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A New Solution to the Strong CP Problem

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Abstract

We suggest a new solution to the strong CP problem. The solution is based on the proper use of the boumdary conditions for the QCD generatimg functional integral. It obeys the principle of renormalizability of Quantum Field Theory and does not involve new particles like axions.

1 Introduction

The strong CP problem for a long time is considered as an outstanding problem of Quantum Field Theory and Elementary Particle Physics. For an excellent review of the subject see [1]. The most popular presently solution [2], [3] to the problem introduces new particles - axions [4],[5]. Axions are considered as real candidates for dark matter of the Univers. But presently only restrictions on their possible properties are established in spite of the numerous experimental efforts to discover such particles, see e.g. [6],[7],[8]. Besides, the axion solution of the strong CP problem violates the principle of renormalizability of Quantum Field Theory. This basic principle presently is one of the most phenomenologically successful principles of Elementary Particles Theory. It ensured in Quantum Electrodynamics the agreement between the theory and the experiment for anomalous magnetic moment of the elctron within ten decimal points. This impressive agreement convinces us that renormalizable Quantum Field Theory is a correct physics heory. Therefore it seems to be interesting to find a solution to the strong CP problem which also obeys the principle of renormalizability. This is the goal of the present paper.

2 Main part

We will deal with the Quantum Chromodynamics (QCD) generating functional of Green functions

$$Z(J) = \int d\Phi \;\; exp\left(i \int d^4x \, (L_{QCD} + J_k \cdot \Phi_k)\right), \tag{1}$$

where $d\Phi$ denotes the integration measure of the functional integral $Z(J)$ over all fields Φ_k of the theory, gluons and quarks. J_k are sources of the fields. The symbol J in $Z(J)$ denotes the full set of sources J_k.

Within perturbation theory the QCD Lagrangian L_{QCD} is invariant, in particular, under the combined symmetry transformations CP, where C is the charge conjugation operator and P is the space reflection.

The essence of the CP problem is that in full nonperturbative QCD one can add to the QCD Lagrangian the CP odd gauge invariant term which seems to be not forbidden from the first principles

$$\Delta L_\theta = \frac{\theta}{32\pi^2} G^a_{\mu\nu} \tilde{G}^a_{\mu\nu}. \tag{2}$$

It is C-invariant and P-noninvariant, hence CP odd. But this term is forbidden by experiments with the rather high precision.

The dual field strength tensor $\tilde{G}^a_{\mu\nu}$ in (2) is defined in the standard way

$$\tilde{G}^a_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} G^a_{\rho\sigma}. \tag{3}$$

The θ-term in (2) is invisible in perturbation theory since it can be rewritten as a total derivative

$$\Delta L_\theta = \theta \partial_\mu K_\mu. \tag{4}$$

Here K_μ is the known Chern-Simons current

$$K_\mu = \frac{1}{16\pi^2} \epsilon_{\mu\nu\rho\sigma} \left(A^a_\nu \partial_\rho A^a_\sigma + \frac{1}{3} f^{abc} A^a_\nu A^b_\rho A^c_\sigma \right). \tag{5}$$

The θ-term can be discarded within perturbation theory since the fields decrease in Euclidean space at infinity and the total derivative (4) does not contribute to the QCD action. But with the discovery of instantons [9] it was realized that the field configurations with the instanton boundary conditions give nonzero nonperturbative contributions to the action. In particular, the one instanton contribution gives

$$\Delta S_\theta = \int d^4x \Delta L_\theta = \theta. \tag{6}$$

The key notion here is the topological charge

$$\mathcal{V} = \int d^4x \partial_\mu K_\mu = \int d^3x K_0(\vec{x}, t)|^{t=+\infty}_{t=-\infty} = \mathcal{K}(t \to +\infty) - \mathcal{K}(t \to -\infty), \tag{7}$$

