GRAVITONS WRIT LARGE; I.E. STABILITY, CONTRIBUTIONS TO EARLY ARROW OF TIME, AND ALSO THEIR POSSIBLE ROLE IN RE ACCELERATION OF THE UNIVERSE 1 BILLION YEARS AGO?

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This document is due to a question by Debasish of the Saha institute of India asked in the Dark Side of the Universe conference, 2010, in Leon, Mexico[1], and also is connected with issues as to the initial configuration of the arrow of time brought up in both Rudn 10 [2], in Rencontres de Blois[3], and Fundamental frontiers of physics 11 [4], in Paris, in July 2010. Further reference is made as to how to reconcile early inflation with re acceleration, partly by dimensional analysis and partly due to recounting a suggestion as by Yurov [5], which the author thinks has merit and which ties into, to a point with using massive gravitons as a re acceleration of the universe a billion years ago enabler, as perhaps a variant of DE.

A Introduction

The supposition advanced in this article is that relic energy flux initially is central to making predictions as to verifying $S_{\text{entropy}} \sim n_f$ [1, 2, 6, 7], where $n_f$ is a ‘particle count’ per phase space ‘volume’ in the beginning of inflation. Having said that, is $n_f$ due to gravitons in near relic conditions? Or is $S_{\text{entropy}} \sim n_f$ due to coherent clumps of gravitons? If so, can the gravitons/ coherent clumps of gravitons carry information? The author in previous manuscripts [1, 2] identified criteria as to $S_{\text{entropy}} \sim n_f \Big|_{\text{start-of-inf}} \propto 10^5 \iff \text{initial information } \propto 10^7$ bits of ‘information’ in line with G. Smoot’s Paris (2007) [8] talk as presented in the Paris observatory. Having said that, the relevant issue as raised in DSU 2010, if gravitons with a small mass are part of the bridge between $S_{\text{entropy}} \sim n_f \Big|_{\text{start-of-inf}} \propto 10^7 \iff \text{initial information } \propto 10^{10}$ bits of ‘information’, can one make a statement about necessary conditions for ‘massive’ graviton stability? Next, if there is a mass associated with What can be said about massive graviton stability? We look at work presented by Maggiorie [9] which specifically delineated for non zero graviton mass, where

$$h \equiv \eta^{uv} h_{uv} = \text{Trace} \cdot (h_{uv}) \quad \text{and} \quad T = \text{Trace} \cdot (T_{uv})$$

that

$$-3m_{\text{graviton}}^2 h = \frac{\kappa}{2} T$$

(1.1)
Our work uses Visser’s [10] 1998 analysis of non zero graviton mass for both T and h. We will use the above equation with a use of particle count \( n_f \) for a way to present initial GW relic inflation density using the definition given by Maggiore [9] as a way to state that a particle count

\[
\Omega_{gw} \equiv \frac{\rho_{gw}}{\rho_c} = \int_{f=0}^{f_{\text{cut}}} d(f \log f) \cdot \Omega_{gw}(f) \Rightarrow h^2 \Omega_{gw}(f) \equiv 3.6 \left[ \frac{n_f}{10^{37}} \right] \left( \frac{f}{1 \text{kHz}} \right)^4
\] (1.2)

where \( n_f \) is the frequency-based numerical count of gravitons per unit phase space. To do so, let us give the reasons for using Visser’s [10] values for T and h above, in Eq. (1.1).

While Maggiore’s explanation [9] and his treatment of gravitational wave density is very good, the problem we have is that any relic conditions for GW involve stochastic background, and also that many theorists have relied upon either turbulence/ and or other forms of plasma induced generation of shock waves, as stated by Duerrer, et. al.[11] and others looking at the electro weak transition as a GW generator. If relic conditions can also yield GW / graviton production, and the consequences exist up to the present era, as Beckwith presented, then the question of stability of gravitons is even more essential. Beckwith write up an early energy flux for GW/ gravitons which he wrote as [13].

\[
E_{\text{init}} \sim \frac{r^2}{64\pi} \left[ \partial^2_{\gamma} h^+ \right]^2 \left[ \mathbf{n} \cdot t_{\text{Planck}} \right]^3 \cdot \Omega_{\text{effective}}
\] (1.3)

The \( n_f \) value obtained, was used to make a relationship, using Y. J. Ng’s entropy [6] counting algorithm of roughly \( S_{\text{entropy}} \sim n_f \). We assert that in order to obtain \( S_{\text{entropy}} \sim n_f \) from initial graviton production, as a way to quantify \( n_f \), that a small mass of the graviton can be assumed. A small mass graviton in four dimensions only makes sense if it is a stable construct. The remainder of this article will be in giving specific cases as to criteria for stability for the low mass 4 dimensional graviton assumed by the author in obtaining his value of \( S_{\text{entropy}} \sim n_f [1,2,3,6,8] \) and resultant information content present in the early universe. In doing so, the author will address if the correspondence principle and the closeness of the links to massless formalism of the graviton as will be brought up is due to ‘tHoofts [12, 13, 14] idea of an embedding of QM within what he calls deterministic quantum theory, involving an embedding of quantum physics within a slightly ‘larger’ highly non linear structure.

