Solvable quintic Equation $X^5 - 45X + 108 = 0$

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Abstract

We have previously proposed a quintic equation [2] that is outside the available arguments of the solvable quintic equations. In this article, we give another quintic equation in Bring-Jerrard form and its root.

Some available arguments:

During the second half ofthe 19th century, john Stuart Glashan, George Paxton Young, and Carl Runge gave such a parameterization : *an irreducible quintic with rational coefficients in Bring-Jerrard form is solvable if and only if either $a = 0$ or it may be written [1]*

$$x^5 + \frac{5\mu^4(4\nu+3)}{\nu^2+1}x + \frac{4\mu^5(2\nu+1)(4\nu+3)}{\nu^2+1} = 0$$

And the theorem. [3] *Let a and b be rational numbers such that the quintic trinomial $x^5 + ax + b$ is irreducible. Then the equation $x^5 + ax + b = 0$ is solvable by radicals if and only if there exist rational numbers $\epsilon(=\pm 1), c(\geqslant 0)$ and $e(\neq 0)$ such that*

$$a = \frac{5e^4(3-4\epsilon c)}{c^2+1}, b = \frac{-4e^5(11\epsilon+2c)}{c^2+1}$$

However, the irreducible equation below :

$$X^5 - 45X + 108 = 0$$

is not satisfy the arguments above, but solvable by radicals.
The roots are:

$$X = \frac{(x_0-3)(21x_0-18)}{-x_0^3+9x_0-18}$$

x_0 are the roots of the equation: $x_0^4 + 45x_0^2 - 270x_0 + 360 = 0$

References

[1] Quintic function - Wikipedia

[2] Quang N V, A new solvable quintic equation of the Bring - Jerrard form $x^5 + ax + b = 0$, Vixra: 2108.0060 (AL)

[3] B.K.Spearman, K.S.Williams: Characterization of solvable quintics $x^5 + ax + b$, The American Math Monthly - Volume 101,1994-Issue 10.

[4] Quang N V, A new solvable quintic equation of the shape $x^5 + ax^2 + b = 0$ Vixra:2011.0165 (AL), Semanticscholar.org:229488183

[5] Quang N V, A proof of the four color theorem by induction Vixra: 1601.0247 (CO)cited by IPJO 2020, London, UK and others - Semanticscholar.org:124682326

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