A Quantum Generalized Evidence Combination Rule Algorithm

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Abstract

In this paper, a quantum generalized combination rule algorithm is proposed to reduce the computational complexity of generalized evidence theory combination rule.

Keywords: Generalized evidence theory, Generalized combination rule, Quantum algorithm, Quantum generalized combination rule

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1. QGCR: Quantum Generalized Combination Rule

QGCR algorithm consists of the following four steps.

Step 1: Transform GBBAs into $\mathcal{M}^a$ and $\mathcal{M}^b$.

Apply a transform function expressed as follow.

$$\mathcal{M}_i = \sqrt{m(\mathcal{F}_i)}. \tag{1}$$

By this transformation, $\mathcal{M}$ now satisfies $\sum_{i=0}^{2^n-1} |\mathcal{M}_i|^2 = 1$.

Step 2: Encode $\mathcal{M}^a$ and $\mathcal{M}^b$ into quantum states.

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Let $G(|\mathcal{F}_t\rangle, i)$ be the *i-th* bit of the quantum basis state $|\mathcal{F}_t\rangle$ and symbol $\mathcal{F}_t$ be the corresponding proposition. Then the correspondence can be characterized as follows:

$$\begin{cases} G(|\mathcal{F}_t\rangle, i) = |1\rangle \Leftrightarrow \theta_i \in \mathcal{F}_t \ , \\ G(|\mathcal{F}_t\rangle, i) = |0\rangle \Leftrightarrow \theta_i \notin \mathcal{F}_t \ . \end{cases} \tag{2}$$

The quantum superposition state could be expressed as follow:

$$|\Psi\rangle = \sum_{i=0}^{2^n-1} \mathcal{M}_i |\mathcal{F}_i\rangle , \tag{3}$$

Step 3: Apply the combination quantum circuit.

Use a specific quantum circuit QC to draw the combination results. The quantum superposition state in target register is expressed as follow:

$$\begin{aligned} |\psi_a\rangle \oplus |\psi_b\rangle = |1\rangle & \sum_{|\mathcal{F}_u \cap \mathcal{F}_v\rangle = |\mathcal{F}_t\rangle, u+v=0} \mathcal{M}_u^a \mathcal{M}_v^b |\mathcal{F}_t\rangle + \\ |0\rangle & \sum_{|\mathcal{F}_u \cap \mathcal{F}_v\rangle = |\mathcal{F}_t\rangle, u+v\neq 0} \mathcal{M}_u^a \mathcal{M}_v^b |\mathcal{F}_t\rangle . \end{aligned} \tag{4}$$

Step 4: Measure target register to estimate combination result.

The combination results could be estimated by following rules:

$$
\begin{aligned}
K &= \sum_{|\mathcal{F}_u \cap \mathcal{F}_v\rangle = |\mathcal{F}_0\rangle} m_a(\mathcal{F}_u) m_b(\mathcal{F}_v) \qquad (5) \\
&= \hat{P}(|1\rangle\,|\mathcal{F}_0\rangle) + \hat{P}(|0\rangle\,|\mathcal{F}_0\rangle),
\end{aligned}
$$

$$
m(\emptyset) = \begin{cases} m_a(\mathcal{F}_0) m_b(\mathcal{F}_0) = \hat{P}(|1\rangle\,|\mathcal{F}_0\rangle), & K \neq 1, \\ 1, & K = 1, \end{cases} \qquad (6)
$$

$$
\begin{aligned}
m(\mathcal{F}_t) &= \frac{(1 - m(\emptyset)) \sum_{|\mathcal{F}_u \cap \mathcal{F}_v\rangle = |\mathcal{F}_t\rangle} m_a(\mathcal{F}_u) m_b(\mathcal{F}_v)}{1 - K} \\
&= \frac{(1 - m(\emptyset)) \hat{P}(|0\rangle\,|\mathcal{F}_t\rangle)}{1 - K}, \quad t \neq 0 \ , \qquad (7)
\end{aligned}
$$

where $\hat{P}(|\mathcal{F}\rangle)$ is the probability of observing a quantum basis state in measurement.

2. Conclusion

The proposed QGCR can exponentially reduce the computational complexity of generalized combination rule.

References

[1] Y. Deng, "Generalized evidence theory," Applied Intelligence, vol. 43, no. 3, pp. 530–543, 2015.