DECELERATION PARAMETER Q(Z) AND THE ROLE OF NUCLEATED GW ‘GRAVITION GAS’ IN THE DEVELOPMENT OF DE ALTERNATIVES

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The case for a four dimensional graviton mass (non zero) influencing reacceleration of the universe in five dimensions is stated, with particular emphasis upon if five dimensional geometries as given below give us new physical insight as to cosmological evolution. A comparison with the quantum gas hypothesis of Glinka shows how stochastic GW/ gravitons may emerge in vacuum nucleated space, with emphasis upon comparing their number in phase space, as compared with different strain values.

I Introduction

1.1 What can be said about gravitational wave density value detection?

We will start with a first-principle introduction to detection of gravitational wave density using the definition given by Maggiore [1]

\[
\Omega_{gw} \equiv \frac{\rho_{gw}}{\rho_c} = \frac{1}{\pi} \int_{f_0}^{f_\infty} d(\log f) \cdot \Omega_{gw}(v) \Rightarrow h_0^2 \Omega_{gw}(v) \equiv 3.6 \left( \frac{n_f}{10^{37}} \right) \left( \frac{v}{1kHz} \right)^4
\]  

(1.1)

where \( n_f \) is the frequency-based numerical count of gravitons per unit phase space. The author suggests that \( n_f \) may also depend upon the interaction of gravitons with neutrinos in plasma during early-universe nucleation, as modeled by M. Marklund et al. [2]. \( \Omega_{gw} \) has the following relic universe candidate values as given by Figure 1 below.

Combining experimental confirmation of Eq. (1.1) with observations and use of different choices for \( H = \frac{\dot{a}}{a} \) and \( \Omega \equiv \rho(t)/\rho_{critical} \) will be tied in, with analysis of the diagram of Figure 1 below.
B. P. Abbott et al. [2] (2009) shows the relation between $\Omega_\gamma$ and frequency. The relation between $\Omega_\gamma$ and the spectrum $h(v_\gamma, \tau)$ is written by Grishchuk, [3] , as

$$\Omega_\gamma \approx \frac{\pi^2}{3} \left( \frac{v}{v_H} \right)^2 h^2(v, \tau), \quad (1.2)$$

We will be using Eq. (1.2) with the range of values as presented in Figure 1, and also prepare for a candidate discriminating criteria for a number count for $\Omega_\gamma$ based upon Glinka’s [4] quantum gas work, namely

$$n_f = \left[ 1/4 \right] \left[ \sqrt{\frac{v(a_{\text{initial}})}{v(a)}} - \sqrt{\frac{v(a)}{v(a_{\text{final}})}} \right] \quad (1.3)$$

As well as, if $h_0 \sim .75$

$$\Omega_{gw}(v) \approx \frac{3.6}{h_0^2} \left[ \frac{n_f}{10^{17}} \right] \left( \frac{v}{1kHz} \right)^4 \quad (1.4)$$
If we take into consideration having \( a \sim a_{\text{final}} \), then Eq. (1.3) above will, in most cases be approximately

\[
    n_f = \left[ \frac{1}{4} \right] \left[ \frac{v(a_{\text{initial}})}{v(a)} - 1 \right] \sim \left[ \frac{1}{4} \right] \left[ \frac{v(a_{\text{initial}})}{v(a)} \right]
\]

(1.5)

For looking at \( \Omega_g \approx 10^{-5} - 10^{-14} \), with \( \Omega_{\text{g}} \approx 10^{-5} \) in pre big bang scenarios, with initial values of frequency set for \( v(a_{\text{initial}}) \approx 10^8 - 10^{10} \) Hz, as specified by Grishuk[5] \( v(a_{\text{final}}) \approx 10^9 - 10^2 \) Hz near the present era, and \( a \sim a_{\text{final}} = 1 \) = \( \delta^* \), i.e. close to the final value of today’s scale value, we can obtain the following table of would be n density values in the regime for which \( a \sim a_{\text{final}} = 1 \) = \( \delta^* \) represents

<table>
<thead>
<tr>
<th>( v(a) \approx v(a_{\text{final}}) \approx 10 - 10^2 )</th>
<th>( v(a_{\text{initial}}) )</th>
<th>( n_f ) (from Eq. 1.5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>***</td>
<td>( 10^3 )</td>
<td>( 10^{32} )</td>
</tr>
<tr>
<td>***</td>
<td>( 10^8 )</td>
<td>( 10^{12} )</td>
</tr>
<tr>
<td>***</td>
<td>( 10^{10} )</td>
<td>( 10^{3} )</td>
</tr>
</tbody>
</table>

Table 1: If one assumes \( \Omega_{\text{g}} \approx 10^{-5} \)

