The differential coordinate transformation in the general relativity theory

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ABSTRACT
In the general relativity theory the gravity field the vacuum, save the line motive
particle coordinate systems the differential coordinate transformation, study the star’s
red shift.

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I. Introduction

In the general relativity, Schwarzschild solution is

$$d\tau^2 = \left(1 - \frac{2GM}{rc^2}\right)dt^2 - \frac{1}{c^2}\left[\frac{dr^2}{1 - \frac{2GM}{rc^2}} + r^2d\theta^2 + r^2\sin^2\theta d\phi^2\right]$$

(1)

If $d\theta = d\phi = 0$ is

$$d\tau^2 = \left(1 - \frac{2GM}{rc^2}\right)dt^2 - \frac{1}{c^2}\frac{dr^2}{1 - \frac{2GM}{rc^2}}$$

$$= \left(1 - \frac{2GM}{rc^2}\right)dt^2\left[1 - \frac{\overline{V}^2}{c^2}\right]$$

$$\overline{V} = \frac{dr}{dt}\frac{1}{1 - \frac{2GM}{rc^2}}$$

(2)

(3)

II. Additional chapter

In this case, think that a particle move line motion in the $d\theta = d\phi = 0$ in the $r$-axis. And think the coordinate systems by the particle’s point and velocity. In this time, think the coordinate systems’ transformation likely the special relativity theory. In the formula (2), in the $r$-axis, on the coordinate system $S(t, r)$, move the other coordinate system $S'(t', r')$ by the velocity $u$, exist a differential coordinate transformation of the coordinate system $S(t, r)$ and the other coordinate system $S'(t', r')$

$$\sqrt{1 - \frac{2GM}{rc^2}}dt = \gamma\sqrt{1 - \frac{2GM}{r'c^2}}dt' + \frac{dr'}{\gamma\sqrt{1 - \frac{2GM}{r'c^2}}}$$

$$\frac{dr}{\sqrt{1 - \frac{2GM}{rc^2}}} = \gamma\left(\frac{dr'}{\sqrt{1 - \frac{2GM}{r'c^2}}} + \overline{u}dt'\frac{1}{\sqrt{1 - \frac{\overline{u}^2}{c^2}}}\right)$$

$$\overline{u} = u/(1 - \frac{2GM}{r'c^2})$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{\overline{u}^2}{c^2}}}$$

(4)

The formula (4) insert the formula (2),

$$d\tau^2 = \left(1 - \frac{2GM}{rc^2}\right)dt^2 - \frac{1}{c^2}\frac{dr^2}{1 - \frac{2GM}{rc^2}}$$
\[ y^2 \left( \frac{1}{1 - \frac{2GM}{r'c^2}} \right)^2 - \frac{1}{c^2} \frac{dr'}{\sqrt{1 - \frac{2GM}{r'c^2}}} + \bar{u}dt' \sqrt{1 - \frac{2GM}{r'c^2}} \]

\[ = y^2 \left( 1 - \frac{\bar{u}^2}{c^2} \right) \left( 1 - \frac{2GM}{r'c^2} \right) - \frac{1}{c^2} \frac{dr'^2}{\sqrt{1 - \frac{2GM}{r'c^2}}} \]

\[ = \left( 1 - \frac{2GM}{r'c^2} \right) dt'^2 - \frac{1}{c^2} \frac{dr'^2}{\sqrt{1 - \frac{2GM}{r'c^2}}} \]

\[ \bar{u} = \frac{u}{(1 - \frac{2GM}{r'c^2})}, \gamma = \frac{1}{\sqrt{1 - \frac{\bar{u}^2}{c^2}}}, \quad d\theta = d\phi = 0 \quad (5) \]

Therefore, the formula (2) treat the light,

\[ d\tau^2 = \left( 1 - \frac{2GM}{rc^2} \right) dt^2 - \frac{1}{c^2} \frac{dr^2}{1 - \frac{2GM}{rc^2}} = 0, \quad d\theta = d\phi = 0 \]

\[ cdt = \frac{dr}{(1 - \frac{2GM}{rc^2})}, \quad ct = r + \frac{2GM}{c^2} \ln \left| r - \frac{2GM}{c^2} \right| \quad (6) \]

In the \( r \)-axis, in the motive coordinate system \( S(t,r) \) and \( S'(t',r') \), treat the light’s formula (6),

\[ cdt = \frac{dr}{(1 - \frac{2GM}{rc^2})}, \quad cdt' = \frac{dr'}{(1 - \frac{2GM}{r'c^2})} \]

\[ cdt \sqrt{1 - \frac{2GM}{rc^2}} = \frac{dr}{\sqrt{1 - \frac{2GM}{rc^2}}}, \quad cdt' \sqrt{1 - \frac{2GM}{r'c^2}} = \frac{dr'}{\sqrt{1 - \frac{2GM}{r'c^2}}} \]

\[ c\sqrt{1 - \frac{2GM}{rc^2}} dt = \gamma \left( 1 - \frac{2GM}{r'c^2} \right) + \bar{u} \frac{c}{d} \left( 1 - \frac{2GM}{r'c^2} \right) \]

\[ = \gamma \left( 1 - \frac{2GM}{r'c^2} \right) \frac{c}{d} \frac{dr'}{\sqrt{1 - \frac{2GM}{r'c^2}}} \]

\[ = \gamma cdt' \sqrt{1 - \frac{2GM}{r'c^2}} \left( 1 + \frac{\bar{u}}{c} \right) \]

\[ = cdt' \sqrt{1 - \frac{2GM}{r'c^2} \left( 1 + \frac{\bar{u}}{c} \right)}, \quad \bar{u} = \frac{u}{(1 - \frac{2GM}{r'c^2})}, \gamma = \frac{1}{\sqrt{1 - \frac{\bar{u}^2}{c^2}}} \quad (7) \]

**III. Conclusion**

Therefore in the \( r \)-axis, in the motive coordinate system \( S(t,r) \)'s and \( S'(t',r') \)'s light’s frequency
\( \nu, \nu' \) is

\[
dt = \frac{1}{\nu}, \quad dt' = \frac{1}{\nu'} \quad (8)
\]

The formula (8) insert the formula (7). Hence light’s Red shift is

\[
\nu' = \frac{1}{\sqrt{\left(1 - \frac{2GM}{r'c^2}\right) \sqrt{\frac{1 + \frac{\nu}{c}}{1 - \frac{\nu}{c}}}}} = \frac{1}{\sqrt{\left(1 - \frac{2GM}{rc^2}\right) \sqrt{\frac{1 + \frac{\nu}{c}}{1 - \frac{\nu}{c}}}}}
\]

\[
\bar{\nu} = \frac{\nu}{\left(1 - \frac{2GM}{r'c^2}\right)} \quad (8)
\]

Reference