**A1**. Defining the Graviton problem and using Visser’s (1998) inputs into \( T_{uv} \)
We begin our inquiry by initially looking at a modification of what was presented by R. Maartens [15], as done by Beckwith [12,13]

\[ m_\alpha (\text{Graviton}) = \frac{n}{L} + 10^{-65} \text{ grams} \]  

(1.4)

On the face of it, this assignment of a mass of about \( 10^{-65} \) grams for a 4 dimensional graviton, allowing for \( m_0 (\text{Graviton} - 4D) \sim 10^{-65} \) grams [12,13] violates all known quantum mechanics, and is to be avoided. Numerous authors, including Maggiore [9] have richly demonstrated how adding a term to the Fiertz Lagrangian for gravitons, and assuming massive gravitons leads to results which appear to violate field theory, as we can call it. Turning to the problem, we can examine what inputs to the Eqn. (1) above can tell us about if there are grounds for \( m_0 (\text{Graviton} - 4D) \sim 10^{-65} \) grams [12,13], and what this says about measurement protocol for both GW and gravitons as given in Eqn. (2) above. Visser [10], in 1998 came up with inputs into the GR stress tensor and also, for the perturbing term \( h_{\mu\nu} \) which will be given below. We will use them to perform a stability analysis of the consequences of setting the value of \( m_0 (\text{Graviton} - 4D) \sim 10^{-65} \) grams [10,12,13], and discuss how T’Hooft’s [12,13,14] supposition of deterministic QM, as an embedding of QFT, and more could play a role if there are conditions for stability of \( m_0 (\text{Graviton} - 4D) \sim 10^{-65} \) grams [10,12,13]

**A2. Visser’s treatment of the stress energy tensor of GR, and its applications**

Visser [10] in 1998, stated a stress energy treatment of gravitons along the lines of

\[
T_{\mu\nu} \bigg|_{m=0} = \left[ \frac{\hbar}{l_p^2 \lambda_g} \cdot \frac{GM}{r} \cdot \exp \left( \frac{r}{\lambda_g} \right) + \frac{GM}{r} \right]^2 \times \begin{bmatrix}
4 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]  

(1.5)

Furthermore, his version of \( g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \) can be written as setting

\[
h_{\mu\nu} \equiv 2 \frac{GM}{r} \left[ \exp \left( \frac{-m_\alpha r}{\hbar} \right) \right] \left( 2 \cdot V_{\mu} V_{\nu} + \eta_{\mu\nu} \right)
\]

(1.6)
If one adds in velocity ‘reduction’ put in with regards to speed propagation of gravitons [10]

\[ v_g = c \cdot \sqrt{1 - \frac{m_g^2 \cdot c^4}{\hbar^2 \omega_g^2}} \]  

(1.7)

As well as setting \((MG/r) \approx 1/5\) for reasons which Visser [10] outlined, one can obtain a real value for the square of frequency \(> 0\), i.e.

\[ h^2 \omega^2 \simeq m_g^2 c^4 \cdot \left[ \sqrt{1 - \frac{1}{A}} \right] > 0 \]  

(1.8)

\[ A = \frac{1}{6m_g c^2} \left( \frac{h^2}{l_p^2 \lambda_g^2} \cdot \exp \left[ - \frac{r}{\lambda_g} + \frac{m_g \cdot r}{h} + \left( \frac{MG}{r} \right) \cdot \exp \left( \frac{m_g r}{h} \right) \right] \right) \]  

(1.9)

According to Jin Young Kim [16], if the square of the frequency of a graviton, with mass, is \(> 0\), and real valued, it is likely that the graviton is stable, at least with regards to perturbations. Kim’s article [16] is with regards to Gravitons in brane / string theory, but it is likely that the same dynamic for semi classical representations of a graviton with mass.

**A3. Conditions permitting Eqn (1.8) to have positive values**

Looking at Eqn. (1.8) is the same as looking at the following, analyzing how

\[ A = \frac{1}{6m_g c^2} \left( \frac{h^2}{l_p^2 \lambda_g^2} \cdot \exp \left[ - \frac{r}{\lambda_g} + \frac{m_g \cdot r}{h} + \left( \frac{MG}{r} \right) \cdot \exp \left( \frac{m_g r}{h} \right) \right] \right) < 1 \]  

(1.10)

I.e. setting

\[ 0 < \frac{1}{6m_g c^2} \left( \frac{h^2}{l_p^2 \lambda_g^2} \cdot \exp \left[ - \frac{r}{\lambda_g} + \frac{m_g \cdot r}{h} + \left( \frac{MG}{r} \right) \cdot \exp \left( \frac{m_g r}{h} \right) \right] \right) < 1 \]  

(1.11)

Note that Visser [10] (1998) writes \( m_g < 2 \times 10^{-28} eV \sim 2 \times 10^{-38} m_{nucleon} \), and a wave length \( \lambda_g \sim 6 \times 10^{-22} \) meters. The two values, as well as ascertaining when one can use \( \frac{MG}{r} \sim 1/5 \), with r the usual distance from a graviton generating source, and M the ‘mass’ of an object which would be a graviton emitter put severe restrictions as to the volume of space time values for which r could be ascertained. If, however, Eq. (1.10) had, in most cases, a setting for which, then in many cases, Eq. (1.8) would hold.
The author believes that such a configuration would be naturally occurring in most generation of gravitons at, or before the Electro Weak transition point in early cosmology evolution.

