Galois Connections on a Brane

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Abstract

On an absolute frame of reference, a Galois connection to a d-brane may be prescribed such that the data of the frame becomes locally presentable. We describe these connections briefly.

1 Introduction

In Einstein's theory of relativity, all physical objects are assumed to be bounded by a local frame of reference, which is relative to all other open neighborhoods of spacetime. To many minds, this discredits the possibility of an absolute frame of reference. However, in the discrete aether action theory of Agnew [1], we can fix a closed pre-topological category $\mathscr{A}$, which serves as the aether or absolute frame of reference. In such a theory, real spacetime emerges as a restriction $\mathscr{A}|_{\mathbb{K}} \longrightarrow \mathbb{R}^{n,m}$, with n time dimensions and m spacial dimensions.

This bears a remarkable resemblence to the theory of Galois connections. Fix a category $\mathscr{C}$, with a projection $\mathscr{C} \xrightarrow{pr} \mathscr{C}_{Set} \simeq \mathfrak{X} \to M \in Man$, where $\mathfrak{X}$ is a topological stack. Now, fix a stratification of M into discrete units, obtaining the manifold $Strat_M^{\delta}$. Suppose that δ is contractible to a point. Then, there is a map

$$Strat_M^{\delta} \longrightarrow Strat_M^{\{*\}}$$

which is faithful.

For each open neighborhood $\{\mathcal{U}(x)\}_{x\in M}$, there is a diffeomorphism class $Diff_x$ on which the holonomy groupoid $Hol_x^{\mathscr{G}}$ acts, where $\mathscr{G}$ denotes the isotropy group of x. Essentially, for every self-map in the class x^x, there is an inclusion $A(x) \in x^x \hookrightarrow x^{-x}(\mathscr{G}_x)$ such that $A(x) \circ (A(x))^{-1} = Id_{\tilde{x}}$, where $\tilde{x}$ is an element of $\mathfrak{X}$.

Thus, there is an exact sequence:

$$\{\mathcal{U}(x)\}_{x\in M} \xrightarrow{\sim} \tilde{x} \twoheadrightarrow x \twoheadrightarrow \{*\}$$

which converges to a single point. This is, essentially, a restriction from a field of truth values to a single stabilizer, such that $x = stab(d(x, \partial M))$, where $\partial M = 0 \times (\tau(x))$, with τ being a truth value. This means that for a measurement in a local co-ordinate patch corresponding to the expected location of x, the value of the measurement will be zero.

For $\hat{Q}$ a collection of quasi-quanta, the contraction of an open neighborhood about $\hat{Q}$ roughly corresponds to a shrinking homotopy path space. This means that the map

$$\{\mathcal{U}(\hat{Q})\}_{\hat{Q}\in Conf_n(M)} \longrightarrow ... \longrightarrow \{*\}$$

equates to the realization of a quantum q at a time $t = k$, where k is the solution to a harmonic function dependent upon one or more "flow" variables.

In [2], we discussed the relationship between this contraction and the set-theoretic notion of "forcing." We shall succinctly restate our results here. Let S be a set with a maximal element s. Then, construct an injective map $s+k \longrightarrow S$ with $k \in \mathbb{N}$. We then obtain an extended set, S^+, and a so-called "forcing notion," $k \Vdash S^+$. As a concrete example, consider the set of super-real numbers, as constructed by Woodin [3].

Example 1. *Let $\mathbb{R}$ denote the field of real numbers. Denote by $\bar{\mathbb{R}}$ a subset of $\mathbb{R}$ whose maximal element is contained in $\mathbb{R}$. Let $sup(\bar{\mathbb{R}})$ be a supercompact cardinal. Define the forcing notion*

$$\aleph^{\omega^{\omega}} \Vdash \bar{\mathbb{R}} \cup \mathbb{H} = \mathbb{S}$$

so that adjoining $\bar{\mathbb{R}}$ with the ultra-powerset of a cardinal $\aleph$ yields a new field, the "super-real" numbers.

