particles. The fields of the composite particles are dynamic superpositions of the fields of their components.
7.17.4 Elementary object samples

With these ingredients we can look for agreements with the standard model. It appears that the coverage is (over)complete. The larger diversity of this HBM table appears to be not (yet) measurable.

For the same generation, the real parts of the QPAD’s (that contain the scalar density distribution) are all born the same way! In this way the Qpatterns become micro states.

Elementary particles are represented by couplings of two QPAD’s that may differ in their discrete symmetries. The differences between the discrete symmetries determine the discrete properties of the particle.

7.17.4.1 Photons and gluons

Photons and gluons modulate the ultra-high frequency fields that constitute particle potentials. Once emitted, they flow freely. When the potential emitting particle oscillates, the photons or particle. When the potential emitting potentials annihilate, then the potentials keep spreading and flee from their original source. In that way special kinds of photons and gluons are created.

In the standard model the eight gluons are constructed from superpositions of the six HBM base gluons.

<table>
<thead>
<tr>
<th>type</th>
<th>s-type</th>
<th>e-charge</th>
<th>c-charge</th>
<th>Handedness</th>
<th>SM Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>{ψ(7)}</td>
<td>boson</td>
<td>0</td>
<td>N</td>
<td>R</td>
<td>photon</td>
</tr>
<tr>
<td>{ψ(9)}</td>
<td>boson</td>
<td>0</td>
<td>W</td>
<td>L</td>
<td>photon</td>
</tr>
<tr>
<td>{ψ(6)}</td>
<td>boson</td>
<td>0</td>
<td>G</td>
<td>L</td>
<td>gluon</td>
</tr>
<tr>
<td>{ψ(2)}</td>
<td>boson</td>
<td>0</td>
<td>B</td>
<td>L</td>
<td>gluon</td>
</tr>
</tbody>
</table>

Only at the instant of their generation or annihilation photons and gluons couple to the emitter or absorber.

Two types of photons exist. One fades away from its point of generation. The other concentrates until it reaches the absorber.

The act of interaction can be interpreted as a Fourier transform. The Fourier transforms converts a distribution in configuration space into a distribution in its canonical conjugated space or vice versa.

For gluons similar things occur.
7.17.4.2 Leptons and quarks
According to the Standard Model both leptons and quarks comprise three generations. They form 22 particles. Neutrinos will be treated separately.

7.17.4.2.1 Leptons

<table>
<thead>
<tr>
<th>Pair</th>
<th>s-type</th>
<th>e-charge</th>
<th>c-charge</th>
<th>Handedness</th>
<th>SM Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\psi^7, \psi^5)</td>
<td>fermion</td>
<td>-1</td>
<td>N</td>
<td>LR</td>
<td>electron</td>
</tr>
<tr>
<td>(\psi^9, \psi^7)</td>
<td>Anti-fermion</td>
<td>+1</td>
<td>W</td>
<td>RL</td>
<td>positron</td>
</tr>
</tbody>
</table>

The generations contain the muon and tau generations of the electrons. The Qpatterns quasi-oscillate asynchronous in three dimensions.

7.17.4.2.2 Quarks

<table>
<thead>
<tr>
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<th>e-charge</th>
<th>c-charge</th>
<th>Handedness</th>
<th>SM Name</th>
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<td>(\bar R)</td>
<td>RR</td>
<td>up-quark</td>
</tr>
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</table>

The generations contain the charm and top versions of the up-quark and the strange and bottom versions of the down-quark. The Qpatterns quasi-oscillate asynchronous in one or two dimensions.

7.17.4.2.3 Reverse quarks

<table>
<thead>
<tr>
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<th>c-charge</th>
<th>Handedness</th>
<th>SM Name</th>
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</table>
The generations contain the charm and top versions of the up-r-quark and the strange and bottom versions of the down-r-quark. The Qpatterns oscillate asynchronous in one or two dimensions.

7.17.4.2.4 Neutrinos
Neutrinos are fermions and have zero electric charge. They are leptons, but they seem to belong to a separate low-weight family of (three) generations. Their quantum state function couples to a QPAD that has the same sign-flavor. The lowest generation has a very small rest mass.

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<th>c-charge</th>
<th>Handedness</th>
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<tr>
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<td>LL</td>
<td>Anti-up-r-quark</td>
</tr>
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</table>
7.17.4.3W-particles
The 18 W-particles have indiscernible color mix. $W_+$ and $W_-$ are each other’s anti-particle.

<table>
<thead>
<tr>
<th>Pair</th>
<th>s-type</th>
<th>e-charge</th>
<th>c-charge</th>
<th>Handedness</th>
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<td>$W_+$</td>
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<tr>
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<td>$W_+$</td>
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<td>$\bar{G}G$</td>
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<td>$G\bar{G}$</td>
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<td>RL</td>
<td>$W_-$</td>
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<td>${\psi^3, \psi^5}$</td>
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<td>+1</td>
<td>$B\bar{G}$</td>
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<td>$W_+$</td>
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<tr>
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<td>$B\bar{B}$</td>
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<td>$W_+$</td>
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</table>

The Qpatterns oscillate differently in multiple dimensions.

