

Spin Angular Momentum Explained by the Classical Quantum Model

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This paper proposes a new picture of spin angular momentum. In the conventional picture of spin, the precession of the axis is based on the assumption that the electron has an acceleration. In this study we first consider the case where the acceleration is expressed as a simple harmonic oscillator and the precession as a sinusoidal function. In this case, a double angle appears in the outer product of the Thomas precession, confirming that an angular velocity of one revolution of space can be obtained with half the circumference of the circle. It also shows the results of operations in which a single electron can take both up and down spin depending on the time transition. Next we consider the case of Lorentz contraction of the circumference in the direction of the axis of rotation. Einstein pointed out that in a rotating coordinate system the ratio of circumference to diameter is not pi. This study propose that the Lorentz contraction is the cause of the anomalous magnetic moment. The anomalous magnetic moment is regarded as a Lorentz contraction of the rotational angular momentum. As a result, the average oscillation velocity of the electron within Compton wavelengths is calculated to be about four percent of the speed of light. Furthermore, the Schwarzschild radius on general relativity included in the scope of the consideration to predict the size of the electron.

I. INTRODUCTION

The depiction of spin as the precession of a piece was largely influenced by arguments derived from Thomas's brilliant work [1]. In this study, a new spin image will be proposed from a different perspective from conventional spin. In 1925, Uhlenbeck and Goudsmit wrote a paper [2] on rotating electronic images. One reason for the dismissal of the classical electron theory was noted by Lorenz. He pointed out that very fast rotation was required to have a rotation angular momentum and that the speed of the electron surface was ten times the speed of light.

Till date, the detailed reasons for the emergence of spin have not been clarified. In physics textbooks, spin is often described by a picture of the precessional motion of a rotating piece when describing spin. In this study, we discuss the classical aspects of the spin picture, going back to the time before spin was imaged by rotational motion.

In 1945 Nobel Lecture, Pauli mentioned,

"... The gap was filled by Uhlenbeck and Goudsmit's idea of electron spin, which made it possible to understand the anomalous Zeeman effect simply by assuming that the spin quantum number of one electron is equal to 1/2 and that the quotient of the magnetic moment to the mechanical angular moment has for the spin a value twice as large as for the ordinary orbit of the electron. Since that time, the exclusion principle has been closely connected with the idea of spin. Although at first I strongly doubted the correctness of this idea because of its classical-mechanical character,

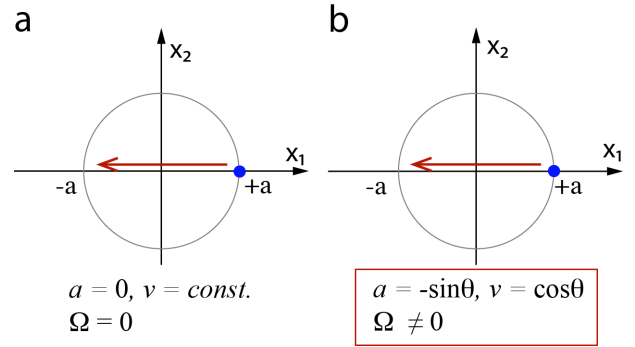


Fig. 1. Point mass observed in the laboratory coordinate system. The blue dots move from $+a$ to $-a$ with different accelerations. (a) As the point mass passes through the origin of the coordinate axes in uniform linear motion ($a = 0$), the angular momentum, Ω , is zero. (b) According to Thomas's study, the angular momentum does not have a zero value when the point mass passes through the origin of the coordinate axes in accelerated motion.

I was finally converted to it by Thomas' calculations on the magnitude of doublet splitting. [3]"

Pauli did not reject quantum mechanics based on the classical manner. We shall go back in time to 1925 and re-produce spin images based on classical quantum theory. The behaviour of an electron travelling between two kernels can be described by a simple sinusoidal function, as shown by the results in Eq. (VI.5) in the Appendix. That is, the central kinetic energy of the virtual photon with simple harmonic oscillation for an electron can be described by a simple sinusoidal function.