\[
0 \leq \exp \left[ -\frac{r}{\lambda_g} + \frac{m_g \cdot r}{\hbar} \right] \ll 1
\]  \hspace{1cm} (1.12)

A 4. Review of if there is a \( n_f \approx 10^5 \text{ to } 10^6 \) initial production of coherent groups of gravitons in relic conditions. And its effect on the arrow of time question

The author, Beckwith, believes, that satisfying Eqn. (1.12) would allow to predict a particle count behavior along the lines where Beckwith [1,2,3] obtained \( n_f \approx 10^5 \text{ to } 10^6 \). This value of \( n_f \approx 10^5 \text{ to } 10^6 \) as given by Beckwith [1,2,12,13] would be put into Eqn. (1.2) above, which would have implications for what to look for in stochastic GW generation. The question to raise, is what “particle” is being counted, in \( n_f \approx 10^5 \text{ to } 10^6 \). Conceivably, it could be coherent packets of gravitons. The reasons for raising this question will be spelled out in the following analysis.

Recently, Beckwith asked [1,2,3] if the following could occur, \( S \equiv [E - \mu N] / T \rightarrow S \propto T^3 \) by setting the chemical potential \( \mu \rightarrow 0 \) with initial entropy \( S \sim 10^4 \) at the beginning of inflation. Conventional discussions of the arrow of time states that as the Universe grows its temperature drops, which leaves less energy available to perform useful work in the future than was available in the past. Thus the Universe itself has a well-defined thermodynamic arrow of time. The problem of the initial configuration of the arrow of time, however, is not brought up. This paper is to initiate how to set up a well defined initial starting point for the arrow of time. Specifically re setting the degrees of freedom of about \( g_* \sim 100 - 120 \) of the electro weak era, to \( g_* \sim 1000 \) at the onset of inflation [1], may permit \( S_{\text{initial}} \propto T^3 \).

If the initial temperature of an emerging universe were very low, scaling \( S \propto T^3 \) may be a way to get an arrow of time, with respect to thermal temperatures, alone, with the graviton count a later, emergent particle phenomenon.

B. What can be said initially about usual arrow of time formulations of early cosmology?

Usual treatments of the arrow of time, i.e. the onset of entropy. The discussion below makes the point that expansion of the universe in itself does not ‘grow’ entropy
The entropy density $s$ of a radiation field of temperature $T$ is $s \sim T^3$. The entropy $S$ in a given comoving volume $V$ is $S = sV$. Since the comoving volume $V$ increases as the universe expands, we have $V \sim R^3$. And since the temperature of the microwave background goes down as the universe expands: $T \sim 1/R$, we have the result that the entropy of a given comoving volume of given space $S \sim R^{-3} \cdot R^3 = \text{constant}$. Thus the expansion of the universe by itself is not responsible for any entropy increase. There is no heat exchange between different parts of the universe. The expansion is adiabatic and isentropic: $dS_{\text{expansion}} = 0$. I.e. a process has to be initiated in order to start entropy production.

This discussion above is to emphasize the importance of an initial process for the onset and the growth of entropy. We will initiate candidates for making sense of the following datum.

To measure entropy in cosmology we can count photons. If the number of photons in a given volume of the universe is $N$, then the entropy of that volume is $S \sim kN$ where $k$ is called here Boltzmann’s constant.

Note that Y. Jack Ng. has [6], from a very different stand point derived $S \sim n$ based upon string theory derived ideas, with $n$ a ‘particle’ count, which in Y. Jack Ng's procedure is based upon the number of dark matter candidates in a given region of phase space. Y. Jack Ng’s idea was partly based upon the idea of quantum ‘infinite’ statistics, and a partition function.

This counting procedure is different from traditional notions. To paraphrase them, one can state that “The reason why entropy is increasing is because there are stars in that ‘box’ (unit of phase space used for counting contributions to entropy). Hydrogen fuses to helium and nuclear energy is transformed into heat.” I.e. the traditional notion would be akin to heat production due to, initially start BBN nucleosynthesis, and then, frankly, star production/nuclear burning. I.e. one would need to have nuclear processes to initiate heat production. This idea of heat production is actually similar to setting $S \propto T^3$, with heat production due to either BBN/hydrogen burning leading to an increase in temperature, $T$. In this manuscript, we make use of, if $S \equiv [E - \mu N]/T \rightarrow S \propto T^3$ by setting the chemical potential $\mu \rightarrow 0$ with initial entropy $S \sim 10^3$ at the beginning of inflation. This entails, as we will detail, having increased number of degrees of freedom, initially, with re setting the degrees of freedom of about $g_* \sim 100-120$ of the electro weak era, to $g_* \sim 1000$ at the onset of inflation, i.e. what will be examined will be the feasibility of the following: $S \equiv [E - \mu N]/T \rightarrow \mu \rightarrow 0 \rightarrow S \propto T^3 \approx n$, with $n$ an initial ‘quantum unit’ count in phase space of Planckian dimensions, where $S \sim 10^5$ at the beginning of inflation. Let us now look at how to initiate such a counting algorithm if one is looking at, say, highly energized gravitons, initially, as part of a counting ‘algorithm’.
B1. Estimating the size of contribution to energy in $S \equiv E/T$, assuming a peak frequency $\nu \sim 10^{10}$ Hertz for relic gravitons, if the standard chemical potential is effectively $\mu = 0$ at the onset of creation.

