The local Doppler effect due to Earth’s rotation and the CNGS neutrino anomaly due to neutrino’s index of refraction through the Earth Crust

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In this brief paper, we show the neutrino velocity discrepancy obtained in [1] may be due to the local Doppler effect between a local clock attached to a given detector at Gran Sasso, say $C_G$, and the respective instantaneous clock crossing $C_C$, say $C_{C'}$, being this latter at rest in the instantaneous inertial frame having got the velocity of rotation of CERN about Earth’s axis in relation to the fixed stars. With this effect, the index of refraction of the Earth crust may accomplish a refractive effect by which the neutrino velocity through the Earth crust turns out to be small in relation to the speed of light in the empty space.

1 Definitions and Solution

Firstly, the effect investigated here is not the same one that was investigated in [2], but, throughout this paper, we will use some useful configurations defined in [2]. The relative velocity between Gran Sasso and CERN due to the Earth daily rotation may be written:

$$\vec{v}_G - \vec{v}_C = 2\omega R \sin \alpha \hat{z}, \quad (1)$$

where $\hat{z}$ is a convenient unitary vector, the same used in [2], $\omega$ is the norm of the Earth angular velocity vector about its daily rotation axis, being $R$ given by:

$$R_E = \frac{R}{\cos \lambda}, \quad (2)$$

where $R_E$ is the radius of the Earth, its averaged value $R_E = 6.37 \times 10^6 m$, and $\alpha$ given by:

$$\alpha = \frac{1}{2} (\alpha_C - \alpha_G), \quad (3)$$

where $\alpha_C$ and $\alpha_G$ are, respectively, CERN’s and Gran Sasso’s longitudes ($\leftarrow WE \rightarrow$). Consider the inertial (in relation to the fixed stars) reference frame $O_{C-x_C y_C z_C} \equiv Oxyc$ in [2]. This is the lab reference frame and consider this frame with its local clocks at each spatial position as being ideally synchronized, viz., under an ideal situation of synchronicity between the clocks of $O_{C-x_C y_C z_C} \equiv Oxyc$. This situation is the expected ideal situation for the OPERA collaboration regarding synchronicity in the instantaneous lab (CERN) frame.

Now, consider an interaction between a single neutrino and a local detector at Gran Sasso. This event occurs at a given spacetime point $(t_r, x_r, y_r, z_r)$ in $O_{C-x_C y_C z_C} \equiv Oxyc$. The interaction instant $t_r$ is measured by a local clock $C_C$ at rest at $(x_r, y_r, z_r)$ in the lab frame, viz., in the $O_{C-x_C y_C z_C} \equiv Oxyc$ frame. But, under gedanken, at this instant $t_r$, according to $O_{C-x_C y_C z_C} \equiv Oxyc$, there is a clock $C_G$ attached to the detector at Gran Sasso that crosses the point $(t_r, x_r, y_r, z_r)$ with velocity given by Eq. (1). Since $C_G$ crosses $C_{C'}$, the Doppler effect between the proper tic-tac rates measured at each location of $C_C$ and $C_G$, viz., measured at their respective locations in their respective reference frames (the reference frame of $C_G$ is the $O_{C-x_C y_C z_C} \equiv Oxyc$ in [2], also inertial in relation to the fixed stars), regarding a gedanken control tic-tac rate continuously sent by $C_C$, say via electromagnetic pulses from $C_C$, is not transverse. Since the points at which $C_C$ and $C_G$ are at rest in their respective reference frames will instantaneously coincide, better saying, will instantaneously intersect, at $t_r$ accordingly to $C_{C'}$, they must be previously approximating, shortening their mutual distance during the interval $t_r - \delta t_r \rightarrow t_r (\delta t_r \rightarrow 0, \delta t_r > 0)$ along the line passing through these clocks as described in the $C_C$ world.