Of interest to us, is that we can actually define the realization of a quantum q as a forcing notion

$$\{*\}_{\tau=1} \Vdash q \in \mathbb{R}^{1,3}$$

Here, we will be substituting the Minkowski manifold $\mathbb{R}^{1,3}$ with a generic d-brane $\mathcal{B}^d$. The realization of q then becomes a Galois connection

$$Gal_{\{*\}} : \mathscr{A} \longrightarrow q \in \mathcal{B}^d_{Set}$$

This gives our forcing notion a bit more concreteness. Firstly, we can construct a d-brane to either be an open or closed cover of a particle's isotropy groupoid, which is the mathematical analogue to its location. This allows us to model either open or closed dynamical systems, with differing boundary conditions. In the case where ∂B^d_{Set} has a truth value $\varepsilon > 0$, we obtain a locally isolated dynamical system $\hat{\mathcal{P}}$. As we have discussed in [4], such a closed system naturally has the property that its quantum mixed states are mutually cobordant with one another. Further, if q is an element of $\mathcal{B}^d_{Set} \sim \hat{\mathcal{P}}$, then q also admits an embedding into a Lagrangian submanifold of the category of necessary moments, $\Box$.

Another advantage of using presentable branes[1] is that we can more easily perform computations involving differential k-forms. Notice, the object $\mathcal{B}^d_{Set}$ may be extended to an arbitrary orbifold by the formula

$$Orb_{\mathcal{B}} = (\mathcal{B}^d_{Set} \otimes T^e) \cup Sing(K)$$

where $Sing(K)$ is an arbitrary singularity, and T^e is an e-dimensional topological object. In the case where $T^e = \mathbb{R}^{1,3}$ is the Minkowski space, we obtain a perfect copy of the Minkowski lightcone, so that $Orb_{\mathcal{B}} = \mathbb{L}^4$; and, whence $T^e = S^{n,m}$, one obtains a chiral superfield $L^\sharp Orb_{\mathcal{B}}$, the free loop superspace of the orbifold.[2]

2 Main Discussion

Let $Caus_\Gamma$ be a partially-ordered causal set, and Γ a graph. Our main objective is to define a Galois connection

$$(\mathscr{A}, Caus_\Gamma) \longrightarrow \lim_{t \to 0} \; t(A(x))$$

where $A(x)$ is an isotropic action on some object x. Throughout, we will let $x \in \mathcal{B}^d_{Set}$, and we will denote $\mathcal{B}^d_{Set}$ simply by $\mathcal{B}^d$.

2.1 Ricci iteration

Let $Curv^d_{\mathcal{B}}$ denote the mean curvature of a d-brane $\mathcal{B}^d$. Let $t \in \mathbb{N}$ for all values of t. Then, one defines the *Ricci iteration* of the brane as a map $Ric_{g_{i+1}} : t \to t+1 \Rightarrow Curv^d_{\mathcal{B}} \longrightarrow Curv^d_{\mathcal{B}}$. Assuming that $\mathcal{B}^d$ is a C^d-space, we obtain a differential k-form ω_{Ric}, with $k \leq d$.

For every open cover of a particle on our brane, we can take the mean curvature of said neighborhood and compare it with a triangle on a flat (Cat(0)) Riemannian manifold. This gives us a relative curvature for an arbitrarily chosen connection on the brane. The Ricci iteration, in effect, measures the difference between potentials over time with respect to a quantum field $\mathscr{F}$.

Here, potential is taken to be the solution for the equation

$$\int_{i=0}^{n} \frac{d\omega_{Ric}}{d(Ric_g)} = \hat{p}$$

The set of all potentials for a particle q gives us the holonomy vector for that particle. Recall that a holonomy groupoid consists of an isotropy groupoid $\mathscr{G}$ and an object x which is acted upon by the groupoid. The holonomy vector, then, is the Ricci iteration

$$Ric_{g_{i+1}} Hol^{\mathscr{G}}_{g_i}$$

[1] Recall that a category C is called presentable if it admits a proper morphism $C \longrightarrow C_{Set}$

[2] See [4] fore more details about this construction.

2.2 Curvature and causality

Fix a Galois connection $\mathbb{T} \longrightarrow Curv^d_{\mathcal{B}}$, where $\mathbb{T} = (\mathscr{A}, Caus_\Gamma)$. The moment map of this connection is given by the map $\mathbb{L}^4 \longrightarrow Ric_{g_{i+1}}\mathbb{F}$, where $\mathbb{F}$ is a frame field. For our purposes, a frame field is simply a collection of covers of a $\tau = 1$ particle along with a pre-defined notion of holonomy.