7.17.4.4Z-candidates
The 12 Z-particles have indiscernible color mix.

<table>
<thead>
<tr>
<th>Pair</th>
<th>s-type</th>
<th>e-charge</th>
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<td>RR</td>
<td>$Z$</td>
</tr>
</tbody>
</table>

The Qpatterns oscillate differently in multiple dimensions.
7.18 Physical fields
Elementary particles conserve their properties in higher level bindings. These properties are sources to fields that are exposed as dedicated potentials. Examples are the gravitational potential field and the electrostatic potential field. As soon as they leave the particle, the corresponding waves start their own life and keep flowing away from their source. These waves feature a fixed ultra-high frequency. If the particle oscillates or annihilates, then their amplitude can be modulated. We know these amplitude modulations as photons and gluons.
If the source stays at rest, then the waves superpose as a static potential. If the source oscillates, then the emitted stream oscillates as well. The corresponding amplitude modulation has a lower frequency.
If in a certain region a coherent distribution of property carriers exist, then that distribution can again be described by a QPAD. These fields are secondary fields. These new fields can be described by quaternionic distributions and when they cover large numbers of particles they can be described with quaternionic distributions that contain a scalar potential and a vector potential like the QPAD's that describe elementary particles.
Besides the photons and the gluons these secondary fields are the physical fields that we know.

7.19 Gravitation field
One of the physical fields, the gravitation field describes the local curvature of the reference Palestra. It equals the scalar potential field that corresponds to the real part of the quantum state function.
Now let $\phi$ represent the quaternionic potential of a set of massive particles. It is a superposition of single charge potentials.
$$\phi = \phi_0 + \phi = \sum_i \phi_i = \sum_i m_i \varphi_i$$
(1)
The particles may represent composites. In that case the mass $m_i$ includes the internal kinetic energy of the corresponding particle. All massive particles attract each other. In superpositions, gravitational fields tend to enforce each other.

7.20 Electromagnetic fields
The electric charge $e_i$ is represented similarly as $m_i$, but where $m_i$ is always positive, the electric charge $e_i$ can be either positive or negative. Equal signs repel, opposite signs attract each other. Superposition of the fields must include the sign. In superpositions, arbitrary electronic fields tend to neutralize each other. Moving electric charges correspond to a vector potential and the curl of this vector potential corresponds to a magnetic field.
$$\phi = \phi_0 + \phi = \sum_i e_i \varphi_i$$
(1)
Here $\phi$ is the quaternionic electro potential. It is a superposition of single charge potentials $\phi_i$.
$\phi_0$ is the scalar potential. $\phi$ is the vector potential. The values of the electric charge sources $e_i$ are included in $\phi$.
$$E = V_0 \phi + \nabla \phi_0$$
$$B = \nabla \times \phi$$
(2)
(3)

7.21 Photons and gluons
Photons and gluons can be described by quaternionic functions.
In configuration space they obey
\[ \nabla \psi = 0 \]
\[ \nabla^2 \psi = 0 \]  
Ensembles of photons and/or gluons are better considered as QPAD’s in the canonical conjugated space of the configuration space.

### 7.22 Anisotropic potentials

The characteristics of the potentials that are emitted or absorbed by elementary particles are determined by the differences between the symmetry set of the quantum state function of the particle and the symmetry set of the coupled QPAD that represents the embedding continuum. This difference determines whether the potentials act in 1, 2 or 3 dimensions. In odd dimensions the resilience of the potentials can be explained by the Huygens principle. This does not work for quarks, W-particles and Z-particles. The corresponding messengers are gluons. For these objects the potentials also act in two dimensions. In even dimensions the Huygens principle does not act in its normal way.

#### 7.22.1 Huygens principle for odd and even number of spatial dimension

The following is taken from http://www.mathpages.com/home/kmath242/kmath242.htm

The spherically symmetrical wave equation in \( n \) spatial dimensions can be written as

\[
\frac{\partial^2 \psi}{\partial r^2} + \frac{n - 1}{r} \frac{\partial \psi}{\partial r} = \frac{\partial^2 \psi}{\partial t^2} \tag{1}
\]

Now suppose we define a new scalar field \( \phi \) by the relation

\[
\phi(r, t) = r^{(n-1)/2} \psi(r, t) \tag{2}
\]

This leads to

\[
\frac{\partial^2 \phi}{\partial r^2} + \frac{(n - 1)(n - 3)}{4r^2} \phi = \frac{\partial^2 \phi}{\partial t^2} \tag{3}
\]

If \( n \) equals 1, meaning that we have just a single space dimension, then the second term on the left hand side vanishes, leaving us with a one-dimensional wave equation, with has the well-known general solution

\[
\psi(r, t) = f(r - t) + g(r + t) \tag{4}
\]

for arbitrary functions \( f \) and \( g \).