Herein, the image of a spinning top with precession in uniform circular motion has been discarded. We abandon the diagram (a) shown on Fig. 1 and seek a new spin

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image within the diagram (b). Instead, the harmonic oscillator has been placed on the coordinate axis and its angular acceleration has been considered. The electron is not assumed to be in uniformly accelerated motion, but to have an acceleration represented by a sinusoidal function, giving a completely new spin picture that has never been seen before.

II. THE ACCELERATION OF THE ELECTRON COULD NOT BE CONSTANT

A. Review the Thomas precession

To make this idea quantitative, this study does not make the assumption of constant acceleration (a : *acceleration* = f : *fource*) in Thomas theory. The acceleration of the electrons can be changing. The electron does not travel in an uniform linear motion but with an intrinsic velocity, which could be expressed by a sinusoidal function. If the velocity \mathbf{v} is expressed by $\mathbf{v} = \cos\theta$, the acceleration is expressed by its derivative, $\mathbf{a} = -\sin\theta$. In this study, at the beginning, we reviewed Thomas's work and substituted $\mathbf{a} = -\sin\theta$ instead of constant value ($\mathbf{a} = \mathbf{f}$) into the Thomas precession.

The discussion begins with the background of the association of spin with precessional motion. In relativity, if the electron is in uniform linear motion, the coordinate system describing the electron's motion can be calculated by Lorentz transformation. However, if the electron is in an accelerated motion, it is calculated that the axis of the coordinate system describing this electron rotates when observed from the laboratory system. Thomas wrote in his paper that the axes of a coordinate system with an origin and translating with the electrons are observed in a laboratory system to rotate with the following angular velocity as in Eq. (II.1),

$$\boldsymbol{\Omega} = \frac{1}{2c^2}[\mathbf{a} \times \mathbf{v}], \quad (\text{II.1})$$

where \mathbf{a} is the acceleration of the electron and \mathbf{v} is the velocity of the electron. Note that in Eq. (II.1), the approximation ($\beta = 1 - v^2/c^2 \doteq 1$) is set in Lorenz transformation. Equation (II.1) can also be applied to the general case where the particles are not in uniform circular motion. As the particles are in uniform circular motion, the following equation is obtained,

$$\Omega = -\frac{1}{2} \frac{v^2}{c^2} \omega_{\text{const}}. \quad (\text{II.2})$$

The spin image in precession that we now recall comes from Eq. (II.2). The angular velocity Ω obtained is a constant proportional to ω_{const} . In this study, however, we will not consider the issue using Eq. (II.2), but rather equation (II.1).

B. Assuming a simple harmonic oscillation instead of uniform circular motion

This section is the innovative part of this study. The quantisation of the orbital angular momentum into units of \hbar reflects the nature of space, which returns to its original state after one rotation. According to the relationship between angular momentum and magnetic moment, if the angular momentum is halved to $\hbar/2$, the magnetic moment should also be $\mu_e/2$. However, the magnetic moment of the spin angular momentum is equal to μ_e , even though the angular momentum is $\hbar/2$. This means that spin rotation can generate magnetic fields twice as efficiently as orbital rotation and responds to magnetic fields with twice the sensitivity. This property could not be explained by theories based on circular currents observed in three-dimensional space.

Consider this discrepancy from the perspective of the Thomas precession. Equation (VI.5) forms an important basis for this paper. The traveling of the virtual photon, γ^* , is represented by a sinusoidal function (cf. Eq. (VI.5) and see yellow line on Fig. 3). The study was described as the 0-Sphere electron model. In this electron model, the thermal potential energy (TPE) of the electron is a set of radiation and absorption, which describes the motion of the electron; the TPE changes partly kinetic energy, which drives the photon. The motion of the photon could be represented by a very simple sinusoidal function in this research model. First, we let the two values as follows;

$$\begin{aligned} (\text{Verocity}) : v_{\gamma^*} &= \cos\omega t, \\ (\text{Accereration}) : a_{\gamma^*} &= -\sin\omega t. \end{aligned} \quad (\text{II.3})$$