where $\mathcal{K}$ is the Pontryagin number. The topological charge is zero for perturbative fields, i.e. in perturbation theory. But instanton fields, e.g. in the $A_0 = 0$ gauge, interpolate between $A_i(\vec{x}, t = -\infty) = 0, \quad i = 1, 2, 3$ and nonzero $A_i(\vec{x}, t = +\infty) = U^+ \partial_i U, \quad i = 1, 2, 3$. Here the matrix U is the Polyakov hedgehog

$$U(\vec{x}) = exp \left(- \frac{i\pi \vec{x} \cdot \vec{\sigma}}{\sqrt{\vec{x}^2 + \rho^2}} \right). \tag{8}$$

For this instanton configuration one has that the Pontryagin number and correspondingly the topological charge are equal to unity

$$\mathcal{V} = \mathcal{K}(t = +\infty) = 1, \tag{9}$$

Thus the θ-term gives the nonzero contribution to the QCD action.

In the full QCD, with quarks, there is also contributions to the CP odd part of the QCD Lagrangian from the imaginary phases of the quark mass matrix. The phases can be rotated away by the chiral transformations of quark fields. But there is the axial anomaly [11],[12]. It generates noninvariance of the measure of the Feynman functional integral under chiral transformations [13]. Therefore the phases of the quark mass matrix arise before the $G\tilde{G}$ term in the Lagrangian. Hence the parameter which determines the value of the CP violation is in fact

$$\theta + arg(det\mathcal{M}), \tag{10}$$

where $\mathcal{M}$ is the quark mass matrix.

Below we shall use the same symbol θ for this parameter to simplify notations.

Probably the most essential effect of the θ-term is a nonzero electric dipole moment of the neutron d_n. The latter is given by the effective Lagrangian

$$L_{nEDM} = \frac{d_n}{2} \bar{n} i \gamma_5 \sigma_{\mu\nu} n F^{\mu\nu}, \tag{11}$$

where $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ is the photon field strength tensor, n is the neutron field , $\sigma_{\mu\nu} = \frac{1}{21}[\gamma_{mu}\gamma_{nu}]$ is the antisymmetric product of Dirac gamma matrices.

The θ-term generates the following d_n

$$< n(p_f)\gamma(k)|eJ_\mu^{em} A^\mu i \int d^4x \Delta L_\theta |n(p_i) >= d_n \bar{n}(p_f)\gamma_5 \sigma_{\mu\nu} n(p_i) k^\mu \epsilon^\mu(k). \tag{12}$$

Here J_μ is the quark electromagnetic current. $k^\mu = p_f^\mu - p_i^\mu$, $\epsilon^\mu(k)$ is the photon polarization.

The matrix element in the left hand side of eq.(12) is calculated within nonperturbative QCD. There are several methods by which d_n was estimated, see e.g. [1]. The results have considerable uncertainties of the order of 50% because of difficulties of nonperturbative QCD culculations. But anyway the average theoretical value for d_n can be estimated as

$$d_{n,theor} \approx \theta \times 10^{-16}\ e \cdot cm. \tag{13}$$

This should be compared with the recent experimental value [14]

$$d_n = (0.0 \pm 1.1) \times 10^{-26}\ e \cdot cm. \tag{14}$$

Thus one gets an extremely strong restriction on the value of θ

$$|\theta| \leq 10^{-10}. \tag{15}$$

The explanation of this practically zero value of the coupling θ is the essence of a solution to the strong CP problem.

The presently popular solution to the problem is the axion solution. It assumes the addition to the QCD Lagrangian the term with the new axion field $a(x)$, which reduces to the shift $\theta \rightarrow a(x)/f_a + \theta$. So the corresponding term ΔL_θ of eq.(2) in the QCD Lagrangian becomes

$$\Delta L_\theta \rightarrow \left(\frac{a(x)}{f_a} + \theta \right) \frac{1}{32\pi^2} G^b_{\mu\nu} \tilde{G}^b_{\mu\nu}. \tag{16}$$

After the spontaneous symmetry breaking of the global symmetry [2],[3] one calculates the effective potential for the axion field $a(x)$ and finds that when the axion rests at the minimum of this porential, the CP violating term (16) nullifies. This is the axion solution to the strong CP problem.

