Four-Variable Jacobian Conjecture in a Topological Quantum Model of Intersecting Fields

Alfonso De Miguel Bueno*

February 17, 2024

Abstract

This preprint introduces in a visual and conceptual way a model of two intersecting curved fields with a shared nucleus, whose quantized dynamics offer potential cases of the four-variable Jacobian conjecture and a nonlinear Hodge cycle.

The Kummer type geometry of the model suggests a unified framework where abstract mathematical developments like Tomita-Takesaki, Gorenstein, and Dolbeault theories, can be conceptually linked to the Jacobian, Hodge, and Riemann conjectures.

Other mathematical physics topics, like the mass gap problem, releflection positivity, the arise of an imaginary time, or t-duality are also described within this context.

The model also lays the foundation of a novel deterministic quantum atomic system with a supersymmetric dual nucleus structure of matter and mirror antimatter.

1 Introduction

1.1 Antisymmetric system

Two intersecting curved fields that vary with opposite phase, when the right field contracts the left expands and vice versa, form in their intersection two transverse and two vertical subfields. The transverse subfields are mirror antisymmetric, meaning that being mirror reflection of each other, they follow opposite phases: when the right subfield expands the left contract and vice versa.

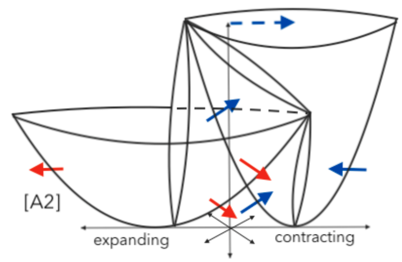

Figure 1: *Antisymmetric system: at moment A_2, the right transverse subfield contracts and the left transverse subfield expands; the vertical subfield moves rightward.A_4*

The curvature of the transverse subfields is half positive and half negative, and they are determined by the forces of pressure caused by the inward displacement of the negative curvature of the contracting field, and by the outward displacement of the positive curvature of the expanding field.

These forces of pressure are represented by four eigenvectors with eigenvalue 1 or -1, all pointing toward right or toward the left.

An inversion of the system A_2, equivalent to a 180 degrees rotation, given by the change of sign of the four positive eigenvectors, is operated when the right-hand contracting field expands and the left-hand expanding field contracts. Then, the right contracting transversal subfield at A_2 is mapped to the left contracting transversal subfield at A_4, and the left expanding transversal subfield at A_2 is mapped to the

*Independent researcher — ademiguelbueno@gmail.com Madrid, Spain. Orcid: 0009-0000-5420-3805

right expanding transversal subfield of A_4.

This operation can be considered as an involution, which is a type of automorphism. The system at A_2 maps to itself at A_3 and viceversa.

The complex conjugate function that describes the continuous evolution from A_2 to A_4 and from A_4 to A_2 is its own inverse.

The yet non-transformed right contracting transverse subfield at time T_1 can be interchanged with the transformed left contracting transversal subfield at T_2, and the non-transformed left expanding transversal subfield at T_1 can be interchanged with the transformed right expanding transverse subfield at T_2. In that way, the left and right transversal subfields exhibit chiral mirror symmetry at different times.

This occurs as half of the system A_2 (and the system A_4) follows a purely imaginary time dimension, delayed with respect to the real time dimension that follows the other half of the system.

In this context, a time dimension is considered to be a necessary reference to measure the periodic fluctuation of space, which can be represented by an axis in the coordinate system.

The top vertical subfield will move left or right, toward the side of the intersecting field that contracts.

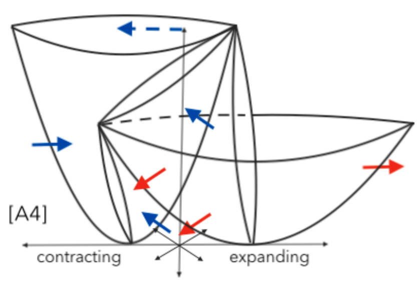

Figure 2: *Antisymmetric system: at moment A_4, the right transverse subfield expands and the left transverse subfield contracts; the vertical subfield moves leftward.*

Both left and right transverse subfields are here described by the same spatial dimensions. These spatial dimensions cannot be the same that are used to describe the intersecting fields, because the Y coordinate of the transverse subfields will be considered a diagonal axis from the point of view of the coordinate system taken as reference to describe the intersecting fields.

Misleading the different coordinate systems without considering the higher spatial dimensions of the transverse subfields will result in a relativistic spacetime metric that appears elongated. This effect would be observed when measuring the transversal subfields from the location of the intersecting fields, using their referential coordinates system.

Given that each eigenvector has two possible directions, pointing toward right or left, the antisymmetric system formed by A_2 and A_4 can be described by a complex function of two variables, and a pair of 2×2 complex matrices whose elements are the mentioned eigenvectors:

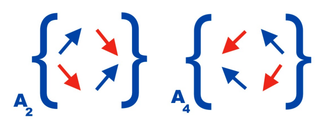

Figure 3: *Pair of 2×2 complex matrices whose elements are eigenvectors with eigenvalue 1 or -1, related to the antisymmetric system.*

1.2 Symmetric system

In the symmetric system represented by A_1 and A_3, the two intersecting fields oscillate in unison: they contract and expand simultaneously.

The transversal subfields exhibit chiral mirror symmetry at the same time. If the system A_1 were inverted in the same way previously seen for the antisymmetric system, equivalent to a 180-degrees rotation in the horizontal plane, both left and right transverse subfields would be interchangeable.

However, in the symmetric system, the positive and negative eigenvectors point upward and downward, respectively. The continuous inversion of system A_1 performed at A_3 implies that all four positive eigenvectors pointing upward revert their sign, becoming negative and pointing downward. In this context, the inversion does not represent a 180-degrees rotation of the vertical plane, as the curvatures do not get inverted. However, the inversion operated at A_3 has consequences similar to that type of planar rotation. All the pressure forces at A_2 are caused by the negative curvatures of both intersecting fields moving inward, and all the pressure forces at A_4 are caused by the positive curvature of both intersecting fields moving outward. This change in the relevant side of the curvature in terms of pushing force is equivalent to an inversion in curvature.

Considering a mapping that takes the system from a state of expansion at A_1 to a state of contraction at A_3, there will exist a reverse mapping that transitions the system from contraction back to expansion. This transformation is a continuous process, and despite differences in shape, the topological structure of the curvatures of the transverse subfields is preserved. Therefore, this mapping can be considered a homeomorphism.

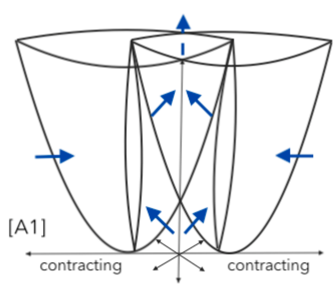

Figure 4: *Symmetric system: at moment A_1, both left and right transversal subfields expand. The vertical subfield moves upward while contracting.*A_4

Observing the dynamics determined by the periodic contraction and expansion of the intersecting fields, it can be observed that at moment A_1, when both intersecting fields undergo contraction, there is a simultaneous expansion of both transverse subfields. Conversely, at A_3, when the previously contracting fields begin to expand, both transverse subfields enter a state of contraction.

The linear, continuous evolution from A_1 to A_3 and vice versa, can be modeled by a complex function of two variables. The changes of the curvatures of the oscillating spaces and subspaces are indicated by the variation in the eigenvectors' direction. The four

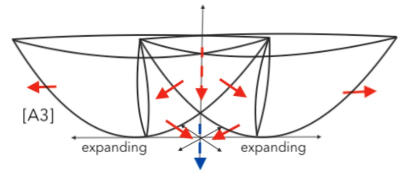

Figure 5: *Symmetric system: at moment A_3, both left and right transversal subfields contract. The vertical subfield moves downward while expanding.*A_4

eigenvectors can point upward (at A_1) or downward (at A_3), depending on their sign, and they can be represented in a pair of 2×2 complex matrices with eigenvalues of either 1 or -1.

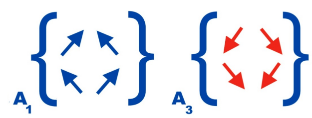

Figure 6: *Pair of 2×2 complex matrices of eigenvectors related to the antisymmetric system.*

1.3 Rotational system

Thus far in the article, the symmetric system A_1 A_3, and the antisymmetric system A_2 A_4 have been treated as independent and unrelated systems, linearly described by two distinct functions related to different pairs of matrices.

However, within a rotational framework, the symmetric and antisymmetric systems may in fact turn out to be the same system that undergoes topological transformations with each 90-degree rotation, alternating between symmetric and antisymmetric states. The nonlinear evolution of such a rotational system would need to be described by two interpolated functions: a complex function and a harmonic partial complex conjugate function.

However, in a rotational framework, the symmetric and the antisymmetric systems may turn to be the same system which is topologically transformed after each 90-degrees rotation, becoming periodically symmetric or antisymmetric.

This would imply that the smooth but non-linear evolution of the system given by $A_1 + A_2 + A_3 + A_4$, must be described by two interpolated functions, a complex function (related to the symmetric moments of the system) and its harmonic partial complex conjugate function (related to the antisymmetric moments of the system).