If \( n \) equals 3, i.e., in the case of three spatial dimensions, the spherically symmetrical wave equation reduces again to a one-parametric wave equation, in the modified wave function \( \phi = r \psi \). Hence the general solution in three space dimensions is

\[
\psi(r, t) = \frac{f(r - t)}{r} + \frac{g(r + t)}{r} \tag{5}
\]
The fact that this solution is divided by $r$ signifies that the magnitude of the wave tends to drop as $r$ increases (unlike the one-dimensional case, in which a wave would theoretical propagate forever with undiminished strength). Focusing on just the "retarded" component of the wave, $f(r - t)/r$, the fact that the time parameter $t$ appears only in the difference $r - t$ implies that the (attenuated) wave propagates in time with a phase velocity of precisely 1, because for any fixed phase $\beta$ we have $r - t = \beta$ and so $dr/dt$ for this phase point is 1. Consequently if $f$ is a single pulse, it will propagate outward in a spherical shell at precisely the speed 1, i.e., on the light cone. Conversely, it can be shown that the wave function at any point in space and time is fully determined by the values and derivatives of that function on the past light cone of the point.

Any wave equation for which this is true (i.e., for which disturbances propagate at a single precise speed) is said to satisfy Huygens' Principle. The connection with Huygens' original statement about secondary wavelets is that each wavelet - with the same speed as the original wave - represents a tiny light cone at that point, and Huygens' principle asserts that light is confined to those light cones.

For $n$ equals 2 the extra term in equation (3) does not vanish. We can still solve the wave equation, but the solution is not just a simple spherical wave propagating with unit velocity. Instead, we find that there are effectively infinitely many velocities, in the sense that a single pulse disturbance at the origin will propagate outward on infinitely many "light cones" (and sub-cones) with speeds ranging from the maximum down to zero. Hence if we lived in a universe with two spatial dimensions (instead of three), an observer at a fixed location from the origin of a single pulse would "see" an initial flash but then the disturbance "afterglow" would persist, becoming less and less intense, but continuing forever, as slower and slower subsidiary branches arrive.

7.22.2 The case of even spatial dimensions
Now again start from equation (1) and try a solution in the form:

$$\psi(r, t) = f(r)g(t)$$  \hspace{1cm} (6)

Inserting this into the wave equation and expanding the derivatives by the product rule gives

$$g \frac{\partial^2 f}{\partial r^2} + \frac{n - 1}{r} g \frac{\partial f}{\partial r} = f \frac{\partial^2 g}{\partial t^2}$$  \hspace{1cm} (7)

Dividing through by $fg$ gives

$$\frac{1}{f} \frac{\partial^2 f}{\partial r^2} + \frac{n - 1}{r} \frac{\partial f}{\partial r} = \frac{1}{g} \frac{\partial^2 g}{\partial t^2}$$  \hspace{1cm} (8)

This decouples into two equations

$$\frac{\partial^2 f}{\partial r^2} + \frac{n - 1}{r} \frac{\partial f}{\partial r} = k f$$  \hspace{1cm} (9)
\[ \frac{\partial^2 g}{\partial t^2} = k \, g \]  

(10)

If \( k \) is positive or zero the right hand equation gives “run-away” solutions for \( g(t) \), whereas if \( k \) is negative we can choose scaling so that \( k = -1 \) and then \( g(t) \) satisfies the simple harmonic equation, whose solutions include functions of the form \( \sin(ct) \) and \( \cos(ct) \). In that case equation (9) can be re-written in the form

\[ r \frac{\partial^2 f}{\partial r^2} + (n - 1) \frac{\partial f}{\partial r} + r \, f = 0 \]  

(11)

This is the form of a Bessel’s equation. In fact for \( n = 2 \) the solution is the zero order Bessel function \( J_0(r) \).

\[ J_0(r) = \frac{2}{\pi} \int_0^\infty \sin(\cosh(\theta) \, r) \, d\theta \]  

(12)

A plot of \( J_0(r) \) is shown below.

Inserting \( g(t) = \sin(ct) \) gives

\[ \psi(r, t) = \frac{1}{\pi} \int_0^\infty [\cos(\cosh(\theta) \, r - ct) - \cos(\cosh(\theta) \, r + ct)] \, d\theta \]  

(13)

Hence, instead of the solution being purely a function of \( r \pm ct \) as in the case of odd dimensions, we find that it is an integral of functions of \( \cosh(\theta) r \pm ct \). Each value of \( \theta \) corresponds to a propagation speed of \( c/\cosh(\theta) \), so the speeds vary from \( c \) down to zero. This
signifies that the wave function at any event is correlated not just with the wave function on its “light cone”, but with the wave function at every event inside its light cone.

**7.23 Discussion**
This particular behavior of the Huygens principle for potential contributions that cover even dimensions might explain the exceptional strength of the corresponding strong force mechanism.

It appears that leptons with electric charges of $\pm n/3 \, e$ produce $n$ dimensional waves that contribute to their electrostatic potential.
For $n=3$ the Green’s function is of form $1/r$.
For $n=2$ the Green’s function is a zero order Bessel function.
For $n=1$ the Green’s function is a constant.

The gravitation potential is not influenced by the discrete symmetries. The corresponding potential contributions are always transmitted isotropic in three dimensions.

The electric potential is controlled by the discrete symmetry sets. Depending on the resulting electric charge of the particle the electric potential contributions are transmitted in 1, 2 or 3 dimensions.

The correlation mechanism applies the Huygens principle for the recreation in each progression step of the corresponding potentials.
8 Inertia
We use the ideas of Denis Sciama\textsuperscript{66,67,68}.