Substitute Eq. (II.3) into Eq. (II.1) then,

$$\begin{aligned} \boldsymbol{\Omega} &= \frac{1}{2c^2}[\mathbf{a}_{\gamma^*} \times \mathbf{v}_{\gamma^*}] \\ &= \frac{1}{2c^2}[-\sin\omega t \times \cos\omega t] \\ &= \frac{1}{2c^2} \cdot \left(-\frac{1}{2} \sin 2\omega t \right). \end{aligned} \quad (\text{II.4})$$

The above discussion yields an extremely important result. Namely, when the outer product of cosine and sine is calculated, $-\sin 2\omega t$ appears. Equation (II.4) is the basis for obtaining a doubled angular velocity cycle. It was found that the displacement, velocity and period of a single oscillation have a cycle of ωt , whereas the angular velocity has a cycle of $2\omega t$. One wave period of single oscillation is determined by the angular velocity. The angular velocity with Thomas precession has a period of half the displacement.

The results of the study of the above equation provide a basis for the quantisation of the spin angular momentum to a value half the Planck constant.

Equation (II.4) provides us with a further important conclusion. The angular velocity of an electron can take both positive and negative values over a range of time transitions, since sine takes values in the range from -1 to 1 . This result does not follow from Eq. (II.2). In conventional quantum mechanics, spin has been described as quantum superposition of up-spin and down-spin states. Equation (II.4) indicates that the electron repeats up-spin and down-spin with time transitions.

III. LORENTZ CONTRACTION CAUSING THE ANOMALOUS MAGNETIC MOMENT

A. Associating the anomalous magnetic moment with Lorentz contraction

This section describes the anomalous magnetic moment of electrons as an application. Einstein made the following point in a paper published in 1912 [4]. Namely, in the rotational coordinate system the ratio of circumference to diameter differs from that in Euclidean geometry. For example, imagine a bicycle wheel circumference spinning at close to the speed of light. In the direction along the rim sidewall, the Lorentz transformation causes a length contraction, whereas no Lorentz contraction occurs in the direction of the tangential spokes from the periphery towards the centre.

In this section, the goal is to use the Lorentz transformation of rotationality to consider that one rotation of space, 2π , becomes shorter than 2π when affected by Lorentz contraction as shown in Fig. 2. Then, since one rotation affected by Lorentz contraction becomes shorter than 2π , the difference is interpreted as anomalous magnetic moment in this study.

The results of the previous discussions showed that if an electron in accelerated motion is significantly slower than the speed of light ($\beta = 1 - v^2/c^2 \approx 1$), one period is halved from 2π to 1π on the basis of Eq. (II.4). This provided a basis for generating a magnetic field twice as efficiently as orbital rotation.

Thomas studied parallel infinitesimal displacement of coordinate axes. He concluded that parallel displacement of axes means that at any instant the axis at that instant is parallel to the axis after an infinitesimal amount of time. Thomas used the Lorentz transformation to make this calculation. When an object is in uniform linear motion, a coordinate transformation can be performed using the special Lorentz transformation, which is non-rotational. On the other hand, the Lorentz transformation that Thomas verified for angular momentum was a rotational transformation.

Under the assumption that the acceleration of electrons is sufficiently slow compared to the speed of light, the Thomas precession would have created a picture of the piece rotating. It has been assumed that electrons in an atom move much slower than the speed of light and are not affected by Lorentz contraction.

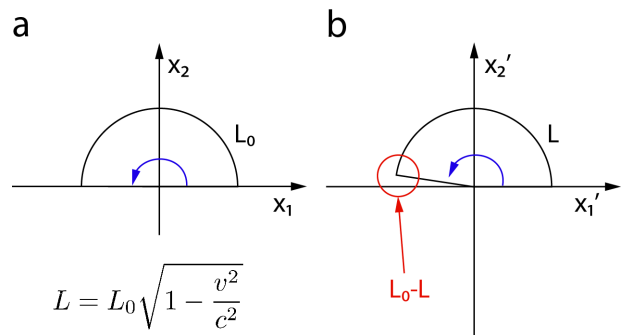


Fig. 2. Presence of rotational Lorentz contraction. Presence of rotational Lorentz contraction. To be precise, rotation should be regarded as the motion of a point through the origin, as shown in Fig. 1. However here it is shown as a circumference for visual clarity. (a) Lorentz contraction was applied to rotational coordinates. If the electrons are travelling significantly slower than the speed of light, the rotational Lorentz contraction can be neglected. ($\beta = 1 - v^2/c^2 \approx 1$) (b) Lorentz contraction cannot be ignored when the speed of electrons travelling approaches the speed of light. Therefore, the length of the π circumference shrinks. This contraction is considered to be the cause of the anomalous magnetic moment.