Suppose $C_C$ sends $N$ electromagnetic pulses to $C_G$. During the $C_C$ proper time interval $(t_r - \delta t_r) - 0 = t_r - \delta t_r$ * within which $C_C$ emits the $N$ electromagnetic pulses, the first emitted pulse travels the distance $c (t_r - \delta t_r)$ and reaches the clock $C_G$, as described by $C_C$. Within this distance, there are $N$ equally spaced distances between consecutive pulses as described in the $C_C$ world, say $\lambda$:

$$N \lambda_C = c (t_r - \delta t_r). \quad (4)$$

Also, since the clocks $C_C$ and $C_G$ will intersect at $t_r$, as described in $O_{C-x_C y_C z_C} \equiv Oxyc$, during the interval $\delta t_r$, the clock $C_G$ must travel the distance $2\omega R \sin \alpha \delta t_r$ in the $C_C$ world to accomplish the matching spatial intersection at the instant $t_r$, hence the clock $C_G$ travels the $2\omega R \sin \alpha \delta t_r$ in the

*The initial instant $C_C$ starts to emit the electromagnetic pulses is set to zero in both the frames $O_{C-x_C y_C z_C} \equiv Oxyc$ and $O_{C-x_C y_C z_C} \equiv Oxyc$; zero also is the instant the neutrino starts the travel to Gran Sasso in $O_{C-x_C y_C z_C} \equiv Oxyc$; hence the instant the neutrino starts the travel to Gran Sasso and the emission of the first pulse by $C_C$ are simultaneous events in $O_{C-x_C y_C z_C} \equiv Oxyc$. These events are simultaneous in $O_{C-x_C y_C z_C} \equiv Oxyc$ too, since they have got the same spatial coordinate $z_r \equiv z = 0$ along the $O_{C-x_C y_C z_C} \equiv Oxyc$ direction as defined in [2]. The relative motion between CERN and Gran Sasso is parallel to this direction. The only one difference between these events is the difference in their $x_r \equiv x$ coordinates, being $x_r = 0$ for the neutrino departure and $x_r = L = 7.3 \times 10^4 m$ for $C_C$, being these locations perpendicularly located in relation to the relative velocity given by the Eq. (1).
$C_C$ world, viz., as described by $C_C$ in $O_{C_C} C_R C_C \equiv O_{xy} C_R$:

$$N \lambda_C = 2 \omega^R \sin \alpha \delta t_r \Rightarrow \delta t_r = N \frac{\lambda_C}{2 \omega^R \sin \alpha}. \tag{5}$$

Solving for $t_r$, from the Eqs. (4) and (5), one reaches:

$$t_r = \frac{N \lambda_C}{c} \left(1 + \frac{c}{2 \omega^R \sin \alpha}\right). \tag{6}$$

Now, from the perspective of $C_G$, in $O_{C_G} y_G z_G \equiv O_{\bar{y} \bar{y} \bar{z}}$, there must be $N$ equally spaced pulses covering the distance:

$$c \left(t^G_r - \delta t^G_r \right) = 2 \omega^R \sin \alpha \left(t^G_r - \delta t^G_r \right), \tag{7}$$

where $t^G_r - \delta t^G_r$ is the time interval between the non-proper instants $t^G_r = t_r = 0$, at which the $C_C$ clock sends the first pulse, and the instant $t^G_r - \delta t^G_r$, at which this first pulse reaches $C_G$, as described by $C_G$ in its world $O_{C_G} y_G z_G \equiv O_{\bar{y} \bar{y} \bar{z}}$. During the time interval $t^G_r - \delta t^G_r$, $C_C$ describes, in its $O_{C_C} y_C z_C \equiv O_{\bar{y} \bar{y} \bar{z}}$ world, the clock $C_C$ approximating the distance:

$$2 \omega^R \sin \alpha \left(\delta t^G_r \right), \tag{8}$$

with the first pulse traveling:

$$c \left(t^G_r - \delta t^G_r \right), \tag{9}$$

giving the distance within which there must be $N$ equally spaced pulses, say, spaced by $\lambda_G$, as described by $C_G$ in its $O_{C_G} y_G z_G \equiv O_{\bar{y} \bar{y} \bar{z}}$ world:

$$N \lambda_G = (c - 2 \omega^R \sin \alpha) \left(t^G_r - \delta t^G_r \right). \tag{10}$$