8.1 Inertia from coupling equation
In order to discuss inertia we must reformulate the coupling equation.

\begin{align*}
\nabla \psi &= m \varphi \\
\nabla_0 \psi_0 - \langle \nabla, \psi \rangle &= m \varphi_0 \\
\nabla_0 \psi + \nabla \psi_0 + \nabla \times \psi &= \mathcal{E} + \mathcal{B} = m \varphi
\end{align*}

We will write \( \psi \) as a superposition

\begin{align*}
\psi &= \chi + \chi_0 \, \mathbf{v} \\
\psi_0 &= \chi_0 \\
\psi &= \chi + \chi_0 \, \mathbf{v}
\end{align*}

\( \chi \) represents the rest state of the object. With respect to progression, it is a constant.

\[ \nabla_0 \chi = 0 \]

For the elementary particles the coupled distributions \( \{ \psi, \varphi \} \) have the same real part.

\begin{align*}
\psi_0 &= \varphi_0 \\
\nabla_0 \psi &= \chi_0 \, \dot{\mathbf{v}}
\end{align*}

Remember

\begin{align*}
\mathcal{E} &= \nabla_0 \psi + \nabla \psi_0 \\
\chi_0 \, \dot{\mathbf{v}} &= \mathcal{E} - \nabla \psi_0
\end{align*}

In static conditions \( \mathbf{v} \) represents a uniform speed of linear movement. However, if the uniform speed turns into acceleration \( \dot{\mathbf{v}} \neq \mathbf{0} \), then an extra field of size \( \chi_0 \, \dot{\mathbf{v}} \) is generated that counteracts the acceleration. The Qpattern does not change, thus \( \nabla \psi_0 \) does not change. Also \( \mathcal{B} \) does not change. This means that the acceleration of the particle corresponds to an extra \( \mathcal{E} \) field that counteracts the acceleration. On its turn it corresponds with a change of the coupling partner \( \varphi \). That change involves the coupling strength \( m \). The counteraction is felt as inertia.

8.2 Information horizon
The terms in the integral continuity equation

\[ \Phi = \int \nabla \psi \, dV = \int \frac{\Phi}{\mathbf{v}} \, dV \]

can be interpreted as representing the influence of a local object onto the rest of the universe or as the influence of the rest of the universe onto a local object. In the second case the influence diminishes with distance and the number of influencers increases such that the most distant contributors together poses the largest influence. These influencers sit at the information horizon. In the history of the model they are part of the birth state of the current episode of the universe. This was a state of densest packaging.

*The local Qpattern that is described by \( \psi \) couples to the historic Qpattern \( \varphi \) for which the RQE acts as a Qpatch and as a Qtarget.* This historic Qpattern resided in the reference page of the HBM.
9 Gravitation as a descriptor
The gravitation field describes the local curvature. The sharp allocation function can act as the base of a quaternionic gravitation theory. The sharp allocation function has sixteen partial derivatives that combine in a differential.

9.1 Palestra
All quantum state functions share their parameter space as affine spaces. Due to the fact that the coupling of Qpatterns affects this parameter space, the Palestra is curved. The curvature is not static. With other words the Qpatches in the parameter space move and densities in the distribution of these patches change. For potential observers, the Palestra is the place where everything classically happens. The Palestra comprises the whole universe.

9.1.1 Spacetime metric
The Palestra is defined with respect to a flat parameter space, which is spanned by the rational quaternions. We already introduced the existence of a smallest rational number, which is used to arrange interspace freedom. The specification of the set of Qpatches is performed by a continuous quaternionic distribution $\wp(x)$ that acts as a (partial) allocation function. This allocation function defines a quaternionic infinitesimal interval $ds$. On its turn this definition defines a metric.

$$ds(x) = ds^\nu(x)e_\nu = d\wp = \sum_{\mu=0}^{3} \frac{\partial \wp}{\partial x_\mu} dx_\mu = q^\mu(x)dx_\mu$$

$$= \sum_{\mu=0}^{3} \sum_{\nu=0}^{3} e_\nu \frac{\partial \wp}{\partial x_\mu} dx_\mu = \sum_{\mu=0}^{3} \sum_{\nu=0}^{3} e_\nu q^\mu_\nu dx_\mu$$

The base $e_\nu$ and the coordinates $x_\mu$ are taken from the flat parameter space of $\wp(x)$. That parameter space is spanned by the quaternions. The definition of the quaternionic metric uses a full derivative $d\wp$ of the (partial) allocation function $\wp(x)$. This full derivative differs from the quaternionic nabla $\nabla$, which ignores the curvature of the parameter space. On its turn $d\wp$ ignores the blur of $\wp$.

The allocation function $\wp(x)$ may include an isotropic scaling function $a(\tau)$ that only depends on progression $\tau$. It defines the expansion/compression of the Palestra.

$ds$ is the infinitesimal quaternionic step that results from the combined real valued infinitesimal $dx_\mu$ steps that are taken along the $e_\mu$ base axes in the (flat) parameter space of $\wp(x)$.