As suggested earlier by Beckwith [12,13], gravitons may have contributed to the re-acceleration of the universe one billion years ago. Here, we are making use of refining the following estimates. In what follows, we will have even stricter bounds upon the energy value (as well as the mass) of the graviton based upon the geometry of the quantum bounce, with a radii of the quantum bounce on the order of $l_{\text{Planck}} \sim 10^{-35}$ meters [1], [5].

$$m_{\text{graviton}} \mid_{\text{RELATIVISTIC}} < 4.4 \times 10^{-22} \text{eV} / c^2$$

$$\Rightarrow \lambda_{\text{graviton}} \equiv \frac{\hbar}{m_{\text{graviton}} \cdot c} < 2.8 \times 10^{-8} \text{meters} \quad (1.13)$$

For looking at the onset of creation, with a bounce; if we look at $\rho_{\text{max}} \propto 2.07 \cdot \rho_{\text{planck}}$ for the quantum bounce with a value put in for when $\rho_{\text{planck}} \approx 5.1 \times 10^{99}$ grams/ meter$^3$, where [1]

$$E_{\text{eff}} \propto 2.07 \cdot l_{\text{Planck}}^3 \cdot \rho_{\text{planck}} \sim 5 \times 10^{24} \text{GeV} \quad (1.14)$$

Then, taking note of this, one is obtaining having a scaled entropy of $S \equiv E/T \sim 10^5$ when one has an initial Planck temperature $T \approx T_{\text{Planck}} \sim 10^{19} \text{GeV}$. One needs, then to consider, if the energy per given graviton is, if a frequency $\nu \propto 10^{10} \text{Hz}$ and $E_{\text{graviton-effective}} \propto 2 \cdot h \nu \approx 5 \times 10^{-5} \text{eV}$, then [1]

$$S \equiv E_{\text{eff}} / T \sim \left[10^{10} \times E_{\text{graviton-effective}}(\nu \approx 10^{10} \text{Hz}) / T \sim 10^{19} \text{GeV}\right] \approx 10^3 \quad (1.15)$$

Having said that, the $E_{\text{graviton-effective}} \propto 2 \cdot h \nu \approx 5 \times 10^{-5} \text{eV}$ is $10^{22}$ greater than the rest mass energy of a graviton if $E \sim m_{\text{graviton}} \left[\text{red-shift} \sim .55\right] \sim \left(10^{-27} \text{eV}\right)$ grams is taken when applied to Eq. (1.2) above.

B II. The electro weak generation regime of space time for Entropy and early universe Graviton production before electo weak transitions

A typical value and relationship between an inflaton potential $V[\phi]$, and a hubble parameter value, $H$ is [1]
\[ H^2 \sim V(\phi)/m_{\text{Planck}}^2 \] (1.16)

Also, if we look at the temperature \( T^* \) occurring about the time of the Electro weak transition, if \( T \leq T^* \) when \( T^* = T_c \) was a critical value, (of which we can write \( v(T_c)/T_c > 1 \), where \( v(T_c) \) denotes the Higgs vacuum expectation value at the critical temperature \( T_c \), i.e. \( v(T_c)/T_c > 1 \) according to C. Balazs et al (2005) [17] and denotes that the electro weak transition was a ‘strongly first order phase transition’) then one can write, by conventional theory that

\[ H \sim 1.66 \cdot \sqrt{g_\ast} \cdot [T^2 / m_{\text{Planck}}^2] \] (1.17)

Here, the factor put in, of \( \sqrt{g_\ast} \) is the number of degrees of freedom. Kolb and Turner [18] put a ceiling of about \( g_\ast \approx 100 - 120 \) in the early universe as of about the electro weak transition. If, however, \( g_\ast \approx 1000 \) or higher for earlier than that, i.e up to the onset of inflation for temperatures up to \( T \approx T_{\text{Planck}} \approx 10^{19} \text{ GeV} \), it may be a way to write, if we also state that \( V(\phi) = E_{\text{net}} \) that if [1]

\[ S \sim 3 m_{\text{Planck}}^2 \left[ H = 1.66 \cdot \sqrt{g_\ast} \cdot T^2 / m_{\text{Planck}}^2 \right]^3 \sim 3 \cdot \left[ 1.66 \cdot \sqrt{g_\ast} \right]^3 T^3 \] (1.18)

Should the degrees of freedom hold, for temperatures much greater than \( T^* \), and with \( g_\ast \approx 1000 \) at the onset of inflation, for temperatures, rising up to, say \( T \approx 10^{19} \text{ GeV} \), from initially a very low level, pre inflation, then this may be enough to explain how and why certain particle may arise in a nucleated state, without necessarily being transferred from a prior to a present universe.