With similar reasoning that led to the Eq. (5), now in the $O_{C_G} y_G z_G \equiv O_{\bar{y} \bar{y} \bar{z}}$ world, prior to the spatial matching intersection between $C_C$ and $C_G$, the $C_C$ clock must travel the distance $N \lambda_G$ during the time interval $\delta t^G_r$, with the $C_C$ approximation velocity $2 \omega^R \sin \alpha$:

$$N \lambda_G = 2 \omega^R \sin \alpha \delta t^G_r \Rightarrow \delta t^G_r = N \frac{\lambda_G}{2 \omega^R \sin \alpha}. \tag{11}$$

From Eqs. (10) and (11), we solve for $t^G_r$:

$$t^G_r = N \frac{\lambda_G}{2 \omega^R \sin \alpha} \left[1 - \frac{c}{1 - \left(2 \omega^R \sin \alpha \right)^2 / c^2}\right]. \tag{12}$$

From the Eqs. (6) and (12), we have got the relation between the neutrino arrival instant $t_r$, as measured by the CERN reference frame, $O_{C_C} y_C z_C \equiv O_{xy} C_R$, and the neutrino arrival instant $t^G_r$ as measured by the Gran Sasso reference frame, $O_{C_G} y_G z_G \equiv O_{\bar{y} \bar{y} \bar{z}}$, at the exact location of the interaction at an interaction location within the Gran Sasso block of detectors, provided the effect of the Earth daily rotation under the assumptions we are taking in relation to the instantaneous movements of these locations in relation to the fixed stars as previously discussed:

$$\frac{t^G_r}{t_r} = \frac{\lambda_G}{\lambda_C} \left[1 - \left(2 \omega^R \sin \alpha \right)^2 / c^2\right]^{-1} = \gamma^2 \frac{\lambda_G}{\lambda_C}, \tag{13}$$

where $\gamma \geq 1$ is the usual relativity factor as defined above.

Now, $\lambda_G / \lambda_C$ is simply the ratio between the spatial displacement between our consecutive gedanken control pulses, being these displacements defined through our previous paragraphs, leading to the Eqs. (4) and (10). Of course, this ratio is simply given by the relativistic Doppler effect under an approximation case in which $C_C$ is the source and $C_G$ the detector. The ratio between the Eqs. (10) and (4) gives:

$$\frac{\lambda_G}{\lambda_C} = \left[1 - \left(2 \omega^R \sin \alpha \right)^2 / c^2\right]^{-1/2}. \tag{14}$$

But the time interval $(t_r - \delta t_r)$ is a proper time interval measured by the source clock $C_C$, as previously discussed. It accounts for the time interval between the first pulse sent and the last pulse sent as locally described by $C_C$ is its $O_{C_C} y_C z_C \equiv O_{xy} C_R$ world. These two events occur at different spatial locations in the $C_G$ detector clock world $O_{C_G} y_G z_G \equiv O_{\bar{y} \bar{y} \bar{z}}$, since $C_C$ is approximating to $C_G$ is this latter world. Hence, $t_r - \delta t_r$ is the Lorentz time contraction of $t^G_r - \delta t^G_r$, viz.:

$$t_r - \delta t_r = \gamma^{-1} \left(t^G_r - \delta t^G_r \right), \tag{15}$$

With the Eqs. (14) and (15), one reaches the usual relativistic Doppler effect expression for the approximation case:

$$\frac{\lambda_G}{\lambda_C} = \sqrt{\frac{1 - \left(2 \omega^R \sin \alpha \right)^2 / c^2}{1 + \left(2 \omega^R \sin \alpha \right)^2 / c^2}}. \tag{16}$$

With the Eq. (16), the Eq. (13) reads:

$$\frac{t^G_r}{t_r} = \left[1 - \left(2 \omega^R \sin \alpha \right)^2 / c^2\right]^{-1/2} \left[1 + \left(2 \omega^R \sin \alpha \right) / c\right]^{-1} = \gamma.$$