$dx_0 = c\ d\tau$ plays the role of the infinitesimal space time interval $ds_{st}$. It is a physical invariant. $d\tau$ plays the role of the proper time interval and it equals the infinitesimal progression interval. The progression step is an HBM invariant. Without curvature, $dt$ in $\|ds\| = c\ dt$ plays the role of the infinitesimal coordinate time interval.

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69 http://en.wikipedia.org/wiki/Quaternion_algebra#Quaternion_algebras_over_the_rational_numbers

70 The intervals that are constituted by the smallest rational numbers represent the infinitesimal steps. Probably the hair of mathematicians are raised when we treat the interspacing as an infinitesimal steps. I apologize for that.

71 Notice the difference between the quaternionic interval $ds$ and the spacetime interval $ds_{st}$.
\[ c^2 \, dt^2 = ds \, ds^* = dx_0^2 + dx_1^2 + dx_2^2 + dx_3^2 \] (2)

\[ dx_0^2 = ds_{st}^2 = c^2 \, dt^2 - dx_1^2 - dx_2^2 - dx_3^2 \] (3)

\( dx_0^2 \) is used to define the local spacetime metric tensor. With that metric the Palestra is a pseudo-Riemannian manifold that has a Minkowski signature. When the metric is based on \( ds^2 \), then the Palestra is a Riemannian manifold with a Euclidean signature. The Palestra comprises the whole universe. It is the arena where everything happens.

For the (partial) allocation function holds

\[ \frac{\partial^2 \varrho}{\partial x_\mu \partial x_\nu} = \frac{\partial^2 \varrho}{\partial x_\nu \partial x_\mu} \] (4)

And similarly for higher-order derivatives. Due to the spatial continuity of the allocation function \( \varrho(x) \), the quaternionic metric as it is defined above is far more restrictive than the metric tensor that is used in General Relativity:

\[ ds^2 = g_{ik} \, dx^i \, dx^k \] (5)

Still

\[ g_{ik} = g_{ki} \] (6)

### 9.1.2 The Palestra step

When nature steps with universe (Palestra) wide steps during a progression step \( \Delta x_0 \), then in the Palestra a quaternionic step \( \Delta s_{\varphi} \) will be taken that differs from the corresponding flat step \( \Delta s_f \)

\[ \Delta s_f = \Delta x_0 + i \, \Delta x_1 + j \, \Delta x_2 + k \, \Delta x_3 \] (1)

\[ \Delta s_{\varphi} = q^0 \, \Delta x_0 + q^1 \, \Delta x_1 + q^2 \, \Delta x_2 + q^3 \, \Delta x_3 \] (2)

The coefficients \( q^\mu \) are quaternions. The \( \Delta x_\mu \) are steps taken in the (flat) parameter space of the (partial) allocation function \( \varrho(x) \).

### 9.1.3 Pacific space and black regions.

If we treat the Palestra as a continuum, then the parameter space of the allocation function is a flat space that it is spanned by the number system of the quaternions. This parameter space gets the name “Pacific space”. This is the space where the RQE’s live. If in a certain region of the Palestra no matter is present, then in that region the Palestra is nearly equal to the parameter space of the allocation function.

The Pacific space has the advantage that when distributions are converted to this parameter space the Fourier transform of the converted distributions is not affected by curvature.

In a region where the curvature is high, the Palestra step comes close to zero. At the end where the Palestra step reaches the smallest rational value, an information horizon is established. For a distant observer, nothing can pass that horizon. The information horizon encloses a black region.

Inside that region the quantum state functions are so densely packed that they lose their identity. However, they do not lose their sign flavor. The result is the formation of a single quantum state function that consists of the superposition of all contributing quantum state functions. The resulting black body has mass, electric charge and angular momentum. The quantum state function of a black region is quantized. Due to the fact that no information can escape through the information horizon, the inside of the horizon is obscure. No experiment can reveal its content. It
does not contain a singularity at its center. All characteristics of the black region are contained in its quantum state function\textsuperscript{72}. The (partial) allocation function $\varphi(x)$ is a continuous quaternionic distribution. Like all continuous quaternionic distributions it contains two fields. It is NOT a QPAD. It does not contain density distributions.

9.1.4 Start of the universe.
At the start of the universe the package density was so high that also in that condition only one mixed QPAD can exist. That QPAD was a superposition of QPAD’s that have different sign flavors. Only when the universe expands enough, multiple individual Qpatterns may have been generated. In the beginning, these QPAD’s were uncoupled.

\textsuperscript{72} See Cosmological history
10 Modularization
A very powerful influencer is modularization. Together with the corresponding encapsulation it has a very healthy influence on the relational complexity of the ensemble of objects on which modularization works. The encapsulation takes care of the fact that most relations are kept internal to the module. When relations between modules are reduced to a few types, then the module becomes reusable. The most Influential kind of modularization is achieved when modules can be configured from lower order modules.
Elementary particles can be considered as the lowest level of modules. All composites are higher level modules.
When sufficient resources in the form of reusable modules are present, then modularization can reach enormous heights. On earth it was capable to generate intelligent species.

10.1 Complexity

**Potential complexity** of a set of objects is a measure that is defined by the number of potential relations that exist between the members of that set.
If there are n elements in the set, then there exist n*(n-1) potential relations.