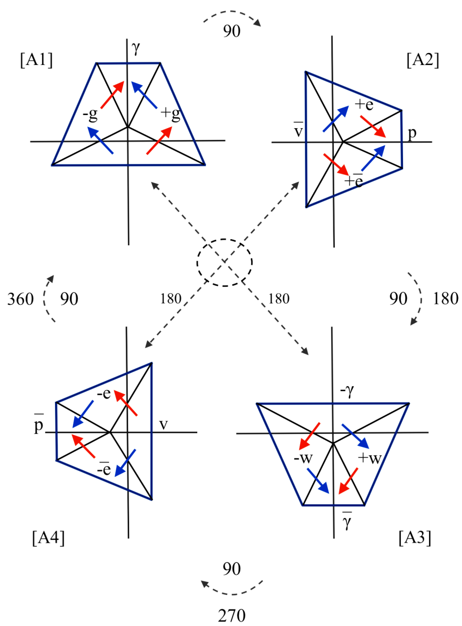

Figure 7: *Rotational interpolation of the vector symmetric and antisymmetric systems represented in a 2D scheme*

The evolution of that interpolated rotational system can be represented by a set of four 2×2 complex rotational matrices of eigenvectors with eigenvalue 1 or -1:

The rotational system's evolution progresses through four stages, each corresponding to a 90-degrees rotation. This evolution is associated with the periodic interpolation of symmetric and antisymmetric moments of the system. In each stage, only two of the four eigenvectors, which signify the variation in the curvature of half of the system, change

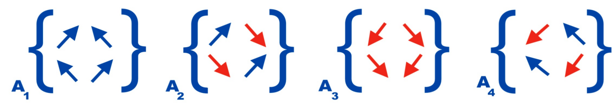

Figure 8: *Set of 2×2 rotational matrices related by pairs to the complex symmetric (A_1 and A_3) and the partial complex conjugate antisymmetric (A_2 and A_4) systems.*

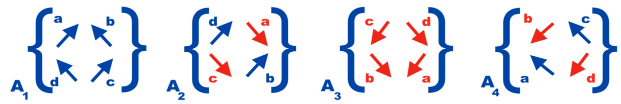

Figure 9: *Actual rotation of the vectors in the context of the rotational matrices, changing their position after each 90-degrees rotation.*

their sign according to their eigenvalues of 1 or -1.

The variation observed in half of the eigenvectors can be interpreted as a partial derivative, suggesting a half-order derivative. The complete first-order derivative is achieved when all four eigenvectors alter their signs following two successive 90-degrees rotations. The eigenvectors that change represent the differentiated complex variables, while the eigenvectors that remain constant represent the undifferentiated complex variables.

The partial derivative also represents a partial complex conjugation.

However, by assigning specific letters to the eigenvectors, it can be observed that all four eigenvectors change their direction at each stage, effectively displacing after each 90-degrees rotation.

The four eigenvectors actually rotate in the complex plane, altering their position and direction four times, each time by 90 degrees. In this context, the function describing the system's evolution can be interpreted as a function of four variables. These four variables represent the four potential directions or positions in the complex plane that the eigenvectors can assume during the system's non linear rotational evolution.

When only two of the four eigenvectors change their sign, the system's evolution will be represented by a function of two variables.

On the other hand, the set of transformation matrices A_1, A_2, A_3, A_4 that describe the evolution of the rotational system result from the operations of transposition, complex conjugation (as the sum of two partial complex conjugations), and inversion:

- *A_1* (0-degrees rotation) represents the eigenvectors in the symmetric system, when both intersecting fields contract. The transversal subspaces have mirror symmetry at the same moment and the top vertical contracting subfield experiences a double force of compression while ascending. A_1 can be taken as the identity matrix.

- *A_1* (90-degrees rotation) represents the eigenvectors when half of the system has delayed its phase, introducing a purely imaginary time dimension. The transversal subfields have mirror antisymmetry (the left expands while the right contracts), and the vertical subfield moves right. A_2 represents the transposition of A_1. The change of sign of half of eigenvectors implies a partial complex conjugation of A_1, and its $\frac{1}{2}$ order derivative.

- *A_1* (180-degrees rotation) represents the partial conjugation of A_2, changing sign the two eigenvectors that had not changed sign at A_2). As its four eigenvectors have already commuted their sign with respect to A_1, A_3 represents the negative reflection of A_1 and its whole first order $(\frac{1}{2}+\frac{1}{2})$ derivative; (A_3 represents the $\frac{1}{2}$ order derivative of A_2).

- *A_1* (270-degrees rotation with respect to A_1, 180-degrees with respect to A_2, and 90-degrees with respect to A_3) represents the transpose of A_3, the$\frac{1}{2}$ order antiderivative of A_3, the second transposition of A_1, and the first order $(\frac{1}{2}+\frac{1}{2})$ derivative of A_2; A_4 is also the negative mirror reflection of A_2, having commuted sign its four eigenvectors.

- An additional 90-degrees rotation produces A_1 which represents the positive reflection of A_3, a $\frac{1}{2}$ order antiderivative of A_4, and the first order $(\frac{1}{2}+\frac{1}{2})$ antiderivative of A_3.

2 Jacobian conjecture

The Jacobian conjecture [1] formulated by Keller in 1939 states that if a polynomial map from an n-dimensional space to itself has Jacobian determinant which is a non-zero constant, then, the function has a polynomial inverse.

Expressed in terms of vectorial functions, it would state that if a vector-valued function map from an n-dimensional space to itself has Jacobian determinant which is a non-zero constant, then the function has a vector-valued inverse.

The Jacobian determinant is a measure of how much a transformation stretches or shrinks the space it maps to, and it is defined for continuous transformations. The Jacobian conjecture applies to maps between homeomorphic spaces.

In the context of the rotational system, the transformations are continuous but not in a linear way. The smooth continuity passes through the interpolation of the complex and the complex partial conjugate function spaces after each 90-degrees rotation.

In that context, the topological structure of the transverse subfields is preserved, being automorphic, even when their size is not identical as it happens in the symmetric system when the contracting subfields map the expanding subfields. Their curvatures are always half positive and half negative, as they are formed by the inner curvature of an intersecting field and by the outer curvature of the other intersecting field.

In the antisymmetric system, the topological structure of the automorphic vertical subfield that maps to itself when moving leftward or rightward is also preserved because it's always formed by a negative curvature formed by the inner curvatures of both left and right intersecting fields.

The top vertical subfield in the antisymmetric system that moves upward while contracting (when both intersecting fields contract), or downward while expanding (when both intersecting fields expand), has a negative curvature. However, the inverted subfield that at moment A_3 exists at the convex side of the system, mapping the top vertical subfield that exists in the concave side at A_1, has a double positive curvature.

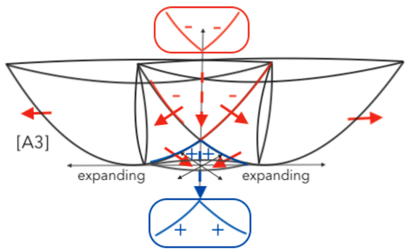

Figure 10: *Antisymmetric system: at moment A_3, when the top vertical subfieldd descends while expanding it has negative curvatures; but its mirror counterpart at the convex side has positive curvatures.*

In that sense, the topological structure of the concave vertical subfield of A_1 is not preserved in the inverse convex subfield of A_3. That would represent that the top vertical subfield is not mapped when the inversion is operated at A_3.

Still in that case, the ascending contracting vertical subfield with negative curvature of A_1 can be considered being mapped to itself at A_3, when it descends while expanding. However, in that case, the mirror reflection property of the vertical subfield would not be being considered by the Jacobian conjecture.

An additional complexity could be introduced if the intersecting fields periodically synchronize and desynchronize their phases of fluctuation while the whole system rotates.

3 Gorenstein Liaison

In algebraic geometry, the Gorenstein theory [2] establishes that two modules in projective space are linked if they are isomorphic.

These isomorphic modules contain curved with a same deficiency. The curvature singularity reflects a lack of regularity. For instance, a curve exhibiting a change in its sign, inverting its direction, represents a defect in that curve.

The deficiency module of a curve is isomorphic to the deficiency module of any other curve with the same deficiency. Consequently, if two curves share the same deficiency, then the modules that encompass them in the projective space will also exhibit the same deficiency module.

This establishes that the two modules are "linked" by their deficiency in a Gorenstein sense.

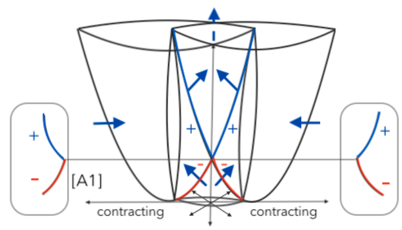

Figure 11: *The left and right modulus, represented by the mirror transversal subfields, encompas curvatures that exhibit a same irregularity, being half + and half − from the point of intersection between the left and right fields.*

To be linked in projective space, it suffices for two modules to preserve the algebraic structure imparted by the deficiency in the curves.

Under the model of intersecting fields proposed in this article, the modules that contain the curves of the Gorenstein theory are interpreted as the transverse subfields in both the symmetric and antisymmetric systems.