But first of all, the term with the axion field $a(x)$ in (16) has the dynamical dimension five instead of necessary for renormalizability four. Hence it violates renormalizability of the Lagrangian. As we already have mentioned in the introduction, renormalizability is a rather important principle of Quantum Field Theory which turned out to be very successful phenomenologically. Therefore it is quite important to preserve it when solving the strong CP problem. Secondly, the axion is not found experimentally in spite of numerous experimental attemps.

Therefore we suggest a new solution to the strong CP problem which preserves renormalizability of the theory and does not involve new exotic particles like axions.

Let us again consider the QCD generating functional (1). In perturbation theory one has the following boundary conditions e.g. for gluon fields

$$A^a_\mu(\vec{x}, t \rightarrow \infty) \rightarrow A^a_{\mu,in}(x), \tag{17}$$

$$A_\mu^a(\vec{x}, t \to \infty) \to A_\mu^{a,out}(x).$$

Here incoming asymptotic fields $A_{\mu,in}^a(x)$ contains only the positive frequency part and outgoing fields $A_{\mu,}^{a,out}(x)$ contain only the negative frequency part:

$$A_{\mu,in}^a(x) = \frac{1}{(2\pi)^{3/2}} \int d^3k \; e^{i(\vec{k}\vec{x}-\omega t)} v_\mu^i(k) a_i(k)/\sqrt{2w}, \tag{18}$$

$$A_{\mu,}^{a,out}(x) = \frac{1}{(2\pi)^{3/2}} \int d^3k \; e^{i(\vec{k}\vec{x}+\omega t)} v_\mu^i(k) a_i^*(k)/\sqrt{2w},$$

where $\omega = \sqrt{\vec{k}^2}$, $v_\mu^i(k)$ are polarization vectors and the sums over polarizations $i = 1, 2$ are assumed.

These are the Feynman boundary conditions. They are necessary to obtain the correct form of the perturbative propagator of the type $1/(k^2 + i\epsilon)$ with the correct $i\epsilon$-prescription. Thus the fields oscillate at time infinities. After transition to the Euclidean space by means of the Wick rotation $t \to ix_4$ the fields decrease at time infinities, so it is easy to see that in perturbation theory total derivatives in the Lagrangian are zero.

One can write perturbative boundary conditions (17) for all fields Φ_i of QCD symbolically as follows

$$\Phi(t \to \pm\infty) \to \Phi_{in}^{out}(x). \tag{19}$$

In our opinion, it is natural to generalize boundary conditions (19) from perturbative fields to all fields of the theory over which the integration in the generating functional integral (1) proceeds. Such a definition nullifies total derivatives in the Lagrangian. Thus it solves the strong CP problem. Besides this definition allows to write the complete generating functional integral as one formula:

$$Z(J) = \int_{\Phi(t\to\pm\infty)\to\Phi_{in}^{out}} d\Phi \; exp\left(i \int d^4x \, (L_{QCD} + J_k \cdot \Phi_k)\right), \tag{20}$$

Let us make now necessary remarks concerning the famous $U(1)$ problem. The essence of this problem is that the mass of the flavour singlet pseudoscalar η' neson $m_{\eta'} \approx 958 MeV$ is surprisingly heavier than the masses of the flavour octet pseudoscalar mesons.

There is the statement [15],[16] that instantons solve the $U(1)$ problem. But there is also the solution [17],[18] of the $U(1)$ problem using the axial anomaly which was suggested before the discovey of instantons.

3 Conclusions

We suggest a new solution to the strong CP problem. The solution is based on the proper use of the boumdary conditions for the generatimg functional integral. It obeys the principle of renormalizability and does not involve new exotic particles like axions.

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