**Actual complexity** of a set of objects is a measure that is defined by the number of relevant relations that exist between the members of the set.

In human affairs and with intelligent design it takes time and other resources to determine whether a relation is relevant or not. Only an expert has the knowledge that a given relation is relevant. Thus it is advantageous to have as little irrelevant potential relations as is possible, such that mainly relevant and preferably usable relations result.

Physics is based on relations. Quantum logic is a set of axioms that restrict the relations that exist between quantum logical propositions. Via its isomorphism with Hilbert spaces quantum logic forms a fundament for quantum physics. Classical logic is a similar set of restrictions that define how we can communicate logically. Like classical logic, quantum logic only describes static relations. Traditional quantum logic does not treat physical fields and it does not touch dynamics. However, the model that is based on traditional quantum logic can be extended such that physical fields are included as well and by assuming that dynamics is the travel along subsequent versions of extended quantum logics, also dynamics will be treated. The set of propositions of traditional quantum logic is isomorphic with the set of closed subspaces of a Hilbert space. This is a mathematical construct in which quantum physicists do their investigations and calculations. In this way fundamental physics can be constructed. Here holds very strongly that only relevant relations have significance.

10.2 Relational complexity

We define relational complexity as the ratio of the number of actual relations divided by the number of potential relations.

10.3 Interfaces

Modules connect via interfaces. Interfaces are used by interactions. Interactions run via (relevant) relations. Relations that act within modules are lost to the outside world of the module. Thus interfaces are collections of relations that are used by interactions. Inbound interactions come from the past. Outbound interactions go to the future. Two-sided interactions are cyclic. They are either oscillations or rotations of the inter-actor.
10.4 Interface types
Apart from the fact that they are inbound, outbound or cyclic the interfaces can be categorized with respect to the type of relations that they represent. Each category corresponds to an interface type. An interface that possesses a type and that installs the possibility to couple the corresponding module to other modules is called a standard interface.

10.5 Modular subsystems
Modular subsystems consist of connected modules. They need not be modules. They become modules when they are encapsulated and offer standard interfaces that makes the encapsulated system a reusable object.
The cyclic interactions bind the corresponding modules together. Like the coupling factor of elementary particles characterizes the binding of the pair of Qpatterns will a similar characteristic characterize the binding of modules.
This binding characteristic directly relates to the total energy of the constituted sub-system. Let $\psi$ represent the renormalized superposition of the involved distributions.

$$\forall \psi = \phi = m \varphi \quad (1)$$
$$\int_{V} |\psi|^{2} \, dV = \int_{V} |\varphi|^{2} \, dV = 1 \quad (2)$$
$$\int_{V} |\phi|^{2} \, dV = m^{2} \quad (3)$$

Here again $m$ represents total energy.
The binding factor is the total energy of the sub-system minus the sum of the total energies of the separate constituents.

10.6 Relational complexity indicators
The inner product of two Hilbert vectors is a measure of the relational complexity of the combination.
A Hilbert vector represents a linear combination of atoms. When all coefficients are equal, then the vector represents an assembly of atoms. When the coefficients are not equal, then the vector represents a weighted assembly of atoms.
For two normalized vectors $|a\rangle$ and $|b\rangle$:
$$\langle a | a \rangle = 1 \quad (1)$$
$$\langle b | b \rangle = 1 \quad (2)$$
$$\langle a | b \rangle = 0 \text{ means } |a\rangle \text{ and } |b\rangle \text{ are not related.} \quad (3)$$
$$\langle a | b \rangle \neq 0 \text{ means } |a\rangle \text{ and } |b\rangle \text{ are related.} \quad (4)$$
$$|\langle a | b \rangle| = 1 \text{ means } |a\rangle \text{ and } |b\rangle \text{ are optimally related.} \quad (5)$$

10.7 Modular actions
Subsystems that have the ability to choose their activity can choose to organize their actions in a modular way. As with static relational modularization the modular actions reduce complexity and for the decision maker it eases control.

10.8 Random design versus intelligent design
At lower levels of modularization nature design modular structures in a stochastic way. This renders the modularization process rather slow. It takes a huge amount of progression steps in order to achieve a relatively complicated structure. Still the complexity of that structure can be orders of magnitude less than the complexity of an equivalent monolith.
As soon as more intelligent sub-systems arrive, then these systems can design and construct modular systems in a more intelligent way. They use resources efficiently. This speeds the modularization process in an enormous way.
11 Functions that are invariant under Fourier transformation.
A subset of the (quaternionic) distributions have the same shape in configuration space and in the linear canonical conjugated space.
We call them dual space distributions. It are functions that are invariant under Fourier transformation\textsuperscript{73}. These functions are not eigenfunctions.
The Qpatterns and the harmonic and spherical oscillations belong to this class.
Fourier-invariant functions show iso-resolution, that is, $\Delta_p = \Delta_q$ in the Heisenberg’s uncertainty relation.

11.1 Natures preference
Nature seems to have a preference for quaternionic distributions that are invariant under Fourier transformation.
A possible explanation is the two-step generation process, where the first step is realized in configuration space and the second step is realized in canonical conjugated space. The whole pattern is generated two-step by two-step. The only way to keep coherence between a distribution and its Fourier transform that are both generated step by step is to generate them in pairs.