Let $\mathbb{F}^{<1>}$ denote the path groupoid over a frame field, and let $f \in \mathbb{F}$ be an arbitrary frame. The path groupoid is then the groupoid consisting of all invertible actions $f \to f'$ between frames in $\mathbb{F}$. In the case where such actions are diffeomorphisms, we obtain the identity $\mathbb{F}^{<1>} = Diff_f$.

Definition 1. *A causal path, $p : f \longrightarrow Ric_{g_{i+1}}(f)$, is a conic section of $\mathbb{F}$ which is identical to an atlas consisting of charts containing both light-like and space-like distinct points.*

Not all of the diffeomorphisms of f are causal. For instance, a diffeomorphism $\varphi : f \longrightarrow f'$ which does not include any timewise distinct points, is degenerate, and therefore acausal. For a particle q, write $T_q(\mathcal{B}^d)$ for the tangent bundle of q on a brane. We will impose the following axiom:

Axiom 1 (Connectedness)**.** *If q is a particle with truth value 1, then the map $Ric_{g_{i+1}} : q \longrightarrow q'$ is contained with in $T_q(\mathcal{B}^d)$, and further, this space is simply connected, assuming q is not bordant with q'.*

Notice the requirement that q and q' not be bordant. This is because of the anomalous case of teleportation of quantum states, in which the collars of a bordism need not necessarily be simply connected with respect to the throat of a holographic wormhole. Under our definition, teleportation is then an acausal event.

2.2.1 Spin Manifolds

Let $Sp_{\mathcal{B}}$ be a spin manifold. Then, the connection $\mathbb{T} \longrightarrow Sp_{\mathcal{B}}$ arises in the following fashion.

Let $\mathfrak{X} \in \mathbb{T}$ be a topological stack, and let $pr : \mathfrak{X} \longrightarrow \mathcal{B}^d$ be a totally lossless projection. Denote the Dehn twist of $\mathfrak{X}$ by $Dehn_{\mathfrak{X}} = \mathfrak{X} \times_{\mathfrak{X} \times \mathfrak{X}} \mathfrak{X} \longrightarrow Man$. Let $Dehn_{\mathbb{T}} = Sp_{\mathcal{B}}$. Then, for a principle bundle E over $\mathbb{T}$, we have $im(E) \sim F$ for some bundle F over Sp_B.

Let F be an orbifold. Then, there is a homeomorphism $Sp_{\mathcal{B}} \cong \mathbb{L}^4$. For a boson, this is all we need. In the fermionic case, however, an additional subtlety arises. We must decompose our manifold into two manifolds:

$$Sp_{\mathcal{B}} = Sp^-_{\mathcal{B}} \oplus Sp^+_{\mathcal{B}}$$

such that, for every $a \in Sp^-_{\mathcal{B}}$ and for every $b \in Sp^+_{\mathcal{B}}$, the map $a \longrightarrow b$ is the annihilation map. Further, the gauge field of (a, b) is given by a *wrapped Fukaya category*, Fuk_{Wr}, which is an anti-chiral modification of the B-field. The realization of a free fermion vibrating on a string is then given by the splitting of the epimorphism $Ric_{g_{i+1}} = q \in Sp^{\pm}_{\mathcal{B}} \overset{+}{\underset{-}{\rightrightarrows}} (q, \emptyset)$, assuming the vacuum expectation value of q is cancellable. We have

$$q \twoheadrightarrow \emptyset \simeq exp(q) \to 0 \simeq E(q) \to E(\mathcal{U}(q))$$

where $E(q)$ is the energy of q. The right-hand-side of the above equation describes the dissipation of a quantum of energy into the ambient space in which the quantum is annihilated. That is to say, the charts containing q are compactified into a non-degenerate atlas lying in $\mathbb{T}$.

3 References

[1] S. Agnew, *Universal Quantum Action with Discrete Aether*, (2021)
[2] R.J. Buchanan, *Realization of quasi-quanta via the forced contraction of loops*, (2023)
[3] H. Woodin, H.G. Dales, *Super-real fields*, (1996)
[4] R.J. Buchanan, P. Emmerson, *Bordisms and Wordlines II*, (2023)