We would like to consider this assumption. This could mean that the oscillation period of the electron is so fast that the Lorentz contraction cannot be ignored. The purpose of this study is to reconsider this assumption. In the 0-Sphere electron model, an electron travels with two kernels. At these two spatially distant kernels, thermal Potential Energy (TPE) radiates and absorbs respectively. These kernels are spatially discrete. In the author's paper [5], this distance was assumed to be the Compton wavelength for a free electron (cf. Fig. 3). Even if the free electron moves in one direction, this model can describe its movement. And importantly, in the 0-Sphere electron model, electrons do not move in a uniform linear motion. Its motion was assumed to be traveling with acceleration expressed as a sinusoidal function.

According to the 0-Sphere model, the electron is traveling discretely in space [6]. The model claimed that during its movement, the TPE is converted to kinetic energy, which is transferred by the virtual photon. Therefore, an important consequence of the application of this study is as follows. That is, the oscillation of the electron described by the model is represented by a sinusoidal function, and the transfer is below the speed of light. Even if it is intuitively possible, assuming that the hypothetical photon moved at the speed of light at the highest speed of the sinusoidal function, it is clear that the overall speed of the electron moving spatially from point $+a$ to point $-a$ (cf. Fig. 3) is on average less than the speed of light, since the acceleration varies.

Note that this study does not recommend having an

image of a virtual photon surrounding an electron in a semicircle for energy transfer. Figure 2 is a schematic diagram. The electron as a simple harmonic oscillator would oscillate linearly. Similarly, the virtual photons would travel in a straight line with the shortest path. See Appendix VIB for the shortest linear path.

In this section, we calculated the average speed of electrons moving spatially from point $+a$ to point $-a$ based on the rotational Lorentz transformation from the values of the anomalous magnetic moment obtained in our experiments. See Appendix Fig. 3 for two points, $+a$ and $-a$. The result was about 0.04047 times the speed of light. The methods and the results are referred to in the following chapters.

B. Average velocity of electron micro-oscillation

The difference is the anomalous magnetic moment, denoted a and defined as,

$$a = \frac{g-2}{2}. \quad (\text{III.1})$$

As can be seen from the fact that this defining equation is divided by 2, we should consider the fraction of the circumference of 1π that is shortened by Lorentz contraction, not the circumference of 2π per circumference.

The current experimental value and uncertainty is,

$$a_e^{\text{exp}} = 0.001\,159\,652\,180\,59(13). \quad (\text{III.2})$$

Let L_0 be the length of a bar in the coordinate system moving with the electrons and L be the length of the bar when the moving electrons are viewed from the laboratory system, the following relationship holds between the two. Lorentz contraction is expressed by the following equation,

$$L = L_0 \sqrt{1 - \frac{v^2}{c^2}}. \quad (\text{III.3})$$

According to Eq. (II.4), the angular momentum moving with acceleration $-\sin\theta$ was expressed by $\sin 2\theta$. This is a strong evidence that spin rotation can generate a magnetic field twice as efficiently as orbital rotation. This was due to the change from θ to 2θ .

In other words, the interpretation was that instead of having to rotate 360 degrees in space to generate a magnetic field, one half of that, 180 degrees, could be used to generate a magnetic field. In this study, we can consider that the anomalous magnetic moment generates the magnetic field at an angle even less than 180 degrees. That is, we reinterpret the 180-degree angle as a rotational Lorentz contraction that can generate a magnetic field at an angle shorter than 180 degrees (Fig. 2).