<table>
<thead>
<tr>
<th>( v(a) \approx v(a_{\text{final}}) \approx 10 - 10^2 )</th>
<th>( v(a_{\text{initial}}) )</th>
<th>( n_f ) (from Eq. 1.5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>***</td>
<td>( 10^3 )</td>
<td>( 10^{27} )</td>
</tr>
<tr>
<td>***</td>
<td>( 10^8 )</td>
<td>( 10^{7} )</td>
</tr>
<tr>
<td>***</td>
<td>( 10^{10} )</td>
<td>( 10^{-2} ) (not measurable)</td>
</tr>
</tbody>
</table>

Table 2: If one assumes \( \Omega_{\text{g}} \approx 10^{-10} \)

<table>
<thead>
<tr>
<th>( v(a) \approx v(a_{\text{final}}) \approx 10 - 10^2 )</th>
<th>( v(a_{\text{initial}}) )</th>
<th>( n_f ) (from Eq. 1.5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>***</td>
<td>( 10^3 )</td>
<td>( 10^{23} )</td>
</tr>
<tr>
<td>***</td>
<td>( 10^8 )</td>
<td>( 10^{3} )</td>
</tr>
<tr>
<td>***</td>
<td>( 10^{10} )</td>
<td>( 10^{-6} ) (not measurable)</td>
</tr>
</tbody>
</table>

Table 3: If one assumes \( \Omega_{\text{g}} \approx 10^{-14} \)
As will be explained in Appendix A, there is a way to make a relation between graviton count and entropy, so then the numbers associated with $n_f$ are a de facto counting algorithm for entropy per unit phase space. Note that the highest counting numbers for entropy are associated with $\Omega_g \approx 10^{-5}$, which according to Fig 1 above is associated with pre big bang GW/graviton production. Having $\Omega_g \approx 10^{-14}$ is associated with usual inflation, as given in Fig 1 above.

I.e. if one is looking for standard creation of entropy paradigms associated with the early universe, a typical phase transition argument for early entropy production is given by A. Tawfik [5], in 2008, which for QCD regimes

\[ S_{total} \equiv V_3 \cdot T^3 \sim 2.05 \cdot 10^{58} \]  

We assume that here, $S_{total} \sim 10^{58}$ may be associated with Graviton/GW and with frequencies initially of the order of about $10^8$ to $10^{10}$ in the beginning of cosmological evolution.

Such a huge burst of graviton production would lead to measurable consequences.

This is for temperatures of the order of $T \sim 174 \cdot MeV$, and if one factors in the volume of space time one is obtaining very likely values in between Tables 1 and 2 above.

Consider if there is, then also a small graviton mass, i.e. as factored in, in

\[ m_g(Graviton) = \frac{n}{L} + 10^{-65} \text{ grams} \]  

Note that Rubakov [11] writes KK graviton representation as, after using the following normalization

\[ \int dz \cdot h_n(z) \cdot h_n(z) = 6(m - \bar{m}) \]  

where $J_{i1}, J_{i2}, N_1, N_2$ are different forms of Bessel functions, to obtain the KK graviton/DM candidate representation along RS dS brane world

\[ h_n(z) = \sqrt{\frac{m/k}{J_i(m/k) \cdot N_i(m/k) \cdot \exp(k \cdot z) + N_i(m/k) \cdot J_i(m/k) \cdot \exp(k \cdot z)}} \]  

This Eq. (1.8)) is for KK gravitons having a TeV magnitude mass $M_\chi \sim k$ (i.e. for mass values at .5 TeV to above a TeV in value) on a negative tension RS brane. What would be useful would be managing to relate this KK graviton, which is moving with a speed proportional to $H^{-1}$ with regards to the negative tension brane with
\( h \equiv h_n(z \to 0) = \text{const} \cdot \sqrt{\frac{m}{k}} \) as an initial starting value for the KK graviton mass, before the KK graviton, as a ‘massive’ graviton moves with velocity \( H^{-1} \) along the RS dS brane. If so, and if \( h \equiv h_n(z \to 0) = \text{const} \cdot \sqrt{\frac{m}{k}} \) represents an initial state, then one may relate the mass of the KK graviton, moving at high speed, with the initial rest mass of the graviton, which in four space in a rest mass configuration would have a mass lower in value, i.e. of \( m_{\text{graviton}}(4-\text{Dim} \, \text{GR}) \sim 10^{-48} \text{eV} \), as opposed to \( M_X \sim M_{\text{KK-Graviton}} \sim 0.5 \times 10^5 \text{eV} \). Whatever the range of the graviton mass, it may be a way to make sense of what was presented by Dubovsky et. al. [12] who argue for graviton mass using CMBR measurements, of \( M_{\text{KK-Graviton}} \sim 10^{-20} \text{eV} \). Also Eq. (1.9) will be the starting point used for a KK tower version of Eq. (1.9) below. So from Maarten’s [14] per,

\[
\dot{a}^2 = \left( \frac{\kappa^2}{3} \left[ \rho + \frac{\rho^2}{2\Lambda} \right] \right) a^2 + \frac{\Lambda \cdot a^2}{3} + \frac{m}{a^2} - K \tag{1.9}
\]