Furthermore, if one assumes that \( S \propto T^3 \) [5] when \( g_\ast \approx 1000 \) or even higher even if \( T \approx 10^{19} \text{ GeV} >> T^* \), then there is the possibility that \( S \propto T^3 \) when \( g_\ast \approx 1000 \) could also hold, if there was in pre inflationary states very LOW initial temperatures, which rapidly built up in an interval of time, as could be given by \( 0 < t < t_{\text{Planck}} \approx 10^{-44} \text{ seconds} \) [1]

**B III. Justification for setting \( g_\ast \approx 1000 \) initially.**

H. de La Vega, in conversations with the author in Colmo, Italy, 2009 [7], flatly ruled out having \( g_\ast \approx 1000 \) initially. What will be presented here will be a justification for taking this step which H. de La Vega says is not measurable and possible. The author points to, among other things, the Wheeler de Witt derivation for a wave function of the
universe, as given by M. Morris [8] (1989) in perturbative super space, with no restriction on the degrees of freedom. While the WdW style of cosmological evolution is now out of fashion, something akin to obtaining an initial ‘wave function of the universe’ as given in his Eq. (3.1) of his article is, by the authors view, necessary, to make sense out of initial conditions appropriate for $S \propto T^3 \sim n$ when $\tilde{g}_* \approx 1000$. The count, $n$, would be in terms of a procedure brought up by both Beckwith, [1] and Mukhanov [9] on page 82 of his book leading to a Bogoluybov particle number density of becoming exponentially large, where $\eta_i$ is a time evolution factor, which we can set $|\eta_i| \sim O(\beta \cdot t_{\text{Planck}})$, with $\beta$ some numerical multiplicative factor for the Planck interval of time $t_{\text{Planck}}$ [1], [9]

$$n \sim \sinh^2[m_0 \eta_i]$$  \hspace{1cm} (1.19)

BIV. Making sense of the factor of $10^{38}$ in Eq. (1.5). I.e. how to reconcile Eq. (1.5) with $S \sim n$ used by Y. Jack Ng for DM particles in his entropy/particle counting algorithm?

Note that J. Y. Ng uses the following. [8] I.e. for DM, $S \sim n$, but this is for DM particles, presumably of the order of mass of a WIMP, i.e. $m_{\text{WIMP}} \approx 100 \cdot GeV \sim 10^{11}$ electron volts, as opposed to a relic graviton mass–energy relationship:

$$m_{\text{graviton}} \left(\text{energy}\right) \approx 10^{10} \text{Hz} \approx \left[100 \cdot GeV \sim 10^{11} eV - \text{WIMP}\right] \times 10^{-16} \sim 10^{-5} eV$$  \hspace{1cm} (1.20)

If one drops the effective energy contribution to $\nu \approx 10^0 \sim 1 \text{Hz}$, as has been suggested, then the relic graviton mass-energy relationship is:

$$m_{\text{graviton}} \left(\text{energy}\right) \approx 10^0 \text{Hz} \approx \left[100 \cdot GeV \sim 10^{11} eV - \text{WIMP}\right] \times 10^{-26} \sim 10^{-15} eV$$  \hspace{1cm} (1.21)

Finally, if one is looking at the mass of a graviton a billion years ago, with

$$m_{\text{graviton}} \left(\text{red–shift–value} \sim .55\right) \approx \left[100 \cdot GeV \sim 10^{11} eV - \text{WIMP}\right] \times 10^{-38} \sim 10^{-21} eV$$  \hspace{1cm} (1.22)

I.e. if one is looking at the mass of a graviton, in terms of its possible value as of a billion years ago, one gets the factor of needing to multiply by $10^{38}$ in order to obtain WIMP level energy-mass values, congruent with Y. Jack Ngs $S \sim N$ counting algorithm. I.e.
the equivalence relationship for entropy and ‘particle count’ may work out well for the WIMP sized DM candidates, and may break down for the graviton mass-energy problem.

**BV. Making an argument for DM/ DE, if there is a small rest graviton mass a billion years ago**

Either there is clumping of gravitons into coherent GW states, as may be the resolution of the $10^{38}$ factor in Eq. (3), and the GW frequency drops dramatically a billion years ago, to take into account having, instead of the energy associated with relic gravitons of value $\approx 5 \times 10^{-5} eV$, as assumed in Eq (1), or else Y. J. Ng’s $S \approx < n >$ will only work for particles with $E_{\text{rel}} \approx 100 \cdot GeV$ which is the energy-mass value of WIMP DM. Needless to say, if the coherent GW state interpretation is correct, for relic GW, as clumped to make $S \approx < n >$ correct, then if there is a drop in frequency a billion years ago, for existing Gravitons, with an effective rest mass per graviton, one may have an explanation for Beckwith’s reacceleration graph when Beckwith found at $z \approx 0.423$, a billion years ago, that acceleration of the universe increased, as shown in Figure 1.

![Figure 1](image)

**FIGURE 1**: Reacceleration of the universe based on Beckwith’s Dark Side of the Universe lecture (note that $q < 0$ if $z < 0.423$)

If a modification of DM along the lines of Eq. (1.4) can be proved, i.e. a small rest graviton mass, instead of treating Eq. (1.23) as a purely 4 dimensional construction as was done by Alves, then one has to consider the following as far as how to get appropriate de acceleration parameter behavior.