The projective space scenario is represented by the left and right intersecting curved fields. These fields have a different weight in regard to the creation of the singularity in the curvature of the transversal subfields.

In that way, the left transverse subfield, embedded in the left intersecting field, exhibits a negative curvature up to a point of inflection, where that curvature inverts its sign becoming positive. That point of singularity is the point of intersection of the left and right fielfds.

That deficient by irregular positive sign of the curvature is given because it corresponds the the outer side of the curvature of the right intersecting field. In that context, the right field has a relevant or determining weight or in the inversion of the sign that causes the singularity.

The same occurs in regard to the right transverse subspace, embedded in the right intersecting field. Its curvature exhibits the same dual structure as the left transversal subfield, being half positive and half negative. In this case, the relevant weight in the curvature singularity will be attributed to the left intersecting field.

In this context, both left and right modular transverse subspaces are isomorphic because the structure of their deficiency is preserved under the map to each other that occurs during the evolution of the projecting arena, creating a Gorenstein-style linkage, as a short of entanglement.

Considered in these terms, the Gorenstein modular linkage can be connected to the Jacobian conjecture previously exposed, in the framework of the intersecting fields model.

4 Tomita-Takesaki theory

Considering the rotational fields system as a specific case of the Jacobian conjecture, it is also possible to conceptually infer some relations to Tomita-Takesaki (TT) modular theory [3].

In TT theory two intersecting algebras form two shared "modular inclusions" (with $+$ and $-$ half sided subalgebras) and a "modular intersection" (with an integer sided subalgebra).

The left and right half handed subalgebras will be images of each other, when they are commutative, or they will not be their mirror image when they are noncommutative.

Mapping the modular inclusion to its reflection image, the left and right subalgebras will be the opposite image of each other (reverting their initial signs) if they are commutative; if they are noncommutative, the initial left sided subalgebra will be the image of the right sided mapped subalgebra, and the initial right-handed subalgebra will be the image of the left sided mapped subalgebra.

TT theory decomposes a linear transformation into its modular building blocks, revealing its automorphisms.

Decomposing the bounded operator, it obtains the modular operator and the modular conjugation (or modular involution) which is a transformation that reverses the orientation, preserving distances and angles.

Translating the abstract algebraic terms to the fields model, two intersecting algebras would represent the two intersecting fields fluctuating with the same or opposite phase.

The half handed subalgebras (or "modular inclusions") will be the transversal subfields of the nucleus shared by the intersecting fields, while the integer handed subalgebra (or "intersection inclusion") will be our vertical subfields. In this context, we identify commutativity and noncommutativity with mirror symmetry and mirror antisymmetry, respectively.

The bounded operator that is decomposed will be the 90-degrees rotational matrix; The modular building blocks are the set of matrices that are obtained when applying the operator.

The modular operator will be the $\frac{1}{2}$ partial complex conjugate A_2 matrix; And the modular conjugation will be its conjugate matrix A_4, which forms the whole conjugation by adding the partial conjugations $(\frac{1}{2}+\frac{1}{2})$ of A_2 and A_3.

Therefore, by separating the partial complex conjugate matrix from the complex one, the automorphism of the antisymmetric partial conjugate system is found.

The half sided algebras that form a modular inclusion are noncommutative, it means we are in the antisymmetric system where the left intersecting field contracts while the right one contracts and vice versa; in that system, the left transversal subfield will be the mirror symmetric image (it will be the mapped image) of the right transversal subfield when, later, the left intersecting field expands and the right one contracts.

In that sense, a past half handed subalgebra is being mapped with its future image. A time delay will exist between both subalgebras.

Considering Δ as the modular operator A_2, J the modular conjugation A_4, and M the intersection of two Von Neumann algebras, $\Delta^{-Yt} M \Delta^{it}$ will represent the positive and negative $\frac{1}{2}$ sided modular inclusions of the modular operator, being t a real time dimension and it an imaginary time dimension given by the partial conjugation of A_1 or A_3.

It is this different time dimension what makes noncommutative, as non-interchangeable, the modular $+$ and $-$ inclusions related to Δ in the antisymmetric system.

Applying the modular involution, yields $J^{yt}M'J^{-it}$.

Δ^{-yt} is transformed into J^{yt} and Δ^{it} is transformed into J^{-it}, being $J^{yt}M'J^{-it}$ the involutive automorphism of $\Delta^{-yt}M\Delta^{it}$.

The noncommutative, as non-interchangeable, Δ^{-yt} and Δ^{it} become commutative or interchangeable through time at $J^{yt}M'J^{-it}$, fixing their antisymmetry (restoring the lost mirror symmetry) in that way .

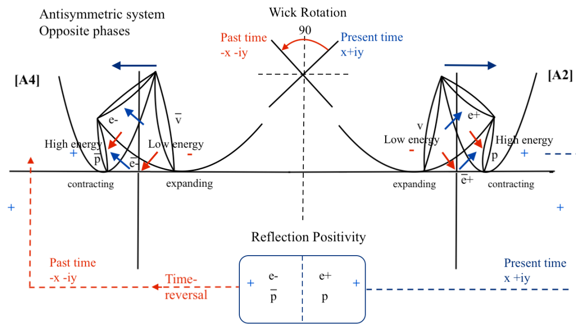

Figure 12: *Reflection positivity in the antisymmetric system.*

5 Reflection positivity

Related to the delay in time in the antisymmetric system, it can also be mentioned a property that all unitary quantum field theories are expected to hold: "reflection positivity" (RP). [4]

The positive increasing energy that appears in one side of the mirror system should also be reflected in the other side. However, in the context of the antisymmetric system, the positive or increasing energy of the contracting right transverse subfield does not mirror simultaneously in the expanding left transverse subfield, which exhibits negative or decreasing energy.

Therefore, to obtain a positive energy reflected at the left side, making the sides of the system virtually symmetric, a time reversal operation is needed.

To observe the positive energy reflected at the left side, it will be needed to go back in time to the moment where the left transversal subfield was contracting and had a positive energy. This operation is performed by a type of "Wick rotation". [5] The main time phase of the symmetric system can be represented with the Y coordinate.

By performing a partial conjugation that involves a fractional derivative, the time coordinate Y undergoes a rotation into the purely imaginary dimension within the complex plane. At that moment, the mirror system becomes antisymmetric as one side of the system keeps following the imaginary time of Y while the other side follows a harmonic phase. A positive or negative time lag has been introduced.

Reversion time on one side of the system serves as a symbolic tool to virtually restore symmetry to the time phases. To revert to the previous time, one could perform a reverse rotation of the complex time axis $(X + iY)$ to achieve a full complex conjugation at $(-X - iY)$.

In the A matrix context, that time backward rotation represents an antiderivative of $-A$.

When the time reverse has been symbolically completed, in the left side of the mirror system the left subfield will be contracting, having an increased positive energy; this is a past reflection of the future positive energy that will emerge a moment later in the right side.

In the reverse past time, at the right side of the system the right subfield will be expanding, having a decreased negative energy.

In regard to the symmetric system, positivity is reflected between the right and left transverse subfields at the same time. In that sense, it's not necessary to use the Wick operation to reverse time.

Both left and right transversal subfields will be the mirror reflection of each other at the same time. However, in the case of the strong interaction in the symmetric system, when the contracting vertical subfield has an increased positive energy while ascending to emit a pushing force, it will be necessary to virtually visit a past moment to look for a previous state where positivity could be reflected.

Going back in time, the vertical subfield will be los-

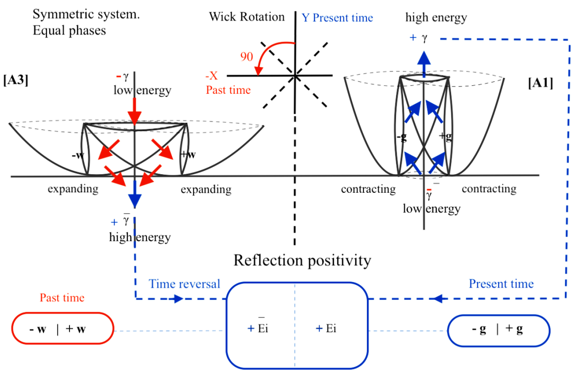

Figure 13: *Reflection positivity the symmetric system.*

ing its energy while expanding, moving downwards. Therefore, at that past moment, the vertical subfield will not display a positive energy.

Reflection positivity, however, can be found at that past moment in the convex side of the system of the two intersecting fields, where an inverted subfield with convex curvatures will be experiencing an increased energy.

That inverted subfield can mirror the vertical subfield which in a future state will be ascending in the concave side of the system through the Y axis.

The missing reflection positivity in the concave side of the system in the strong interaction can be related to a mass gap problem when it comes to the weak interaction.

6 Mass gap problem

There will be a mass gap [6] in the system when the two intersecting fields simultaneously expand, and the vertical subfield experiences a decay of energy.

This case represents the ground state with the lowest possible energy of the vertical subfield, which is always greater than 0 because the highest rate of expansion of the intersecting fields prevents them from having zero curvature.

The zero point of the vacuum, where there should be no energy or mass, is placed at the point of intersection of the XY coordinates, and that point is never reached by the vertical subfield that descends through the Y axis while expanding during its decay.