Maartens [14] gives a 2nd Friedman equation, as

\[
\dot{H}^2 = \left[ -\frac{\kappa^2}{2} \left[ p + \rho \right] \left[ 1 + \frac{\rho^2}{\Lambda} \right] + \frac{\Lambda \cdot a^2}{3} - \frac{2m}{a^2} + \frac{K}{a^2} \right] \tag{1.10}
\]

Also, if we are in the regime for which \( \rho \geq -P \), for red shift values \( z \) between zero to 1.0-1.5 with exact equality, \( \rho = -P \), for \( z \) between zero to 5. The net effect will be to obtain, due to Eq. (1.10), and use \( a \equiv [a_o = 1]/(1+z) \). As given by Beckwith [8]

\[
q = -\frac{\ddot{a}}{a} = -1 + \frac{2}{1 + \kappa^2 [\rho/m] \cdot (1+z) \cdot (1+\rho/2\Lambda)} \approx -1 + \frac{2}{2 + \delta(z)} \tag{1.11}
\]

Eq. (1.10) assumes \( \Lambda = 0 = K \), and the net effect is to obtain, a substitute for DE, by presenting how gravitons with a small mass done with \( \Lambda \neq 0 \), even if curvature \( K = 0 \)

\section{Consequences of small graviton mass for reacceleration of the universe}

In a revision of Alves et al., [13] Beckwith [8] used a higher-dimensional model of the brane world and Marsden [10]KK graviton towers. The density \( \rho \) of the brane world in the Friedman equation as used by Alves et al. [13] is use by Beckwith [8] for a non-zero graviton
\[
\rho \equiv \rho_0 \cdot (1 + z)^3 - \left[ \frac{m_g \cdot (c = 1)^6}{8\pi G(h = 1)^2} \right] \left( \frac{1}{14 \cdot (1 + z)^3} + \frac{2}{5 \cdot (1 + z)^2} - \frac{1}{2} \right) \tag{1.12}
\]

I.e. Eq. (1.12) above is making a joint DM and DE model, with all of Eq. (1.12) being for KK gravitons and DM, and \(10^{-65}\) grams being a 4 dimensional DE. Beckwith [15] found at \(z \sim 4\), a billion years ago, that acceleration of the universe increased, as shown in Fig. 1. This would be a very good verification of Ng. hypothesis [15], and would check work done by Buonnano [16].

Fig. 2: Reacceleration of the universe based on Beckwith (note that \(q < 0\) if \(z < 0.423\))

Conclusion. We need to determine if GW/Gravitons can do double duty as DM/DE candidates in cosmic evolution.

Beckwith [17,18,19], investigated if gravitons could be a graviton gas for a substitute for a vacuum energy, as well as considered a suggestion by Yurov [20], of double inflation which if verified would justify Fig 1 above. He looks forward to presenting elaborations of these ideas in for coming conferences in 2010. It would be highly significant if semi classical treatments of the graviton can be shown to be consistent with Fig 2 above.

Appendix A NTROPY GENERATION VIA NG’S INFINITE QUANTUM STATISTICS

Information counting ties in with information packing as brought up in the use of small graviton creation volume, \(V\); for relic gravitons of a high frequency (short wave length) right after the big bang would be consistent Graviton volume \(V\) for nucleation is tiny, well inside inflation. So the log factor drops out of entropy if \(V\) is chosen properly for both Eq. (A.1) and Eq. (A.2). Ng’s [15] result begins with modification of the entropy/partition function Ng used in an approximation of temperature, starting with early temperature \(T \approx R_H^{-1}\) (\(R_H\) can be thought of as a representation of the region of space of the particles in question). Furthermore, assume that the volume of space is of the form \(V \approx R_H^3\) and look at a numerical factor \(N \sim \left(\frac{R_H}{l_p}\right)^2\), where the denominator is Planck’s length (on the order of \(10^{-35}\) centimeters). We also specify a
“wavelength” $\lambda \approx T^{-1}$. So the value of $\lambda \approx T^{-1}$ and of $R_H$ are the same order of magnitude. Note Ng [46] changed conventional statistics: he outlined how to get $S \approx N$, or $S < n >$ (where $<n>$ is graviton density). Begin with a partition function

$$Z_N = \left( \frac{1}{N!} \right) \left( \frac{V}{\lambda^3} \right)^N$$

(A.1)

This, according to Ng, leads to an entropy of the limiting value of, if $S = \langle \log[Z_N] \rangle$ will be modified by

$$S \approx N \cdot \langle \log[V / N \lambda^3] + 5/2 \rangle \rightarrow N \cdot \langle \log[V / \lambda^3] + 5/2 \rangle \approx N$$

(A.2)

References

15. Y. Ng, Entropy 2008, 10(4), 441-461; DOI: 10.3390/e10040441.