According to the above view, the equation since expresses the relationship between Lorentz contraction and anomalous magnetic moment,

$$\frac{L}{L_0} = \frac{1}{1 + a_e^{\text{exp}}}. \quad (\text{III.4})$$

We further modify Eq. (III.4). We take the root-mean-square (RMS) value of a_e^{exp} because we are trying to find the average velocity; multiplying by the RMS is similar to the reason why the maximum and effective voltages of an AC voltage are different. In other words, the Lorentz contraction is also subject to fluctuations in its length because the harmonic oscillator would constantly produce varying accelerations. Therefore, in order to determine the average speed of the electron motion, the anomalous magnetic moment should probably be converted to an RMS value. The revised formula is;

$$\frac{L}{L_0} = \frac{1}{1 + \frac{1}{\sqrt{2}} a_e^{\text{exp}}}. \quad (\text{III.5})$$

The rationale for this modified idea is that the value of the anomalous magnetic efficiency might be calculated from the highest value of the acceleration caused by the harmonic oscillator. Further observations will confirm the correctness of this idea.

Furthermore, from the following relationship,

$$\frac{L}{L_0} = \sqrt{1 - \frac{v^2}{c^2}}. \quad (\text{III.6})$$

From these two equations, we obtained,

$$\sqrt{1 - \frac{v^2}{c^2}} = \frac{1}{1 + \frac{1}{\sqrt{2}} a_e^{\text{exp}}}. \quad (\text{III.7})$$

It should be noted that Eq. (III.7) derived here will be modified to Eq. (IV.2) in the next chapter to take account of general relativity.

Substituting the anomalous magnetic moment obtained experimentally for a_e^{exp} , $\beta^2 = (v/c)^2$ is obtained,

$$\beta^2 = \left(\frac{v\gamma^*}{c}\right)^2 = 0.00163798087 \quad (\text{III.8})$$

$$v_{\text{electron}}^{\gamma^*} \doteq 0.04047197635 \times c. \quad (\text{III.9})$$

With this beta value, the average speed was calculated to be approximately 12,133 km/s. For reference, we can compare the values of the muon with the results of Eq. (III.9).

Combining the beta implications of the above equation with the 0-Sphere electron model yields the following

consequence. This means that the energy of the electron is moving from point $+a$ to point $-a$ with an average 0.04047 times the speed of light. Applying this result, the wavelength of this electron is also extended from Compton's wavelength by the factor of 0.04047. The modified frequency is calculated as follows,

$$\begin{aligned}\nu_{\text{electron}} &= \frac{\beta c}{\lambda_{\text{compton}}} \\ &= 0.04047 \times 299792458 \div 2.42631 \times 10^{-12} \\ &= 5.007057 \times 10^{18} (\text{Hz}).\end{aligned}\tag{III.10}$$

The above frequency could be equivalent to that of X-rays. Originally, the frequency derived from Compton wavelengths was 1.24×10^{20} . The modified electron frequency is 5.007057×10^{18} when the anomalous magnetic moment is calculated from a consideration that relies on the Lorentz contraction of rotationality in this paper.

However, the opinion that the angular momentum is a continuous value but the actual observations are of upward and downward spin has the following interpretation. Namely, imagine a device in which a spring is retracted and a sphere is placed at its end. When the finger holding the retracted spring is released, the spring will extend and the ball will be launched. The timing of this launch is when the acceleration of the spring decreases. The timing at which the ball leaves the spring is not continuous. Consider this as the timing at which the virtual photon leaves the electron.

The discrete observation of electron spin can be interpreted as this spring/sphere relationship. The spring corresponds to the thermal gradient and the sphere to the virtual photon with kinetic energy. In a simple oscillator, the position at which the acceleration is slowed down during a round trip occurs only twice, once to the right and once to the left, if the spring is placed horizontally; the time at which the acceleration is slowed down, which occurs twice in a cycle, can be regarded as corresponding to the observation of upward and downward spin. Equivalently, the timing at which a virtual photon is ejected out of the electron as an actual photon is considered to be twice per cycle, i.e., when the value of Ω is at its maximum in the Eq. (II.4).