Beckwith [12,13] used a version of the Friedman equations as inputs into the deceleration parameter using Maarten’s [15]...
\[ a^2 = \left( \frac{\kappa^2}{3} \left[ \rho + \rho^2 \right] \right) a^2 + \frac{\Lambda \cdot a^2}{3} + \frac{m}{a^2} - K \]  

(1.24)

Maartens [12,13,15] also gives a 2nd Friedman equation, as

\[ \frac{\ddot{a}}{a^2} = -\left( \frac{\kappa^2}{2} \left[ p + \rho \right] \left[ 1 + \frac{\rho^2}{\lambda} \right] \right) + \frac{\Lambda \cdot a^2}{3} - 2 \frac{m}{a^2} + \frac{K}{a^2} \]  

(1.25)

Also, if we are in the regime for which \( \rho \cong -P \), for red shift values \( z \) between zero to 1.0-1.5 with exact equality, \( \rho = -P \), for \( z \) between zero to \( z \) and using \( a = a_0 = 1/(1 + z) \). Then Eq. (1.23) is as given by Beckwith [12,13]

\[ q = \frac{\ddot{a}}{a^2} \equiv -1 - \frac{\ddot{H}}{H^2} = -1 + \frac{2}{1 + \kappa^2 \left[ \rho / m \right] \left[ 1 + z \right]^2 \left[ 1 + \rho / (2 \lambda) \right]} \]  

(1.26)

Eq. (1.26) assumes \( \Lambda = 0 = K \), and the net effect is to obtain, a substitute for \( \Lambda = 0 \), by presenting how gravitons with a small mass done with \( \Lambda = 0 \), even if curvature \( K = 0 \). Furthermore the density would as in four dimensions, be given by [1.2,12,13, ] -

\[ \rho = \rho_0 \cdot (1 + z)^3 \left[ - \frac{m_g \cdot (c = 1)^6}{8 \pi G(h = 1)^2} \cdot \left( \frac{1}{14 \cdot (1 + z)^3} + \frac{2}{5 \cdot (1 + z)^2} - \frac{1}{2} \right) \right] \]  

(1.27)

Note, that Eq. (1.26) is for gravitons, with a very low rest mass. According to section BIV, the only way to account for keeping \( S \sim <n> \) doable in a phase space rendition would be to make each unit of \( n \), with \( S \sim <n> \sim 10^5 \) would be to have each counted component of \( S \sim <n> \sim 10^5 \) as a coherent bunch of gravitons, i.e. perhaps forming the basic component of a gravitational wave. But the argument as presented in Section B IV does not rule out the possibility that there may be a way to make a functional inter connection between gravitons with mass, and that between the beginning of inflation to the re acceleration of the universe. Note, that, if this is occurring, one probably would be forced to look at more than four dimensions of space time, in line with Eq. (1.23) to Eq. (1.26) above.

To sum it up. If there is a de facto linkage between DM and DE, as implied by Figure 1, then more than four space time dimension may be necessary. That would be for re acceleration of the universe one billion years ago.

Note, the entire premise of the initial work and the first part of the article was on density from LQG affecting , after working with \( E_{eff} \approx 2 \hbar \nu \), [1,2,3] with the frequencies rising
as high as $10^{10}$ Hz for relic gravitons, the amount of gravitons which could be transferred to the present universe from a prior universe. That was assuming a density (energy) proportional to 2.07 times the Planck density value, for a Plank length dimension of a LQG bounce leading to ‘super inflation’. The conclusion was that there was a multiplicative factor of $10^{38}$ necessary. The explanation of how to deal with the $10^{38}$ factor was to appeal to coherent states of gravitons leading to a gravitational wave. i.e. a coherent packet counted as one unit to the $S \sim n = 10^5$. with up to $10^{38}$ gravitons per unit ‘GW’ wave. i.e. that is a LOT of gravitons. The second assumption coming up was that, if $S \sim n$ were to be used by Y. Jack Ng’s counting algorithm, that if it applies to DM, with up to 100 GeV per WIMP particle, that the counting algorithm $S \sim n$ may require substantially higher energy per counted energy packet, than what could be expected by $E_{\text{eff}} \approx 2\hbar\nu$. Given these constraints, the conclusion is, tentatively that fulfilling the $S \sim n$ counting algorithm may necessitate using coherent states of gravitons grouped into GW, and that each count of $n$ is for a packet with up to $10^{38}$ gravitons per unit ‘GW’ “wave” with up to $S \sim 10^5$ of these coherent groups of gravitons, i.e. $10^5$ unit ‘GW’ “waves”

This is for four dimensions. If we wish to analyze if there could be a connection between initial $S \sim n = 10^5$ as the initial configuration for an arrow of time for the start of cosmological evolution and re acceleration of the universe a billion years ago, we will be having to probably look at adding additional space time dimensions about the usual 4 dimensions associated with Einstein's GR.

So, can there be a connection between initial inflation, and re acceleration of the universe a billion years ago. If so, higher dimensions may be necessary. If so, consider what Yurov derived as a possible inter connection between the initial inflation and re acceleration a billion years ago.