12 Events

12.1 Generations and annihilations
At the instant of generation or annihilation, the enumerator generator will change its mode and the Qpattern that will be generated changes its mode as well. If the number of enumerator creations per step that contributes to a Qpattern is left open and if this number is larger than one, then it is difficult to understand that at a given instant the whole Qpattern changes its mode. The Qpattern has no knowledge of the mode that its members are in. The individual members might have that knowledge. In that case it is part of their charge. So, from now on we suppose that the Qpatterns will be generated such that one member, the Qtarget, is generated per progression step. An event then indicates that the enumeration generator changes its generation mode. For example, when a particle is annihilated the generator switches from generating a Qpattern in configuration space to generating an equivalent pattern in the canonical conjugated space. The result is that the pattern is no longer coupled and becomes a photon or a gluon. Of course the reverse procedure occurs at the generation of a particle. In the original space, the object that corresponds to the Qpattern is annihilated while in the new space the transformed object is generated. Since the Qpattern is generated with a Qtarget at each progression step the event has immediate consequences. Conservation laws govern the annihilation and creation processes.

12.2 Emissions and absorptions
When only a part of a composite annihilates, then a similar process can take place. A sub-module is annihilated and either the whole energy is emitted in the form of radiation or only part of the energy is emitted and the rest is used to constitute a new particle at a lower energy level.

\textsuperscript{73} Q-Formulæ contains a section about functions that are invariant under Fourier transformation.
It is also possible that a complete sub-module is emitted. This can be done in a two-step mode, where first the sub-module or part of it is converted into radiation and subsequently the sub-module is regenerated. Absorption is described as the reverse process.

12.3 Oscillating interactions
Oscillating interactions are implemented by cyclic interfaces. They consist of a sequence of annihilations and generations, where the locations alternate.

12.4 Movements
The fact that a particle moves, and the fact that a Qpattern is generated with only one Qtarget per progression step means that during a movement the Qpattern is spread along the path of movement.

12.5 Curvature
When the generator operates in one space and produces there a compact distribution then it affects the curvature of that space. It also has consequences in the canonical conjugated space. However, there the corresponding distribution will be spread out. Its effect on space curvature will also be spread. As a result the effect on space curvature in this canonical conjugated space will be negligible.

12.6 Tsunami
After annihilation in configuration space the new generation in the canonical conjugated space looks like the build-up of a 3D tsunami wave in configuration space. It races with light speed away from the point of annihilation. The process is described in the section on the enumeration process. This lasts until a new generated Qtarget is captured and the generation mode switches again. This happens most probably somewhere at a high amplitude of the tsunami wave. The next Qtarget is generated in configuration space.
13 Entanglement
In the Hilbert Book Model, entanglement enters the model only after a huge number of extension steps. It is due to the fact that nature’s building blocks have a set of discrete properties that can be observed via indirect means, while the building block may extend over rather large distances. So measuring the same property at nearly the same instant at quite different locations will give the same result. When the property is changed shortly before these measurements were performed, then it might give the impression that an instant action at a distance occurred, because light could not bridge these locations in the period between the two measurements. The explanation is that the building block at each progression instant moves to a different step stone and that these step stones may lay far apart. Apart from the property measurements, in this process no information transfer needs to take place. The measurements must be done without affecting the building block. At each arrival at a step stone the building block transmits contributions to its potentials. If the measurement uses these potentials, then the building block is not affected. According to this explanation, at least one progression step must separate the two measurements.

14 Cosmology
14.1 Cosmological view
Even when space was fully densely packed with matter (or another substance) then nothing dynamic would happen. Only when sufficient interspacing comes available dynamics becomes possible. The Hilbert Book Model exploits this possibility. It sees black regions as local returns to the original condition.

14.2 The cosmological equations
The integral equations that describe cosmology are:

\[
\frac{d}{d\tau} \int_V \rho \, dV + \oint_S \hat{n} \rho \, dS = \int_V s \, dV \tag{1}
\]

\[
\int_V \nabla \rho \, dV = \int_V s \, dV \tag{2}
\]

Here \(\hat{n}\) is the normal vector pointing outward the surrounding surface \(S\), \(v(\tau, q)\) is the velocity at which the charge density \(\rho_0(\tau, q)\) enters volume \(V\) and \(s_0\) is the source density inside \(V\). In the above formula \(\rho\) stands for

\[
\rho = \rho_0 + \rho = \rho_0 + \frac{\rho_0 v}{c} \tag{3}
\]

It is the flux (flow per unit of area and per unit of progression) of \(\rho_0\). \(t\) stands for progression (not coordinate time).

14.3 Inversion surfaces
An inversion surface \(S\) is characterized by:

\[
\oint_S \hat{n} \rho \, dS = 0 \tag{1}
\]
14.4 Cosmological history
The inversion surfaces divide universe into compartments. Think that these universe pockets contain matter that is on its way back to its natal state. If there is enough matter in the pocket this state forms a black region. The rest of the pocket is cleared from its mass content. Still the size of the pocket may increase. This represents the expansion of the universe. Inside the pocket the holographic principle governs. The black region represents the densest packaging mode of entropy.