Hence, the velocity at which the acceleration of the virtual photon undergoes maximum acceleration from the thermal gradient produced inside the electron can be given by Eq. (III.4). See Appendix VIC Eq. (VI.5) for the thermal energy gradient created in an electron. This velocity is given by the light speed ratio without the RMS value applied. This idea would be a major step forward in solving the problem of ultraviolet divergence in quantum field theory.

IV. GENERAL RELATIVITY'S GEODETIC EFFECT ON ELECTRON SPIN

A. Considering geodetic precession applying Schwarzschild metrology

In addition to the results obtained in the previous section we further consider the influence of general relativity. Namely, gravity. As is well known, attempts have been made to generate a theory that integrates quantum mechanics and gravity. It is called quantum gravity theory. The reason why this attempt has not been fulfilled is that attempts to relate the gravitational field to the quantum field have not been successful. This is due to the properties of the fields, which take on continuous values.

The geodesic and frame-dragging effect predicted by general relativity has been successfully and accurately confirmed by NASA's Gravity Probe B satellite. GP-B final experimental results were announced on May 4, 2011 [7]. This chapter attempts to apply the geodesic effect to electron spin. Note that the quantisation of gravity is beyond the scope of this paper.

Up to the previous section, electron spin has been analysed using classical models. In this section, the concept of geodesic precession of general relativity is applied to electron spin. To calculate the geodetic precession on electron spin, we refer to the following geodetic precession equation already found [8],

$$\Delta\phi_{\text{geodetic}} = 2\pi \left[1 - \left(1 - \frac{3M}{R} \right)^{1/2} \right] (\text{per orbit}),\tag{IV.1}$$

where M is mass and R is the Schwarzschild radius.

Equation (IV.1) is expressed in units of radians. As considering the anomalous magnetic efficiency of the electron, we should consider half the circumference of a circle to be a unit. Based on our discussion of Eq. (III.1), we use the value of the above equation without multiplying it by 2π ,

$$\frac{L}{L_0} = \frac{1}{1 + \frac{1}{\sqrt{2}} a_e^{\text{exp}} - \frac{\Delta\phi_{\text{geodetic}}}{2\pi}}.\tag{IV.2}$$

The size of the electrons is not currently determined. Observation of a single electron in a Penning trap suggests the upper limit of the particle's radius to be 1.0×10^{-22} meters [9]. That means the following equation (IV.3) is obtained from Eqs. (IV.1) and (IV.2) to calculate Table I. Substitute the mass of the electron for m_{electron} and the radius of the electron for r_{electron} , the result is,

$$\frac{L}{L_0} = 1 / \left(1 + \frac{1}{\sqrt{2}} a_e^{\text{exp}} - \left[1 - \left(1 - \frac{3m_{\text{electron}}}{r_{\text{electron}}} \right)^{1/2} \right] \right).\tag{IV.3}$$

Table I. The velocity and the radius of an electron

$r_{\text{electron}}(\text{m})$	$\Delta\phi_{\text{geodetic}}/2\pi$	$v_{\text{electron}}^{\gamma^*}/c$
1×10^{-22}	1.3670000×10^{-8}	0.040471633
1×10^{-23}	1.3665000×10^{-7}	0.040468601
1×10^{-24}	1.3664100×10^{-6}	0.040438275
1×10^{-25}	1.3664170×10^{-5}	0.040133758
1×10^{-26}	1.3666501×10^{-4}	0.036949906

Once one of the values is measured, the other can be calculated. Table I shows that once the electron radius has been determined, the average speed at which electrons travel can be calculated. For example, if the speed-of-light ratio of $v_{\text{electron}}^{\gamma^*}$ is measured to be $0.0400c$, the radius of the electron would be between $1.0 \times 10^{-25}(\text{m})$ and $1.0 \times 10^{-26}(\text{m})$.