C. Is there a linkage from early inflation, to conditions for re acceleration of the universe a billion years ago?

The following is speculative, and if confirmed through additional research would be a major step toward a cosmological linkage between initial inflation, and re acceleration of the universe one billion years ago [1,2,3] . Look at A. Yurov’s [5] double inflation hypothesis, i.e. Claim: there exist one emergent complex scalar field $\Phi$ and that its evolution in both initial inflation and re acceleration is linked. I.e. he states that this scalar field would account for both 1st and 2nd inflation Potential in both cases chaotic inflation of the type [5]

$$V = \bar{m}^2 \Phi^* \Phi$$  

(1.23)
The “mass” term would be, then, as Beckwith [5,12,13] understands it, for early universe versions of the Friedman equation

\[ \dot{m} \approx \frac{3}{8} \left[ \frac{3H^2}{4\pi G_{\text{time}} - 10^{-35}\text{sec}} + \frac{3H^2}{4\pi G_{\text{time}} - 10^{-34}\text{sec}} \right] \]  

(1.24)

Furthermore, its bound would be specified by having

\[ |\dot{m}| \leq \left[ \frac{l^2}{4} \right] \]  

(1.25)

The term, \( l \) would be an artifact of five dimensional space time, as provided in a metric as given by Maarten’s [15] as

\[ dS^2_{5-\text{dim}} = \frac{l^2}{z^2} \left[ g_{\alpha\beta} dx^\alpha dx^\beta + dz^2 \right] \]  

(1.26)

The 2\textsuperscript{nd} scalar fields as Yurov [5] writes them contributing to the 2\textsuperscript{nd} inflation, which Beckwith represents [2,13] is

\[ \phi_{b,c} = \sqrt{2/3} \cdot \dot{m} \cdot t_{1\text{st-EXT}} \approx 10^{-35}\text{sec} \]  

(1.27)

And

\[ \phi_{v} = \left[ \phi_{b,c} - \sqrt{3/2} \cdot \frac{3M^2}{\dot{m}} \right]^{1/3} \]  

(1.28)

As Beckwith sees it, making a full linkage between Yurov’s formalism [5] for double inflation, Beckwith’s re acceleration graphics [2,12,13], and initial inflationary dynamics, as referenced by obtaining \( n_f \approx 10^{6} \text{to} \cdot 10^{7} \) would be to make the following relations between Yurov’s [5] versions of the Friedman equations, and what Beckwith [2,12,13] did,

\[ H^2 = \frac{1}{6} \left[ \dot{\phi}^2 + \ddot{m} \phi^2 + \frac{M^2}{\phi^3} \right] \leftrightarrow \left( \frac{\kappa^2 \rho}{3} + \frac{\rho^2}{2\lambda} \right) + \frac{m}{a^4} \]  

(1.29)

As well as having:

\[ \dot{H} = V - 3H \leftrightarrow \dot{H} \approx \frac{2m}{a^4} \]  

(1.30)
The left hand side of both Eq (1.14) and Eq (1.15) are Yurov’s [10], and the right hand side of both Eqn. (1.14) and Eq (1.15) above are Beckwith’s adaptation [1,2,12,13] of modification of Maarten’s brane theory [15] work which was used in part to obtain the re acceleration of the universe graphics Beckwith obtained [1,2,12,13] a, i.e. the behavior of massive gravitons one billion years ago to mimic DE in terms of the re acceleration parameter IN any case, the following would be needed to be verified to make the linkage [1,2,10].

\[
\frac{3H^2}{4\pi G} \gg V(t) \quad \text{time} \approx 10^{-44}\text{sec}
\]  

(1.31)

i.e. that the potential energy, V, of initial inflation is initially over shadowed by the contributions of the Friedman equation, H, at the onset of inflation.

We should note, that the potential energy as stated would be assuming that Eq. (1.31) has consistency with Eq. (1.17), for very large temperatures. If, as an example, there were, low initial pre inflation temperatures, then Eq. (1.17) and Eq. (1.31) would not be commensurate with each other and the entire idea would then be falsified and wrong.

D. Revisiting Ng’s counting algorithm for entropy, and Graviton mass

The wave length for a graviton as may be chosen to do such an information exchange would be part of a graviton as being part of an information counting algorithm as can be put below, namely: Argue that when taking the log, that the 1/N term drops out. As used by Ng [6, 12,13]

\[
Z_N \sim \left(\frac{1}{N!}\right) \cdot \left(V/\lambda^3\right)^N
\]

(1.32)

This, according to Ng, [6,12,13] leads to entropy of the value of, if \(S = \left(\log[Z_N]\right)\) will be modified by having the following done, namely after his use of quantum infinite statistics, as commented upon by Beckwith [6,12,13]

\[
S \approx N \cdot \left(\log\left[V/\lambda^3\right] + 5/2\right) \approx N
\]

(1.33)

Eventually, the author hopes to put on a sound foundation what ’tHooft [14] is doing with respect to ’tHooft [12,13,14] deterministic quantum mechanics and equivalence classes embedding quantum particle structures. Furthermore, making a count of clumps of gravitons, with each coherent bunch contributing to a GW with \(S \approx N \sim 10^5\) [1,2,3] with Seth Lloyd’s [1,2,12,13, 16]

\[
I = S_{total} / k_B \ln 2 = \left[\# \text{operations}\right]^{1/4} \approx 10^4
\]

(1.34)
as implying at least one operation per unit graviton, with gravitons being one unit of information, per produced coherent clump of gravitons [1,2,3] Note, Smoot [8] gave initial values of the operations as

\[
\text{[#operations]}_{\text{initial}} \sim 10^7
\]  

(1.35)

The author’s work tends to support this value, and if gravitons are indeed stable in initial conditions, information exchange between a prior to a present universe may become a topic of experimental investigation.