The pockets may merge. Thus at last a very large part of the universe may return to its birth state, which is a state of densest packaging of entropy. Then the resulting mass which is positioned at a huge distance will enforce a uniform attraction. This uniform attraction will install an isotropic extension of the central package. This will disturb the densest packaging quality of that package. The motor behind this is formed by the combination of the attraction through distant massive particles, which installs an isotropic expansion and the influence of the small scale random localization which is present even in the state of densest packaging. This describes an eternal process that takes place in and between the pockets of an affine space.

14.5 Entropy
As a whole, universe expands. Locally regions exist where contraction overwhelms the global expansion. These regions are separated by inversion surfaces. The regions are characterized by their inversion surface. Within these regions the holographic principle resides. The fact that the universe as a whole expands means that the average size of the encapsulated regions increases. The holographic principle says that the total entropy of the region equals the entropy of a black region that would contain all matter in the region. Black regions represent regions where entropy is optimally packed. Thus entropy is directly related to the interspacing between enumerators. With other words, local entropy is related to local curvature.
15 Recapitulation

The model starts by taking quantum logic as its foundation. Next quantum logic is refined to Hilbert logic. It could as well have started by taking an infinite dimensional separable Hilbert space as its foundation. However, in that case the special role of base vectors would not so easily have been brought to the front. It appears that the atoms of the logic system and the base vectors of the Hilbert space play a very crucial role in the model. They represent the lowest level of objects in nature that play the theater of our observation. The atoms are only principally unordered at very small “distances”. They have content. The Hilbert space offers built-in enumerator machinery that defines the distances and that specifies the content of the represented atoms. The same can be achieved in a refined version of quantum logic that we call Hilbert logic.

In fact we focus on a compartment of universe, where universe is an affine space. The isotropic scaling factor that was assumed in the early phases of the model appears to relate to mass carrying particles that exist at huge distances. In the considered compartment an enumeration process establishes a kind of coordinate system. The master of the enumeration process is the allocation function $\mathcal{P}$. This function has a flat parameter space.

$$\mathcal{P} = \phi \circ \psi$$

At small scales this function becomes a spread function $\psi$ that governs the quantum physics of the model. The whole function $\mathcal{P}$ is a convolution of a sharp part $\phi$ and the spread function $\psi$. The differential of $\phi$ delivers a local metric. The spread function appears to be generated by a Poisson generator which produces Qpatterns.

After a myriad of progression steps the original ordering of the natal state of the model is disturbed so much that the natal large and medium scale ordering is largely lost. However, this natal ordering is returning in the black regions that constitute pockets that surround them in universe. When the pockets merge into a huge black region, the history might restart enforced by the still existing low scale randomization and by the isotropic expansion factor, which is the consequence of the existence of massive particles at huge distances in the affine space.

The model uses a first part where elementary particles are formed by the representatives of the atomic propositions of the logic.

In a second part the formation of composites is described by a process called modularization. In that stage, in places where sufficient resources are present, the modularization process is capable of forming intelligent species.

This is the start of a new phase of evolution in which the intelligent species get involved in the modularization process and shift the mode from random design to intelligent design. Intelligent design runs much faster and uses its resources in a more efficient and conscientious way.
16 Conclusion

With respect to conventional physics, this simple model contains extra layers of individual objects. The most interesting addition is formed by the RQE’s, the Qpatches, the Qtargets and the Qpatterns. They represent the atoms of the quantum logic sub-model. The model gives an acceptable explanation for the existence of an (average) maximum velocity of information transfer. The two prepositions:

- Atomic quantum logic fundament
- Correlation vehicle

Lead to the existence of fuzzy interspacing of enumerators of the Hilbert space base vectors and to dynamically varying space curvature when compared to a flat reference continuum. Without the freedom that is introduced by the interspacing fuzziness and which is used by the dynamic curvature, no dynamic behavior would be observable in the Palestra.

In the generation of the model the enumeration process plays a crucial role, but we must keep in mind that the choice of the enumerators and therefore the choice of the type of correlation vehicle is to a large degree arbitrary. It means that the Palestra has no natural origin. It is an affine space. The choice for quaternions as enumerators seems to be justified by the fact that the sign flavors of the quaternions explain the diversity of elementary particles.

Physicist that base their model of physics on an equivalent of the Gelfand triple which lacks a mechanism that creates the freedom that flexible interspaces provide, are using a model in which no natural curvature and fuzziness can occur. Such a model cannot feature dynamics. Attaching a progression parameter to that model can only create the illusion of dynamics. However, that model cannot give a proper explanation of the existence of space curvature, space expansion, quantum physics or even the existence of a maximum speed of information transfer. Physics made its greatest misstep after the nineteen thirties when it turned away from the fundamental work of Garret Birkhoff and John von Neumann. This deviation did not prohibit pragmatic use of the new methodology. However, it did prevent deep understanding of that technology because the methodology is ill founded.

Doing quantum physics in continuous function spaces is possible, but it makes it impossible to find the origins of dynamics, curvature and inertia. Most importantly it makes it impossible to find the reason of existence of quantum physics. Only the acceptance of the fact that physics is fundamentally countable can solve this dilemma. Please attack these statements with your criticism.