The conclusion of this theory is to calculate the size of the electron kernel, but for this we have to wait for the results of the two variable experiment. The first is the development of more powerful measuring instruments capable of detecting electron magnitudes below $1.0 \times 10^{-22}(\text{m})$, and the second is a technique for measuring the velocity of micro-oscillation of an electron. The validity of the results of the above equations would be verified if the radius, r_{electron} , of the electron and the value of its average traveling velocity, $v_{\text{electron}}^{\gamma^*}$, could be measured experimentally.

V. CONCLUSION

We discarded the image of the electron spinning on its own axis and offered the view that spin occurs when it moves back and forth between two kernels in an electron as a simple harmonic oscillator. Whereas spin has traditionally been thought of as a uniform circular motion, in this study it is replaced by a simple harmonic motion. When a point of mass passes through the origin and moves between two kernels, no angular momentum is generated if the motion is uniformly linear. However, when an electron moves back and forth between two kernels, this assumption is negated and accelerated motion occurs between the two kernels.

Three important results were achieved in this work. The first was the withdrawal of the classical basis for the motion of the piece. This precessional motion is a picture that results from the assumption that the electrons are in uniformly accelerated motion. In this paper, the perspective of uniformly accelerated motion is reviewed. Instead, the electron is an oscillator, and the velocity

and acceleration, described by trigonometric functions, are adapted to Thomas's theory. As a result, a factor of 1/2 was calculated in the Eq. (II.4). This means that it is twice as efficient as the magnetism generated by rotation in space.

The second issue raised was whether the anomalous magnetic moment of the electron could be caused by a rotational Lorentz contraction. Calculations based on the experimentally measured anomalous magnetic moment showed that the oscillations of the electrons repeatedly travel at an average speed of about four percent of the speed of light.

The micro-oscillation of an electron would be in accelerated motion between two kernels. That was obtained as a consequence of the 0-Sphere model in the paper by the author [5]. To put it bluntly, the model would allow an electron to behave like an inchworm. Its footprints are discrete as the inchworm moves.

There, when thermal energy was transferred between two kernels by radiation and absorption, the kinetic energy could be represented by a simple sinusoidal function. The electron model obeyed the law of conservation of energy. The centroid of the kinetic energy of the electron moving between the two kernels, or the energy gradient formed by the thermal energy of radiation and absorption, could be represented as the reciprocating motion of a simple harmonic oscillator. In this behaviour, the electrons have spin even though they do not move in a uniform circular motion.

And the third, in addition to the Lorentz contraction, we have corrected each electron's traveling velocity for the geodetic precession of general relativity. This attempt allowed us to predict the size of the electrons.

The 0-Sphere model provides that within a single electron, it is like a pair of black holes and white holes. Furthermore, their respective functions are periodically reversed. The model shows how an electron repeatedly travels as a harmonic oscillator because the energy bodies in an electron repeatedly radiate and absorb its energy. However, the Schwarzschild radius could be introduced to the model for further discussion. The discussion has been continued with the application of the concept of general relativity, which is incompatible with the elementary particle, the electron. We examined and provided the process of predicting the radius of the electron.

The emitting side kernel in the electron could be considered a white hole, and the one on the absorbing side could be considered a black hole. The discussion is based on the assumption that the electron radius, whose size has not yet been determined, would be comparable to that of a black hole. The validity of the calculated average velocity of the electron micro-oscillation and the radius of the electron will have to await further development of the experimental reports.

[1] L. H. Thomas. The Motion of the Spinning Electron : *Nature*, **117** (1926), 514; *Philosoph Mag.*, 3 (1927), 1.

B. What is the 0-sphere

A 0-sphere is a pair of points and has no area. The general form of 0-sphere is represented as n -sphere.

In this subsection, we will review the electronic model with the 0-sphere. A 0-sphere is a pair of points at the ends of a one-dimensional line segment. A 1-sphere is a circle as shown in Fig. 4 (a, b). Alternatively, the 0-sphere is indicate an intersection of a straight line and a circle put on the same plane. In other words, by expanding a two-dimensional circle into three dimensions, the 0-sphere is an intersection points with a straight line passing through a hollow sphere.

In this paper, the Lorenz contraction and the geodetic precession are explained by semicircles. In reality, however, light travels by the shortest path, the virtual photon would travel the shortest distance between the blue and green points.