An "upper" mass gap would refer to the highest possible mass of a particle in the strong interaction. Its limit would be given by the greatest rate of contraction of the intersecting spaces.

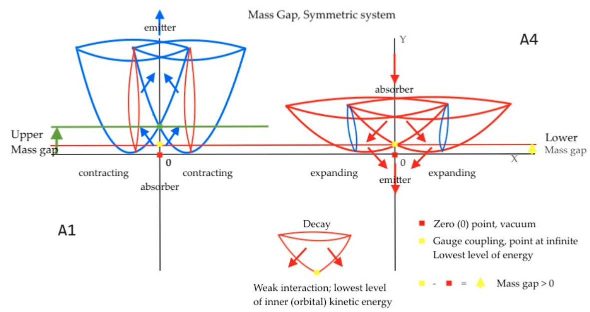

Figure 14: *Mass gap in the symmetric system; the upper gap occurs in the compressed photonic subfield when both intersecting fields contract, while the lower gap occurs in the decompressed subfield when both intersecting fields expand.*

The zero point of the vertical subfield is marked in yellow on the above diagram, at the point of intersection of the left and right intersecting fields.

The gap is given by the distance from that point to the zero point where the X and Y coordinates intersect, represented by a red mark. An arrow shows the gap distance between those critical points.

However, in this model, the zero point does not represent a vacuum where neither energy nor mass exists.

When the mass and energy of the vertical subfield reach their weakest level in the concave side of the symmetric system, an equivalent amount of energy and mass arises in the convex side, where the zero point is located, as the result of the double pushing force caused by the displacement of the positive curvature of the expanding intersecting fields.

That mass and energy at this zero point will be considered dark from the point of view of the concave side of the system.

In the antisymmetric system, the lowest energy level occurs when a transverse subfield experiences

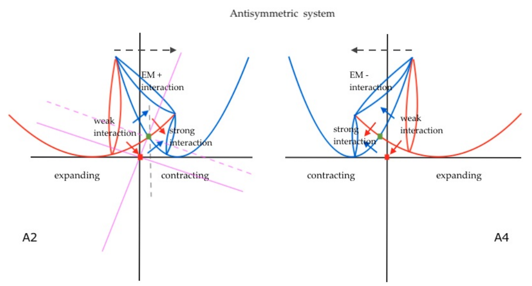

Figure 15: *Mass gap in the antisymmetric system, with the left and right displacements of the point of intersection.*

a double decompression due to the displacement of the concave curvature of the contracting intersecting field and the displacement of the positive curvature of the expanding intersecting field.

The corresponding double compression is then experienced by its mirror antisymmetric transverse subfield.

7 N=1 Supersymmetric atomic model with dual nucleus of matter and antimatter

The fields model emerges in the context of the heuristic development of a novel supersymmetric quantum model of an atom composed of two intersecting curved fields. These fields share a nucleus consisting of two transverse and two vertical subfields, symbolizing the mirror matter and antimatter of this dual structure. [7]

The following sections describe in a general way the dual nature of the nucleus, treating its symmetric and antisymmetric states as separate systems.

The atomic antisymmetric nucleus's composition varies based on the system's evolutionary stage. It could comprise a proton, a positron, and a antineutrino, or conversely, a neutrino, an electron, and an antiproton.

7.1 Fermionic antisymmetric system: the left intersecting field expands while the right one contracts: A_2

- The right contracting transversal subspace represents a proton.
- The left expanding transversal subspace represents a neutrino.
- The vertical subspace moving toward the right represents a positron.

7.2 Antisymmetric system, the left intersecting field contracts while the right one expands: A_4

- The right contracting proton expands, becoming a right expanding antineutrino.
- The left expanding neutrino contracts, becoming a left-handed contracting antiproton.
- The vertical positron moves toward the left, becoming an electron.

The right transverse proton at moment A_2, and the left transverse antiproton at moment A_4 are contracting subspaces that experience a dual compression force, originating from the positive curvature of the expanding field and the negative curvature of the encompassing contracting field.

When the expanding field contracts and the contracting field expands, the side of the curvature that determines the force of pressure of the field changes, inverting the dynamics of the forces and the direction of the energies of the system.

It implies that when the right contracting proton of A_2 is transformed into a right expanding neutrino at A_4, and the left contracting antiproton of A_4 is transformed into a left expanding antineutrino at A_2, the neutrino and antineutrino will experience a double decompression force.

The topological transformation of the contracting proton in an expanding neutrino (and also of the expanding antineutrino in a contracting antiproton) can be described as an automorphic map, as the transverse subfield maps to itself through its periodic contraction and expansion, preserving the structure of its double curvature even though its size and physical properties change.

The curvature of each transverse subfield is half positive and half negative, and this irregular singularity is preserved during the whole evolution of the antisymmetric and symmetric systems.

The A_1 Proton and the A_3 antiproton exhibit chiral mirror reflection symmetry at different moments. Being identical with an inverse sign, they can be considered as the homeomorphic map of each other. The same applies to neutrino and antineutrino.

In this context, proton and antiproton, (and neutrino and antineutrino), are considered to be Dirac antiparticles at different times.

When the right intersecting field contracts and the left expands, the top vertical subfield will move rightward, acting as a positron at A_2. When the left intersecting field contracts and the right expands, the top vertical subfield will move leftward, acting as a positron at A_4.

In this dual fields model, the electron and positron are the same automorphic subfield that maps onto itself at different moments, while oscillating left or right in a pendulum-like motion. Being their own antiparticles, they are modeled as Majorana antimatter.

When the vertical subfield moves toward the left acting as electron, it can be considered that it simultaneously exists in a virtual way on the system's right side, in the Aristotelian sense that although it actually does not exist yet it has the potential of becoming existent a moment later, when the right intersecting field contracts and the right field expands.

The existence of an electron and a positron in the same atom, also known as *positronium*, was predicted by Dirac in 1928. However, the positronium was formulated as an exotic atom with no proton in its nucleus.

In a similar way, the coexistence of proton and antiproton in the same atom is currently accepted as an exotic structure called *protonium*, with no electrons or positrons.

In the dual model introduced in this article, matter and antimatter would coexist in any electromagnetic nucleus, being related to each other by means of their chiral mirror reflection symmetry or antisymmetry.

That mirror symmetry would operate at different times in the fermionic antisymmetric system A_2 A_4, or at the same time in the bosonic symmetric system A_1 and A_3.

Fig. 16 visually represents the limit states of the evolution of the antisymmetric system; however, they do not reflect the moment when the curvatures of the intersecting fields, having an opposite phase, are coincident. At that moment, the top vertical subfield will be passing through the central Y axis, which is the reference zero center of symmetry of the system, and the left and right transverse subfields will exhibit the same curvature and charge, canceling each other.

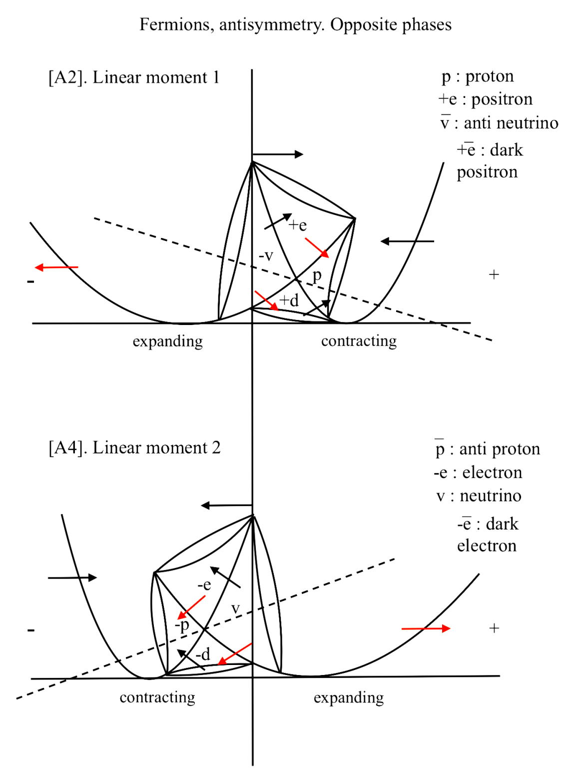

Figure 16: *Fermionic antisymmetric system, lineal evolution.*

This state of neutrality can be considered analogous to a balance scale, where the equal weights on each side cancel each other. It is at this moment that the neutron concept would emerge.

All the subfields in the antisymmetric system are considered fermions with noninteger $\frac{1}{2}$ spin.

Each field is determined by two forces of pressure or decompression, each one being represented by a vector in the fields model. The spin of each subfield is determined by the force of pressure of only one of the intersecting fields, which is represented only by the vector that changes its sign.

These fermionic subfields are governed by the Pauli exclusion principle: when at the right side of the system the transverse subfield contracts acting as a proton, the left transverse subfield cannot be contracting acting as an antiproton, it must be expanding, acting as an antineutrino, as a consequence of the opposite phases that govern the antisymmetric system.

Similarly, when the vertical subfield moves toward the left, acting as an electron, it cannot simultaneously move toward the right, acting as a positron.