In a colloquium presentation done by Dr. Smoot in Paris [8] (2007); he alluded to the following information theory constructions which bear consideration as to how much is transferred between a prior to the present universe in terms of information ‘bits’.

0) Physically observable bits of information possibly in present Universe - $10^{180}$
1) Holographic principle allowed states in the evolution / development of the Universe - $10^{120}$
2) Initially available states given to us to work with at the onset of the inflationary era- $10^{10}$
3) Observable bits of information present due to quantum / statistical fluctuations - $10^8$

Actually, the $10^8$ figure is within an order of magnitude close to the $10^7$ figure of Eq. (1.35). Our guess is as follows. That the thermal flux from a prior to the present universe may account for up to $10^7$ to $10^8$ bits of information. These could be transferred from a prior universe to our universes present big bang itself. Smoot and others gave a red shift figure for the existence of the $10^7$ to $10^8$ bits of information emerging from a prior universe as of about red shift $z \sim 10^{25}$.

E. Conclusion, giant graviton stability possible, and may allow for survival of gravitons with mass in early universe conditions. Contributions to having DE duplicated, as given by Figure 1 are possible, but require more than four dimensions.

The author pursued this question, partly due to wishing to determine if a non brane theory way to identify graviton stability existed. Secondly, note that the initial entropy state was done via an assumption of LQG, with a maximized density value, with the density of the quantum bounce of the order of 2.07 times the Planck density value.

Should this LQG idea be, in any fashion experimentally confirmed, it would probably give a way forward to give , if initial degrees of freedom could be boosted up to $g^2 \sim 1000$
above the electro weak usual limit of $g^* \sim 100$-120, as well as a temperature dependent way of setting the initial arrow of time hypothesis. The question to ask, if does Eq. (1.17) permit a linkage of gravitons as information carriers, and can there be a linkage of information, in terms of the appearance of gravitons in the time interval of, say $0 < t < t_{Planck}$ either by vacuum nucleation of gravitons / information packets.

Appropriate values / inputs into $\rho$ are being considered along the lines of graviton mass/ contributions along the lines brought up in this paper already.

An alternative to applying $S \sim <n>$ if one sees no way of implementing what Ng. suggested via his infinite quantum statistics [3] would be to look at thermal inputs from a prior to the present universe, as suggested by L. Glinka[20, 21]

\begin{equation}
S = \frac{1}{4} \left[ \sqrt{\frac{v(a_{initial})}{v(a)}} - 1 \right]
\end{equation}

As well as, if $h_0 \sim 0.75$

\begin{equation}
\Omega_{gw}(v) \equiv \frac{3.6}{h_0^2} \left[ \frac{n_f}{10^{37}} \right] \left( \frac{v}{1kHz} \right)^4
\end{equation}

If we take into consideration having $a \sim a_{final}$, then Eq. (1.36) above will, in most cases be approximately

\begin{equation}
S \approx n_f = \left[ \sqrt{\frac{v(a_{initial})}{v(a)}} - 1 \right] \sim \left[ \frac{a(a_{initial})}{a_{final}} \right]
\end{equation}

For looking at $\Omega_g \approx 10^{-5} - 10^{-14}$, with $\Omega_g \approx 10^{-5}$ in pre big bang scenarios, with initial values of frequency set for $v(a_{initial}) \approx 10^8 - 10^{10}$ Hz, as specified by Grishkuk [15] $v(a_{final}) \approx 10^0 - 10^2$ Hz near the present era, and $a \sim [a_{final} = 1] - \delta^*$, i.e. close to the final value of today’s scale value. Filling in/ choosing between either implementation of Eq. 1.7, or Eq. 1.38 will be what the author is attempting to do in the foreseeable future. I.e. if one can use it in the near present era, i.e. up to a billion years ago

\begin{equation}
S \approx n_f = \left[ \sqrt{\frac{v(a_{initial})}{v(a)}} - 1 \right] \sim \left[ \frac{a(a_{initial})}{a_{final}} \right]
\end{equation}

Finally if $S \neq n$, using Eq. (1.39) for $n = n_t$, but we instead uses $S \propto T^3$, with temperature rapidly increasing from a low value to $T_{Planck} \approx 10^{19}$ GeV in about a time interval during the onset of inflation, for the beginning of the arrow of time, in cosmology. Beckwith views determining if the degrees of freedom initially could go as high as $\delta_{s}^* \approx 1000$ or even higher even if $T \sim 10^{19}$ GeV as essential in determining the
role of $S \propto T^3$ as, as temperatures go from an initial low point, to $T \sim 10^{19}$ GeV for understanding the role of thermal heat transfer in the arrow of time issue.

References
6. Y. Ng, Entropy 2008, 10(4), 441-461; DOI: 10.3390/e10040441