A rewriting system applied to the simplest algebraic identities

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A rewriting system applied to the simplest algebraic identities is shown to yield second- and third-order equations that share a property associated with 137.036.

I. TWO SYMMETRIC IDENTITIES

The symmetry of this second-order identity
\[ M^2 = M^2 \]
and this third-order identity
\[ \left( \frac{M}{N} \right)^3 + M^2 = \left( \frac{M}{N} \right)^3 + M^2 \]
will be “broken” by making the substitution
\[ M \rightarrow M - y \]
on their left hand sides, and the substitution
\[ M^n \rightarrow M^n - x^p \]
on their right hand sides, where \( p \) equals the order of each identity. Above, \( y \) and \( x \) are variables such that
\[ 0 < y \leq 0.1 \quad \text{(1a)} \]
\[ 0 < x \leq 0.1 \quad \text{(1b)} \]
whereas \( M \) and \( N \) are positive integer constants fulfilling
\[ M = \frac{N^3}{3} + 1 \quad \text{(1c)} \]
so that necessarily
\[ M \geq 10 \quad \text{(1d)} \]
The reason for altering these identities using the above rewriting system (an admittedly unusual thing to do) is to change them from related identities that are true for all values of \( M \) and \( N \), into slightly asymmetric conditional equations that are true for some values of \( M \) and \( N \). It will be shown that the resultant equations share an interesting property involving their derivatives, where this property is associated with 137.036.

II. TWO CONDITIONAL EQUATIONS

Begin with the second-order identity
\[ M^2 = M^2 \]
and break its symmetry by making the substitution
\[ M \rightarrow M - y \]
on its left hand side, and the substitution
\[ M^n \rightarrow M^n - x^p \]
on its right hand side, where \( p = 2 \), the identity’s order. This produces
\[ (M - y)^2 = M^2 - x^2 \quad \text{(2a)} \]
Similarly, for the third-order identity
\[ \left( \frac{M}{N} \right)^3 + M^2 = \left( \frac{M}{N} \right)^3 + M^2 \]
apply the same substitutions, where \( p = 3 \), to get
\[ \left( \frac{M - y}{N} \right)^3 + (M - y)^2 = \frac{M^3 - x^3}{N^3} + M^2 - x^3 \quad \text{(2b)} \]

III. THEIR SHARED PROPERTY

Theorem 1 will show that for Eq. (2a)
\[ \frac{dy}{dx} \approx \frac{x}{M} \]
whereas Theorem 2 will show that for Eq. (2b)
\[ \frac{dy}{dx} \approx \frac{x^2}{M} \]
Accordingly, for both equations
\[ \frac{dy}{dx} \approx \frac{1}{M^p} \quad \text{(3)} \]
at
\[ x = \frac{1}{M} \]
where \( p \) equals the order of each equation.

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Theorem 1. Assume Eq. [2a]

\[(M - y)^2 = M^2 - x^2 \] .

Then

\[ \frac{dy}{dx} \approx \frac{x}{M} . \] (4a)

Proof. Equation [2a]

\[(M - y)^2 = M^2 - x^2 \] expands and simplifies to

\[ 2My - y^2 = x^2 . \]

It follows that

\[ 2Mdy - 2ydy = 2dx . \]

But by Eq. [1a] \( y \leq 0.1 \) and by Eq. [1d] \( M \geq 10 \), so that \( 2ydy \) is small compared to \( 2Mdy \). Hence,

\[ \frac{dy}{dx} \approx \frac{x}{M} \]

holds.

Theorem 2. Assume Eq. [2b]

\[ \left( \frac{M - y}{N^3} \right)^3 + (M - y)^2 = \frac{M^3 - x^3}{N^3} + M^2 - x^3 . \]

Then

\[ \frac{dy}{dx} \approx \frac{x^2}{M} . \] (4b)

Proof. Equation [2b]

\[ \frac{(M - y)^3}{N^3} + (M - y)^2 = \frac{M^3 - x^3}{N^3} + M^2 - x^3 \]

expands and simplifies to

\[ - \frac{3M^2y}{N^3} + \frac{3My^2}{N^3} - \frac{y^3}{N^3} - 2My + y^2 = - \frac{x^3}{N^3} - x^3 , \]

or

\[ 3M^2y - 3My^2 + y^3 + 2MN^3y - N^3y^2 = (N^3 + 1)x^3 . \]

It follows that

\[ (3M^2 - 6My + 3y^2 + 2MN^3 - 2N^3y)dy = 3(N^3 + 1)x^2 dx \]

so that

\[ \frac{dy}{dx} = \frac{3(N^3 + 1)x^2}{3M^2 - 6My + 3y^2 + 2MN^3 - 2N^3y} . \]

We now want to identify and remove the smallest terms from the denominator. As Eq. (1c) requires that

\[ N^3 = 3M - 3 \] ,

substituting for \( N^3 \) gives

\[ \frac{dy}{dx} = \frac{3(3M - 3 + 1)x^2}{3(3M - 2)x^2} \]

\[ = \frac{3M^2 - 6My + 3y^2 + 2M(3M - 3) - 2(3M - 3)y}{3M - 2} \]

\[ = \frac{9M^2 - 12My + 3y^2 - 6M + 6y}{(3M - 2)x^2} \]

\[ = \frac{3M^2 - 4My + y^2 - 2M + 2y}{(3M - 2)x^2} \]

\[ = \frac{(3M - 2)M - 4My + y^2 + 2y}{(3M - 2)x^2} . \]

But by Eq. [1a] \( y \leq 0.1 \) and by Eq. [1d] \( M \geq 10 \), so \( 4My, y^2, \) and \( 2y \) are small compared to \((3M - 2)M\). So,

\[ \frac{dy}{dx} \approx \frac{3 - 2}{3M - 2} \times \frac{x^2}{M} . \]

Accordingly, the approximation

\[ \frac{dy}{dx} \approx \frac{x^2}{M} \]

holds.

Remark 1. When comparing Eq. [4a] of Theorem 1 against Eq. [4b] of Theorem 2 we see that for both

\[ \frac{dy}{dx} \approx \frac{x^p - 1}{M} , \] (4c)

with only their values for \( p \) differing (2 and 3, respectively). At \( x = 1/M \) this gives Eq. [3], the shared property introduced in the previous section.

IV. THE MINIMAL CASE AND 137.036

As promised at the beginning of Section [III], Theorems 1 and 2 show that for Eqs. [2a] and [2b]

\[ \frac{dy}{dx} \approx \frac{1}{M^p} \text{ at } x = \frac{1}{M} , \] (5)

where \( p \) equals the order of each equation. But we know that \( M = N^3/3 + 1 \) for Eq. [2b], and that the smallest positive integers \( (M, N) \) fulfilling this condition are \((10, 3)\). So for this minimal case the right hand side of Eq. [2b] at \( x = 1/M \) gives

\[ \frac{M^3 - x^3}{N^3} + M^2 - x^3 = \frac{10^3 - 10^{-3}}{3^3} + 10^2 - 10^{-3} \]

\[ = 999.999 + 99.999 \]

\[ = 137.036 . \]

This makes 137.036 the smallest value at which the third-order equation behaves like the second-order equation in fulfilling Eq. [5], which makes 137.036 a fundamental constant for Eqs. [2a] and [2b].