Although this unconventional fields model is causal and deterministic, it should be possible to be described in probabilistic terms. In that case, these fermionic fields should follow Fermi-Dirac statistics.

In that same context, considering Schrödinger's cat in the context of the dual atom model, it could be figuratively said that the right alive contracting cat will be the delayed reflection of the left dead expanding cat, and vice versa.

Here, there will not be a single alive and dead cat, but two identical cats with opposite states and positions that are mirror reflections of each other.

Considering there is a unique cat, the simultaneous states of being "alive" and "dead" will be considered "superposed". But in the context of two mirroring cats, their opposite states will be considered entangled with an opposite phase.

7.3 Bosonic symmetric system, when the left and right intersecting fields contract: A_1

- The right and left expanding transversal subspaces represent a right-handed positive and a left-handed negative gluon.
- The top vertical ascending subspace that contracts receiving a double force of compression will be the electromagnetic subfield that emits a photon while pushing upward.
- The inverted bottom vertical subspace at the convex side of the system represents the dark decay of a previous dark antiphoton.

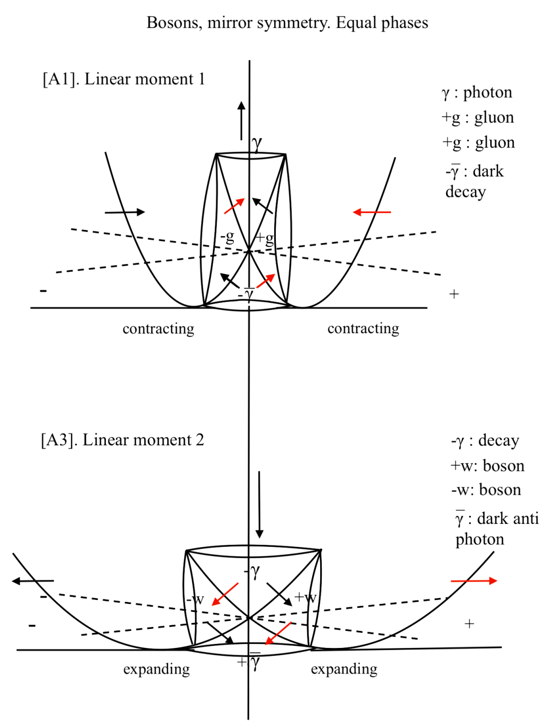

Figure 17: *Bosonic Symmetric system, lineal evolution.*

7.4 Symmetric system, when the left and right intersecting fields expand: A_3

- The right and left expanding transverse subspaces may represent −W and +W bosons.
- The top vertical descending subspace will be the electromagnetic subfield losing its previous energy, after having emitted a photon.
- The inverted bottom vertical subspace at the convex side of the system is the dark anti electromagnetic subfield that emits a dark antiphoton.

The identity of the symmetric transversal subfields, labeled before as "W bosons" and "gluons", requires further clarification in this model.

The left and right transversal subspaces will be mirror symmetric antimatters at the same time. As both experience simultaneously the same state of being contracting (or later being expanding), they are considered to be bosons not ruled by the Pauli exclusion principle. They should then obey the Fermi-Dirac statistics.

However, photons and antiphotons cannot exist simultaneously in the same state. When a photon emerges in the concave side at A_1, an antiphoton decays in the convex side. Conversely, when a photon decays at A_3, an antiphoton emerges. From here, it is inferred that photon and antiphoton are governed by the Pauli exclusion principle.

This occurs because the curvatures of both the left and right transverse bosonic subfields follow the same phase of variation, which is opposite to the phase of the intersecting fields that encompass them.

7.5 Supersymmetric system

While the Standard Model describes fermions and bosons as fundamentally distinct particles, string theory suggests a connection between them through additional supersymmetric particles that would act as superpartners between them.

In the two intersecting fields model proposed in this article, supersymmetry emerges from the dynamic evolution of intersecting fields. As the system rotates, the same four subfields manifest as fermions or bosons according to the phase difference between the intersecting fields. This phase difference determines the fermionic or bosonic topological states that the subfields manifest at different stages of the rotational evolution.

This rotational framework seamlessly unifies fermionic and bosonic systems through achieving a N=1 supersymmetry.

Although in this model the fermionic partners of the photonic subfield are the electron and positron subfields, it still can be considered as an N=1 correspondence. This is because in this model positron and electron are the same subfield placed at opposite sides at different moments. In that sense, the $\frac{1}{2}$ top spin of the positron at A_2 and the $-\frac{1}{2}$ top spin of the electron at A_4 simultaneously converge at A_1 in the vertical photonic field.

Supersymmetry could also be explained in terms of the periodic synchronization and desynchronizationm of the intersecting fields. However, the interpolation caused by the rotational dynamics still seems necessary to introduce a quantization of the space time. When considered independently, the bosonic symmetric and the fermionic antisymmetric systems exhibit the classical linear continuity of longitudinal waves. However, when interpolated within the rotational context, an apparent quantum discontinuity arises, periodically breaking and restoring the symmetry of the rotational system. This smooth quantum phenomenon can be interpreted as a non-linear rotational continuity.

However, the nonlinear interpolation introduced by rotational dynamics seems to play a crucial role in introducing quantization to spacetime. When considered independently, the bosonic symmetric and fermionic antisymmetric systems exhibit a classical continuity. However, within the rotational context, a quantum discontinuity arises, periodically breaking and restoring the symmetry of the system. This smooth quantum phenomenon can be interpreted as a non-linear rotational continuity.

In that context, a single complex function or, separately, its harmonic complex function, cannot describe in a non probabilistic way the whole dynamics of this rotational system, as the fermionic states appear as the harmonic partial conjugation of the bosonic states, and vice versa, being the symmetric and the antisymmetric systems essentially intertwined.

In the context of this rotational system, a single complex function or its harmoic partial complex conjugate cannot capture the complete dynamics in a non-probabilistic manner. This is due to the harmonic partial conjugation of fermionic and bosonic states, which renders the symmetric and antisymmetric moments of the system inherently interdependent through time.

The bosonic and the fermionic states can be considered as the intertwined electric and magnetic moments of the system, respectively.

The next diagram represents the signs of the curvatures of the four nuclear subfields. A colored curvature indicates the intersecting field that changes its curvature, contracting or expanding, creating a force of pressure from the side of the curve represented by a circle on one of the curvature's sign.

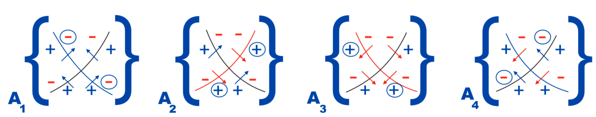

Figure 18: *Vector forces of pressure and signs of curvatures in the nuclear subfields of the interpolated symmetric and antisymmetric systems.*

On the other hand, the nuclear subfield in the symmetric and antisymmetric subfields can be described as cobordant [8] subspaces.

The vertical subspaces share borders with the left and right transversal subspaces, and they all share borders with the two intersecting spaces.

These borders can be thought of as one-dimensional lines described by the curvatures of the intersecting fields.

From that point of view, the fields model could be related to string theories.

7.6 T-duality and SYZ conjecture

In string theory, the SYZ conjecture [9] states that there exists a special type of Calabi-Yau manifold [10] that is related to another Calabi-Yau manifold by a T-duality [11] transformation.

T-duality is a concept that relates the spaces described by type IIA and type IIB string theories.

In Type IIA string theory, the strings can move freely in the Calabi-Yau transverse space with a larger radius, while in type IIB string theory, the strings are confined to the boundaries of the transverse space of smaller radius. This means that a string theory compactified on a large Calabi-Yau space is equivalent to a string theory compactified on a small Calabi-Yau space.

T-duality relates these two different types of larger and smaller transversal spaces by means of a type of inversion that exchanges the roles of the large and small radii transverse spaces.

In the context of the dual fields model, the Calabi-Yau spaces of smaller or larger radius can be considered equivalent to the transverse contracting or expanding subspaces that are mapped to each other by means of their topological transformation through time, as described before in the antisymmetric rotational system.

The elliptic orbits inside of the transversal subfields, caused by their periodic expansion and contraction, can be visually related to the notion of elliptic fibrations.

The inner orbits of the transverse subfields would be elliptic fibrations.

7.7 Unified interactions. Higgs fields

When one of the subfields contracts, its internal kinetic energy increases, and when it expands, its internal kinetic energy decreases, accelerating or decelerating its inner orbital motion.

This increase or decrease in energy is considered here to be a strong or weak interaction, forming the stronger or weaker bond that unites the nuclear subfields and the whole dual system.

The electromagnetic interaction is represented by the vertical subfield, which moves left or right in the antisymmetric magnetic moment or upward or downward in the symmetric electric moment.

The gravitational interaction is represented by the fluctuating curvatures of the intersecting fields, which determine the mass of the subfields acting as a Higgs mechanism.

8 Two-genus geometry and Kummer surfaces

The geometry of intersecting fields model can also be described in terms of a two-genus torus or two related tori.

The outer positive and the inner negative curvatures of the torus can be interpreted as simultaneous representations of the expanding or contracting moments of the oscillating fields. This can be observed when looking at them from above in a top view, using an orthographic projection.