Since $(2 \omega^R \sin \alpha) / c \ll 1$, we may apply an approximation for the Eq. (17), viz.:

$$\gamma \approx 1 + \frac{1}{2} \frac{\left(2 \omega^R \sin \alpha \right)^2 / c^2}{2}, \tag{18}$$

and:

$$\left[1 + \left(2 \omega^R \sin \alpha \right) / c\right]^{-1} \approx 1 - \left(2 \omega^R \sin \alpha \right) / c, \tag{19}$$

from which, neglecting the higher order terms, the Eq. (17) reads:

$$\frac{t^G_r}{t_r} \approx 1 - \frac{2 \omega^R \sin \alpha}{c}, \tag{20}$$

$$t^G_r - t_r = - \frac{2 \omega^R \sin \alpha}{c} t_r. \tag{21}$$
From this result, the clock that tag the arrival interaction instant \( t^G_{\nu} \) in Gran Sasso turns out to measure an arrival time that is shorter than the correct one, this latter given by \( t_{\nu} \). With the discrepancy, \( \epsilon \), given by the value measured by the OPERA Collaboration [1], since \( t_{\nu} \) is simply given by \( L/v_{\nu} \), where \( L \) is the baseline distance between the CERN and Gran Sasso, \( v_{\nu} \) the speed of neutrino through the Earth crust, one obtains a value for \( v_{\nu} \). We rewrite the Eq. (21):

\[
\epsilon = t^G_{\nu} - t_{\nu} = -\frac{2\omega R \sin \alpha}{c} \frac{L}{v_{\nu}}. \tag{22}
\]

With the values \( \omega = 7.3 \times 10^{-5} \text{s}^{-1}, R = R_E \cos \lambda \approx 6.4 \times 10^{6} \text{m} \times \cos(\pi/4) = 4.5 \times 10^{6} \text{m}, \sin \alpha \approx \sin(7\pi/180) = 1.2 \times 10^{-1}, c = 3.0 \times 10^{8} \text{ms}^{-1} \) and \( L = 7.3 \times 10^{6} \text{m} \), also with the discrepancy \( \epsilon \), given by the Eq. (22), being, say, \( \epsilon = -62 \times 10^{-9} \text{s} \), the neutrino velocity through the Earth crust reads:

\[
v_{\nu} \approx 3.1 \times 10^{6} \text{ms}^{-1}, \tag{23}
\]

being the refraction index of the Earth crust for neutrino given by:

\[
n_{c|\nu} = \frac{c}{v_{\nu}} \approx 97. \tag{24}
\]

In reference to the matching PDF (probability distribution function) in the OPERA experiment, one would have a discrepancy between the maximum likelihood distribution obtained from the block of detectors at Gran Sasso and the translation of the PDF due to the protons distribution by TOF, given by, in virtue of the Eq. (22):

\[
\text{TOF}_{\nu} = \text{TOF}_{c} + \epsilon = \text{TOF}_{c} - 2\omega R \sin \alpha \frac{L}{c} \frac{1}{v_{\nu}}.
\]

\[
\text{TOF}_{c} - \text{TOF}_{\nu} \approx -62 \text{ ns}, \tag{25}
\]

under the reasoning and simplifications throughout this paper. One should notice the reasoning here holds if the discrepancy turns out to be encrusted within the time translation of the PDF data, but such effect would not arise if the time interval \( \text{TOF}_{\nu} \) were directly measured, since, in this latter situation, such interval would only read \( L/v_{\nu} \).

**Conclusion**

It is interesting to observe that even with a velocity having got two orders of magnitude lesser than \( c \) a neutrino may be interpreted as having got a velocity greater than \( c \), depending on the method used to measure neutrino’s time of flight, with the Earth crust presenting a index of refraction \( n_{c|\nu} > 1 \), due, also, to the local Doppler effect between the clocks attached to Gran Sasso and the respective intersecting ones in the CERN reference frame, as discussed throughout this paper, due to the Earth daily rotation.

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**Acknowledgments**

A.V.D.A is grateful to Y.H.V.H and CNPq for financial support.

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**References**
