Geometric Probability Model for $\alpha_{\rm em}$ and α_s

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Abstract

A simple geometric probability model for the EM and Strong force coupling constants.

Basic Building Blocks

In this model, the integral

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos\theta_1}{2\pi}\, d\theta = \frac{1}{\pi}, \tag{1}$$

represents an average probability of interaction for two dimensions, based on the cosine of a random distribution for angle θ over a full circle, but with potential interactions limited to $\pm\frac{\pi}{2}$ radians. The angle could vary over time or represent many simultaneous sub-components of a particle. This can be extended to an arbitrary higher dimension d with

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^n\theta}{2\pi}\, d\theta = \frac{n-1}{n}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^{n-2}\theta}{2\pi}\, d\theta\ \ (n>0)\,, \tag{2}$$

where $n = d-1$. In this model the higher dimension integrals include the 1-sphere factor of $1/2\pi$ instead of an n-sphere factor as θ is always between two arbitrary dimensions (not a solid or higher degree angle). The $n-$degree cosine is required as the distribution across "unused" dimensions does affect probabilities on the two arbitrary dimensions.

Initial Attempt

An initial attempt at constructing this model used a simple combination of four 1-spheres which is within 0.523% of $\alpha_{\rm em}$ when divided by $\sqrt{2}$:

$$\alpha_{\rm em} \simeq \frac{1}{\sqrt{2}}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos\theta_1}{2\pi}\, d\theta_1 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos\theta_2}{2\pi}\, d\theta_2 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos\theta_3}{2\pi}\, d\theta_3 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos\theta_4}{2\pi}\, d\theta_4 = \frac{1}{\sqrt{2}\pi^4} \simeq 7.259146\times 10^{-3}. \tag{3}$$

I had actually found this $1/\left(\sqrt{2}\alpha^4\right)$ result in an empirical search years ago (just the result, not derived from anything) but disregarded it as numerological/coincidence.

Interesting Result

A search of hybrid-dimensional combinations found this equation which matches $\alpha_{\rm em}$ to within 0.0398%. It's a better match, but more interesting is the geometry which is analogous to the symmetries of the Standard Model. I've moved the empirical factor of 3 to the second term where it seems to make the most sense:

$$\alpha_{\rm em} \simeq \underbrace{\underbrace{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^5\theta_1}{2\pi}\, d\theta_1}_{S^5}\ \ 3\underbrace{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^3\theta_2}{2\pi}\, d\theta_2}_{S^3}}_{\rm SU(3)} \underbrace{\underbrace{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^3\theta_3}{2\pi}\, d\theta_3}_{S^3}}_{\rm SU(2)} \underbrace{\underbrace{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos\theta_4}{2\pi}\, d\theta_4}_{S^1}}_{\rm U(1)} = \frac{32}{45\pi^4} \simeq 7.300254\times 10^{-3}. \tag{4}$$

Of additional interest, the SU(3) terms are close to $\alpha_s(M_Z)$ and possibly even closer at $M_{\rm H}$ or $v/\sqrt{2}$ energy scale:

$$\alpha_{\rm s}(M_Z^2) \simeq \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^5\theta_1}{2\pi}\, d\theta_1\ \ 3\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^3\theta_2}{2\pi}\, d\theta_2 = \frac{16}{15\pi^2} \simeq 0.108.$$

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