Furthermore, the transversal subspaces in the fields model could be considered equivalent to the transversal lattices in a Weil-type torus model.

On another note, K3 surfaces represent another type of geometry that can be related to the intersecting fields model. Specifically, the subfields that constitute the nuclear manifold of the intersecting fields system could be considered analogous to Kummer surfaces [12].

A main characteristic of Kummer surfaces is that they have a maximum of 16 double points, each representing a singularity that arises from the convergence of two branches of the system. These double points participate in the Kummer inversion, where each double point maps to its inverse.

In the context of the intersecting fields model, each subfield arises from the convergence of the left or right "branches" of the curved intersecting fields, and their double curvature given by that convergence can be interpreted as a double point.

In the symmetric system, where both fields contract, a double point will appear in each of the four subfields. When both fields expand a moment later, four additional double points emerge. Similarly, when the right field contracts while the left field expands, four distinct double points appear; and when the left field contracts while the right field expands, four more double points are formed.

In this way, the dynamics of the system encompass 16 double points, each associated with one of the four subfields and their periodic transformations. Each double point in these subfields maps to its inverse during the evolution or involution of the system, as it has been seen before.

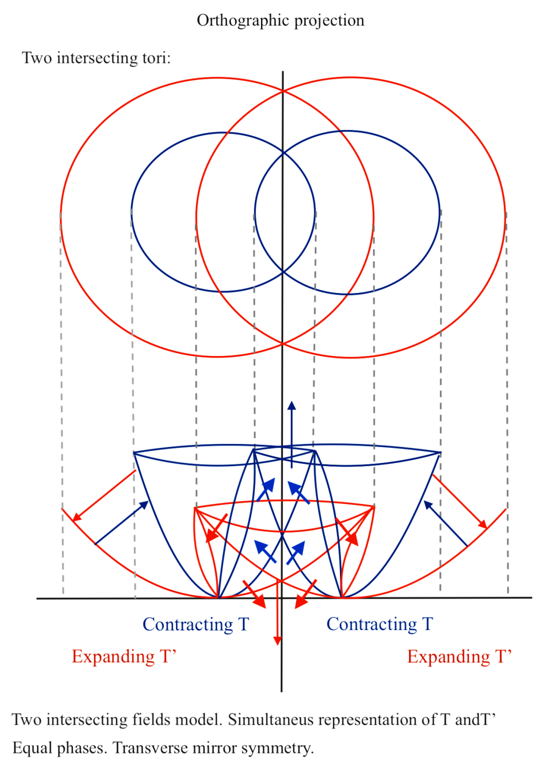

Figure 19: *Two intersecting fields that expand and contract are mapped onto two static intersecting tori using orthographic projection.*

9 Appendix 1. Hodge cycles.

The supersymmetric dual nucleus model, previously described as a Kummer-type system with 16 double points, may also be linked to Hodge theory.

Kummer surfaces are classified as algebraic varieties. In this context, the transversal subfields of A_1 could be interpreted as a projective algebraic variety. This variety is related to another projective algebraic variety represented by the transversal subfields of A_3.

Both varieties are interconnected in a cohomology group through an algebraic cycle.

The same can be said with respect to the cohomology group of the the transversal subfields of A_2 and A_3, and their respective algebraic cycle.

The algebraic cycle within each cohomology group would establish a cyclic mapping between its respective varieties, making each variety the inverse of the other.

By combining the algebraic cycles of both groups, it can be constructed a type of super-cycle that relates in a smooth way all the associated algebraic varieties and the distinct symmetries they carry.

Following the rotational context described earlier, the algebraic cycles would be smoothly combined or interpolated through 90-degrees rotations of the complex plane, giving rise to the cycle $A_1 + A_2 + A_3 + A_4 + A_1 + ...$, where each algebraic cycle is the harmonic partial conjugation of the other.

The cycle formed by combining in this way those algebraic varieties would constitute a Hodge cycle, which links the symmetry of each cohomology group in a dual Hodge structure.

In this sense, the mechanics of the dual atomic model can represent a specific case of the Hodge conjecture, which states that "for particularly nice types of spaces called projective algebraic varieties, the pieces called Hodge cycles are actually rational linear combinations of geometric pieces called algebraic cycles". [13]

However, as it was previously seen, the combination of systems in the rotational fields model, although smooth, cannot be considered linear.

This suggests that the supersymmetric dual nucleus model may point towards a broader interpretation of the Hodge conjecture beyond the classical linearity.

However, as it was previously seen, the combination of systems in the rotational fields model, although smooth, cannot be considered linear.

This suggests that the supersymmetric dual nucleus model may point towards a broader interpretation of the Hodge conjecture beyond the classical linearity.

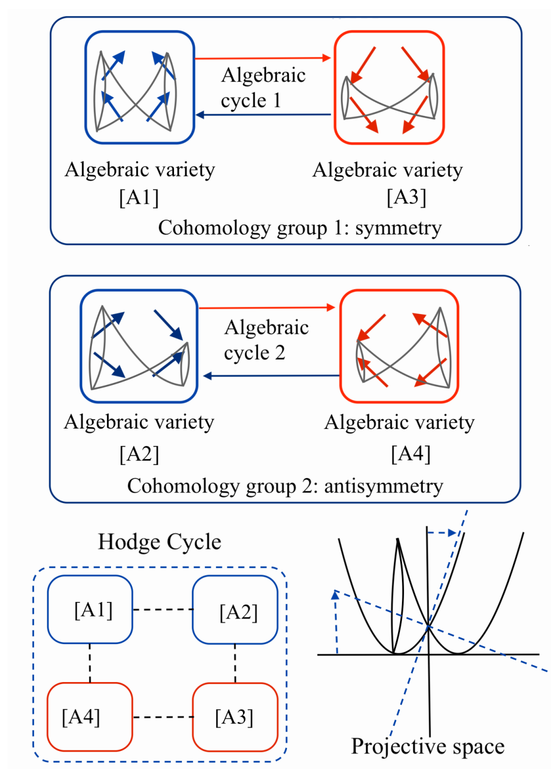

Figure 20: *Algebraic varieties and Hodge cycle. The left- and right-transverse subfields of the dual nucleus can be considered as algebraic varieties of projective spaces. An algebraic variety and its inverse form a cohomology group of symmetry and are linked by a linear cycle of mapping that connects them through time. The nonlinear combination of the algebraic varieties of two groups of cohomology gives rise to a supergroup of cohomology whose cycle would represent a nonlinear quantized Hodge cycle.*

10 Appendix 2. Wirtinger Partial Derivatives. Dolbeault Cohomology.

10.1 Wirtinger Partial Derivatives.

In the framework of the rotational model of intersecting fields, the partial derivatives performed by the rotational operator exhibit properties analogous to those of Wirtinger partial derivatives [cite], introduced by W. Wirtinger in 1927 in the context of functions of several complex variables.

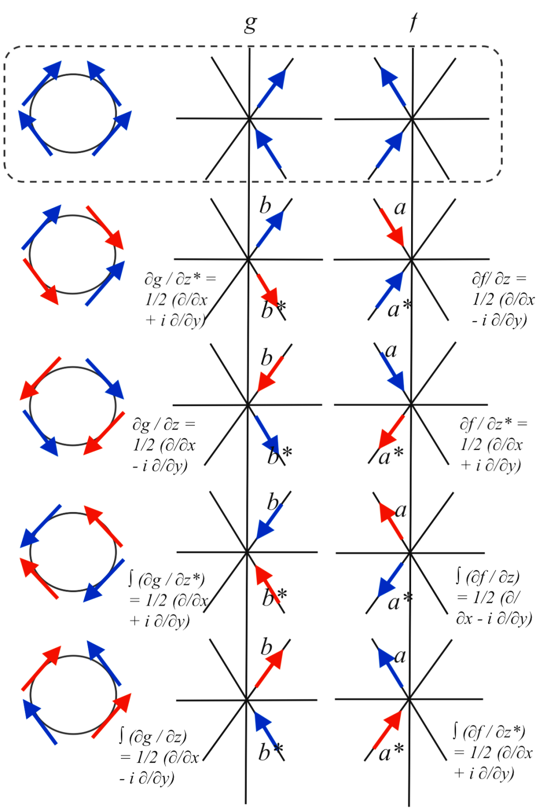

Figure 21: *Wirtinger partial derivatives derivatives in the context of two intersecting functions.*

They are defined in terms of the complex variable $z = x + yi$ and its complex conjugate $\bar{z} = x - yi$ as follows:

$$\frac{\partial}{\partial z} = \frac{1}{2}\left(\frac{\partial}{\partial x} - i\frac{\partial}{\partial y}\right), \quad \frac{\partial}{\partial \bar{z}} = \frac{1}{2}\left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right)$$

The derivative $\frac{\partial}{\partial z}$ acts on the complex variable $z = x + yi$ while treating its conjugate $\bar{z} = x - yi$ as a constant. Conversely, the derivative $\frac{\partial}{\partial \bar{z}}$ acts on the conjugate variable $\bar{z}$ while treating z as a constant.

Geometrically, the complex conjugate $\bar{z}$ represents the reflection of the complex variable z across the real x axis.