C. Thermal energy gradient caused by two kernels

The Appendix quotes from the paper [5] on how the energy gradient arises from two kernels. To maintain the law of conservation of energy, we take each of the two kernels or bare electrons as a thermal potential energy. These two kernels act as both emitters and absorbers in turn. To meet the requirements for simultaneous emission and absorption, assign T_{e1} and T_{e2} as follows;

$$\begin{aligned} (\text{Oscillator } 1) : T_{e1} &= E_0 \cos^4 \left(\frac{\omega t}{2} \right), \\ (\text{Oscillator } 2) : T_{e2} &= E_0 \sin^4 \left(\frac{\omega t}{2} \right), \end{aligned} \quad (\text{VI.1})$$

where E_0 is the ground state of quantised energy. Set the two electrons as paired oscillators with $T_{e1} = E_0 \cos^4 \omega t/2$ and $T_{e2} = E_0 \sin^4 \omega t/2$. The temperature gradient between the two kernels is calculated as,

$$\text{grad } T_e = \text{grad } (T_{e2} - T_{e1}). \quad (\text{VI.2})$$

Since the values of thermal energy at both thermal kernels vary with time, the temperature gradient changes with time. Let the previous ωt is θ ,

$$\begin{aligned} \text{grad } T_{e1} &= \frac{d}{d\theta} \left(E_0 \cos^4 \left(\frac{\theta}{2} \right) \right) \\ &= -2E_0 \cos^3 \left(\frac{\theta}{2} \right) \sin \left(\frac{\theta}{2} \right). \end{aligned} \quad (\text{VI.3})$$

$$\begin{aligned} \text{grad } T_{e2} &= \frac{d}{d\theta} \left(E_0 \sin^4 \left(\frac{\theta}{2} \right) \right) \\ &= 2E_0 \cos \left(\frac{\theta}{2} \right) \sin^3 \left(\frac{\theta}{2} \right). \end{aligned} \quad (\text{VI.4})$$

$\text{grad } T_{e1}$ and $\text{grad } T_{e2}$ include only time derivative terms; their space derivatives are zero, because the kernels do not change in position with time. That is,

$$\begin{aligned} \text{grad } (T_{e2} - T_{e1}) &= 2E_0 \cos \left(\frac{\theta}{2} \right) \sin^3 \left(\frac{\theta}{2} \right) \\ &\quad + 2E_0 \cos^3 \left(\frac{\theta}{2} \right) \sin \left(\frac{\theta}{2} \right) \\ &= 2E_0 \cos \left(\frac{\theta}{2} \right) \sin \left(\frac{\theta}{2} \right) \\ &= E_0 \sin \theta. \end{aligned} \quad (\text{VI.5})$$

Equation (VI.5) shows that the temperature gradient between $\text{grad } T_{e1}$ and $\text{grad } T_{e2}$ produces a force \mathbf{F} . The force drives the velocity of the virtual photon along with simple harmonic motion. On the basis of the above assumption, the virtual photon swing back and force spatially between the two kernels.

Interaction between thermal and kinetic energy is essential in the 0-Sphere electron model, because the interaction between the two kinds of energy, i.e., the thermal potential energy of the spinors and the kinetic energy of the virtual photon, drives the virtual photon along with the harmonic oscillator. See yellow line on Fig. 3.

D. Geodetic precession

“Suppose at the start of an orbit the observer orients the gyro in a direction in the equatorial plane (say in the direction of a distant star). General relativity predicts that on completion of an orbit, the gyro will generally point in a different direction making an angle $\Delta\phi_{\text{geodetic}}$ with the starting one. That change in direction is called *geodetic precession* and its illustrated schematically in Fig. 5.” [8].

The spin comes back after one orbit rotated by an angle,

$$\Delta\phi_{\text{geodetic}} = 2\pi \left[1 - \left(1 - \frac{3M}{R} \right)^{1/2} \right] \text{ (per orbit)}, \quad (\text{VI.6})$$

in the direction of motion, as illustrated in Fig. 5.