Wirtinger differentiation separates the real and imaginary components of the complex variables. When differentiating z, it uses the real part $\frac{d}{dx}$ of the complex variable and the imaginary part $\frac{-id}{dy}$ of its conjugate variable. Conversely, differentiating $\bar{z}$ uses the real part $\frac{d}{dy}$ of the conjugate variable and the imaginary part $\frac{+id}{dx}$ of the complex variable itself.

In the context of the dual model of intersecting fields, two additional complex variables represented by $-X + iY$ and $-X - iY$ are also considered.

The $\frac{1}{2}$ differentiating of Wirtinger derivatives can be represented in the intersecting fields model using two intersecting functions, a right f and a left g function.

When applying the differential operator, only one function f or g is differentiated: the function related to the expanding field in the holomorphic differenciation, or the function related to the contracting field in the anti-holomorphic direction.

This can be represented using two coordinate systems related to the f and g functions, each containing two variables. The variables of this dual system are clockwise identify as follows: $Z1(a) = X + Yi$, $Z2(a^*) = X - Yi$, $Z3(b^*) = X - -Yi$, $Z4(b) = -X + Yi$,

These variables are related to the double curvature of the transversal subspaces which are determined by the expansion and contraction of the intersecting spaces.

The f variables are $Z2$ and $Z4$, and the g variables are $Z1$ and $Z3$.

$Z2(f)$ is the conjugate of the complex $Z1(g)$, being its reflection across the real axis in the right side of the dual system. $Z3(g)$ is the conjugate of $Z4(f)$ at the left side of the system.

When the contracting field related to the left function g expands and the contracting field of the right f function remains contracting, only the variables $Z1$ and $Z3$ of the g function are differentiated, while the variables $Z2$ and $Z4$ of f remain constant.

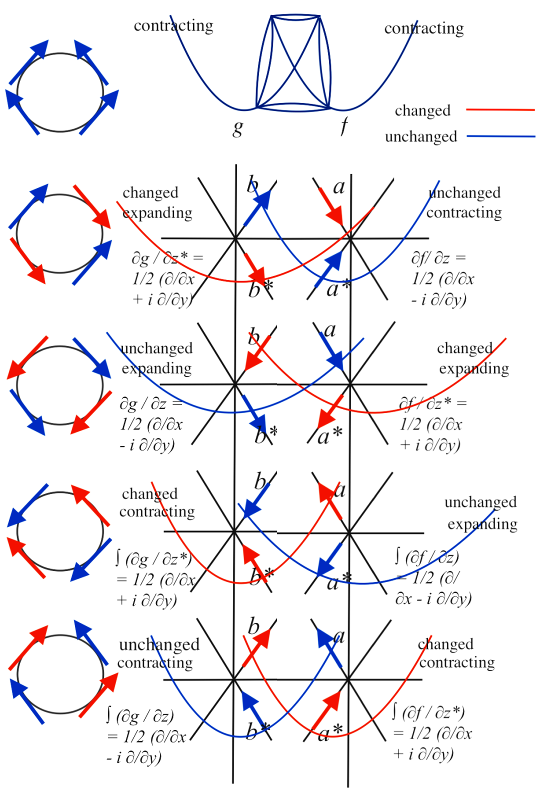

Figure 22: *Dual functions and their Wirtinger type partial derivatives.*

The consequence of this is that, at the right side of the dual system, the complex variable $Z1$ is differentiated while its conjugate variable $Z2$ remains constant. This aligns with the $\frac{1}{2}$ Wirtinger derivative:

$$\frac{\partial g}{\partial z} = \frac{1}{2}\left(\frac{\partial}{\partial x} - i\frac{\partial}{\partial y}\right)$$

Similarly, at the left side of the system, the conjugate variable $Z3$ is differentiated while its complex variable $Z4$ remains constant. This relates to the $\frac{1}{2}$ Wirtinger derivative:

$$\frac{\partial g}{\partial \bar{z}} = \frac{1}{2}\left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right)$$

From the point of view of a single complex function of two variables, the $\frac{1}{2} + \frac{1}{2}$ Wirtinger derivatives operate a complete order of differentiation. But in the context of a composite system with four variables, half of the system still remains undifferentiated.

To operate the complete first order of differentiation of the whole system, it's necessary to apply the differential operator again. A second differentiation given by the rotational operator will imply that the left expanding field of g will remain expanding, and the contracting field of f will expand. In this way, only the variables $Z2$ and $Z4$ of the f function will change while the $Z1$ and $Z3$ variables of g remain constant. Consequently, the $\frac{1}{2}$ derivative will be applied on the $Z2$ conjugate variable of f while its related complex variable $z1$ of g remains constant at the right side of the system. Conversely, at the left side, the other $\frac{1}{2}$ derivative will be applied on the complex $Z4$ (using a clockwise notation) while its conjugate variable $Z3$ remains constant.

This second differential operation inverts the roles of f and g. This inversion stems from the conjugation of the f and g functions that has been completed at this stage. From this point on, subsequent applications of partial derivatives introduce an anti-holomorphic differentiation pattern, reversing the previous changes with the same alternating structure by means of partial integrals.

This is the same reversion mentioned before related to the partial anti-derivative that $A4$ operates on $A3$ and that $A1$ operates on $A4$.

10.2 Dolbeault Cohomology.

In algebraic geometry, Dolbeault cohomology [14] employs Wirtinger partial derivatives in the role of differential operators.

In that sense, $\frac{\partial}{\partial z}$ derivative is related to the Dolbeaut p index, and $\frac{\partial}{\partial \bar{z}}$ is related to the q index.

The (p,q) notation is used to denote the degree of a differential form, where p is the number of holomorphic differentials and q is the number of antiholomorphic differentials.

(p,q) represents the degree of differentition of a differential form in the holomorphic and antiholomorphic directions. Specifically, it represents the instances or number of times that the partial derivatives have been applied. The p index is the number of applications in the holomorphic direction (increasing when applied to $\frac{\partial}{\partial z}$), and q index is the number of applications in the antiholomorphic direction (increasing when applied to $\frac{\partial}{\partial \bar{z}}$.

In that way, $(p+1,q)$ means that a partial derivative has been applied once in the holomorphic direction, while $(p,q+1)$ means that a partial derivative has been applied once in the antiholomorphic direction.

$(p+2,q+1)$ will mean that the operator that applies the partial derivative has been performed twice in the holomorphic direction and once in the antiholomorphic direction.

These indices are used in The Dolbeault theory [14] to form groups of cohomology that are used in the study of Hodge theory.

In Doulbeaut theory the partial derivatives are applied to differential forms. The nuclear manifold of curved subspaces in the intersecting fields model would be equivalent to these differentiable forms. In the fields model the operator is represented by the 90 degrees rotation, that will be applied four times: two in the holomorphic directions as two partial derivatives of the previous stage, and two in the antiholomorphic direction as two partial derivatives in the antiholomorphic direction, which will be partial antiderivatives from the holomorphioc point of view.

In that sense, when a first 90-degrees rotation is operated at A_2, the upper right vector of A_1 related to $X+Yi$ and the lower left vectors related to $-X-Yi$ are differenciated. It impliesd the transposition of matrix A_1 by matrix $A_2$is changed at A_2, while the lower right vector related to $X-Yi$ and the upper left vector related to $-X+Yi$ remain unchanged at A_2.

In the Doulbeault notation, it could be said that A_2 represent a $(p+1,q)$ partial differenciation with respect to A_1 in the holomorphic direction.

An additional 90-degrees rotation operates a second partial derivative, changing at A_3 the two vectors that remained unchanged at A_2. A_3 represents a $(p+2,q)$ partial differenciation in the holomorphic direction, and a complete differenciation with respect to $A1$. A_3 is the complete conjugate solution of A_1.

It can be thought as a first order derivative in the sense that it implies $\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)$ derivatives.

A third 90-degrees rotation operates a first partial derivative at A_4 in the antiholomorphic direction related to the complex variables of A_3, which is the conjugate form of A_1. It will notated as $(p+2,q+1)$, or just as $(p,q+1)$ if considering A_3 as a reverse starting point.

From the point of view of A_1, $(q+1)$ represents a partial antiderivative.

A last 90-degrees rotation operates a second partial derivative at A_4 in the antiholomorphic direction as it operates on the complex variables of A_3. It will be represented as $(p+2,q+2)$ or just as $(p,q+2)$ from the pint of view of A_3.

This way of representing the partial differentiations allows us to relate two forms in the same cohomology group whose instances have the same number: $(p+1,q)$ and its inverse $(p,q+1)$ for the antisymmetric group A_2 A_4; and to relate in a second cohomology group $(p{+}2,q)$ and $(p,q{+}2)$ referred to the symmetric group A_1 A_3.

In Dolbeault theory, the holomorphic $(p+1,q)$ and the antiholomorphic $(p,q+1)$ differential forms are considered to belong to different cohomology groups.

However, in the context of the rotational fields model, the holomorphic and antiholomorphic forms or stages are combined by pairs in the same cohomology group:

The antisymmetric group implies a combination of the holomorphic and antiholomorphic partial differentiations given by A_2 and A_4 respectively, with A_4 being the negative reflection of A_2 around the imaginary y axis.

Similarly, to form a mapping cycle between A_1 and A_3, and vice versa, it will be necessary to combine the holomorphic and antiholomorphic partial differentiations, as the mapping of A_3 to A_1 necessarily passes through the antiholomorphic inversion represented by the partial antiderivatives performed by A_4 on A_3 and by A_1 on A_4.

This combination could be related to a nonlinear case of Hodge conjecture as mentioned before.

11 Appendix 3. Riemann Z function.

The Riemann zeta function [15] is a complex-valued function introduced by Bernhard Riemann. He showed that the zeros of this function are intricately connected to the distribution of prime numbers.

In the context of the Riemann Zeta function, the critical strip refers to a vertical strip parallel to the imaginary axis Y. The width of the critical strip is 1. The critical line is a vertical line that cuts through the middle of the critical strip, also parallel to the imaginary Y axis. In that sense, the real value (the value measured on the real axis X) of the critical line is $\frac{1}{2}$ or 0.5.

The real zeros are placed on the critical line, while complex zeros are in the critical strip.

Non-trivial zeros are those that lie within the critical strip.

Riemann conjectured that only the non-trivial zeros that lie on the critical line, where the real part of the complex number is $\frac{1}{2}$, are related to the prime numbers distribution. This conjecture is known as the "Riemann hypothesis".

From the perspective of the intersecting fields model previously presented, which is based on mirror reflection across the X and Y coordinates, a dual interpretation of the Riemann Z function can be proposed.

Let f and g be two intersecting functions positioned respectively on the right and left sides of a vertical axis denoted as Y. The imaginary axis of f, $Y(f)$, can be represented as $Y+1$, and the imaginary axis of g, $Y(g)$, can be represented as $Y-1$. Consecuwently, Y represents the center of reflective symmetry between both functions.

The vertical strip between $Y-1$ and $Y+1$ represents the "critical strip", where complex zeros are distributed, extending to infinity. The vertical line

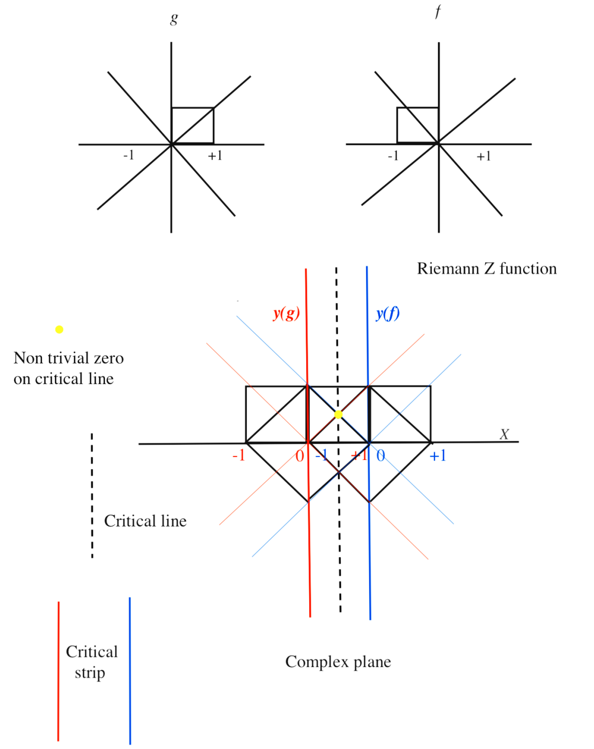

Figure 23: *Dual Riemann Zeta function. The Zero point colored in yellow on the Critical line, in the middle of the critical strip, has a $\frac{1}{2}$ value projected on the real coordinate with respect to the right f function, and it has projected $\frac{1}{2}$ real value with respect to the left g function. The real zero of the right f function corresponds to the right border of the critical strip where the left g function has a $+1$ real value. The real zero of the g function corresponds to the left border of the critical strip where the right f function has a -1 real value. This zero point on the critical line, being relevant to the prime numbers distribution, is the same previously mentioned as the point that determines the gap mass in both the symmetric and antisymmetric systems.*

Y that divides in the middle the critical strip represents a "critical line" of the system extending from an initial point on the real $X = 0.5$ axis to infinity.

In this dual function system, the zeros are represented by the points of intersection of the left and right intersecting functions. The positions of these zeros will change depending on the transformations of the left and right functions.

In the fields context, f and g are associated with changes in the curvature of the left and right intersecting fields. These changes create four subfields at their intersection, each with a double point singularity in its curvature given by the point of intersection of the two fields.

As previously explained, these curvatures will transform through four different possible stages, depending on the phases of variation of the intersecting fields:

- When the intersecting fields vary with the same phase, the intersecting zero point will be located on the vertical critical line of the system, moving upwards or downwards.

- When the intersecting fields vary with opposite phases, the intersecting zero point will be located on the right or left sides of the critical line, always within the critical strip.

Some authors [16] have already researched the connection between the Riemann zeta function and prime distributions with the dynamics of quantum mechanics and string models. This connection would also be present in the dual atomic nucleus model presented in this article.

The non-trivial zeros that lie on the critical line are only those related to the symmetric system, where the left and right transverse subfields simultaneously contract, or when they both simultaneously expand, and the vertical subfield moves upward or downward.

The non-trivial zeros that lie in the critical strip but at the left or right side of the critical line, not over it, are those related to the antisymmetric system, where the right and left transverse subfields vary with opposite phase, and the vertical subfield moves left or right.

This zero point determines the singularity of the double curvature of the four subfields, and the critical line marks the centre of symmetry or asymmetry of the dual system.

The symmetric system is related to the nuclear bosonic particles not governed by the Pauli exclusion principle. The bosonic transverse subfields expanding (or later contracting) at the same time have a strong duality, being isometric mirror subspaces with the same shape and size.

The antisymmetric system is related to the nuclear fermionic particles not governed by the Pauli exclusion principle. The transverse subfields have a delayed duality, being isometric mirror subspaces at different, past or future, times.

In relation to the Riemann hypothesis, only the non-trivial zeros related to the positive and negative symmetric system would be strongly connected to the distribution of prime numbers, although the reason is not clear.

In the dual context of two intersecting or overlapping functions, the reason may be related to the isometry of the mirror transverse subspaces. Being identical elements, dividing the left and right transverse subfields by each other is equivalent to dividing a transverse subfield by itself or by 1.

That operation cannot be done in the antisymmetric system where the isometry appears when the transverse subfields are mapped at different times.

In the rotational framework, as mentioned before, the dual system alternates between symmetric and antisymmetric states every 90 degrees. This pattern of alternation repeats itself in an infinite cycle due to the circular nature of the path.

Keywords: Jacobian, Gorenstein, supersymmetry, mirror symmetry, Tomita-Takesaki, modularity, mass gap, reflection positivity, quantum field theory, dual nucleus, antimatter, t-duality, SYZ conjecture, elliptic fibration, Calabi-yau, Higgs field, Hodge cycles, Kummer surfaces, Wirtinger derivatives, Dolbeault cohomology, Riemann Z function.

References

[1] Wikipedia. Jacobian Conjecture. https://en.wikipedia.org/wiki/Jacobian_conjecture, 2024.

[2] Rosa M. Miró i Roig. Gorenstain Liaison. https://link.springer.com/article/10.1007/BF02938678, 1999.

[3] Wikipedia. Tomita-Takesaki Theory. https://wikipedia.org/wiki/Tomita%E2%80%93Takesaki_theory, 2023.

[4] Arthur M. Jaffe and Zhengwei Liua. Reflection positivity. https://https://www.pnas.org/doi/pdf/10.1073/pnas.1710707114, 2017.

[5] Wikipedia. Wick rotation. https://en.wikipedia.org/wiki/Wick_rotation, 2023.

[6] Wikipedia. Mass gap problem. https://en.wikipedia.org/wiki/YangE28093Mills_existence_and_mass_gap, 2023.

[7] A. de Miguel Bueno. N=1 Supersymmetric Dual Quantum Field Model. https://vixra.org/abs/2311.0037, 2023.

[8] Wikipedia. Cobordism. https://en.wikipedia.org/wiki/Cobordism, 2023.

[9] Wikipedia. SYZ (Strominger, Yau, and Zaslow) conjecture. https://en.wikipedia.org/wiki/SYZ_conjecture, 2023.

[10] Shing-Thu Yau and Steve Nadis. *The Shape of Inner Space*. Basic Books, 2010. ISBN 978-0465020232.

[11] Wikipedia. T-duality. https://en.wikipedia.org/wiki/T-duality, 2023.

[12] Wikipedia. Kummer surfaces. https://en.wikipedia.org/wiki/Kummer_surface, 2022.

[13] Clay Institute. Hodge Conjecture. https://www.claymath.org/millennium/hodge-conjecture/, 2023.

[14] Wikipedia. Dolbeault cohomology. https://en.wikipedia.org/wiki/Dolbeault_cohomology, 2023.

[15] Wikipedia. Riemann Zeta function. https://en.wikipedia.org/wiki/Riemann_zeta_function, 2024.

[16] Michael L. Lapidus. In Search of the Riemann Zeros, 2007.

[17] Wikipedia. Wirtinger derivatives. https://en.wikipedia.org/wiki/Wirtinger_derivatives, 2023.

Contents