Spin Angular Momentum Explained by the Classical Quantum Model

Satoshi Hanamura*
(Dated: September 9, 2023)

This paper proposes a new picture of spin angular momentum. In the conventional picture of spin, the precession of the axis is based on the assumption that the electron has an acceleration. In this study we first consider the case where the acceleration is expressed as a simple harmonic oscillator and the precession as a sinusoidal function. In this case, a double angle appears in the outer product of the Thomas precession, confirming that an angular velocity of one revolution of space can be obtained with half the circumference of the circle. Next we consider the case of Lorentz contraction of the circumference in the direction of the axis of rotation. Einstein pointed out that in a rotating coordinate system the ratio of circumference to diameter is not pi. This study propose that the Lorentz contraction is the cause of the anomalous magnetic moment. The anomalous magnetic moment is regarded as a Lorentz contraction of the rotational angular momentum. As a result, the oscillation of the electron at Compton wavelengths is calculated to be about four percent of the speed of light.

I. INTRODUCTION

The depiction of spin as the precession of a piece was largely influenced by arguments derived from Thomas's brilliant work [1]. In this study, a new spin image will be proposed from a different perspective from conventional spin. In 1925, Uhlenbeck and Goudsmit wrote a paper [2] on rotating electronic images. One reason for the dismissal of the classical electron theory was noted by Lorenz. He pointed out that very fast rotation was required to have a rotation angular momentum and that the speed of the electron surface was ten times the speed of light.

Till date, the detailed reasons for the emergence of spin have not been clarified. In physics textbooks, spin is often described by a picture of the precessional motion of a rotating piece when describing spin. In this study, we discuss the classical aspects of the spin picture, going back to the time before spin was imaged by rotational motion.

In 1945 Nobel Lecture, Pauli mentioned,

> "... The gap was filled by Uhlenbeck and Goudsmit's idea of electron spin, which made it possible to understand the anomalous Zeeman effect simply by assuming that the spin quantum number of one electron is equal to 1/2 and that the quotient of the magnetic moment to the mechanical angular moment has for the spin a value twice as large as for the ordinary orbit of the electron. Since that time, the exclusion principle has been closely connected with the idea of spin. Although at first I strongly doubted the correctness of this idea because of its classical-mechanical character, I was finally converted to it by Thomas' calculations on the magnitude of doublet splitting. [3]"

* hana.tensor@gmail.com

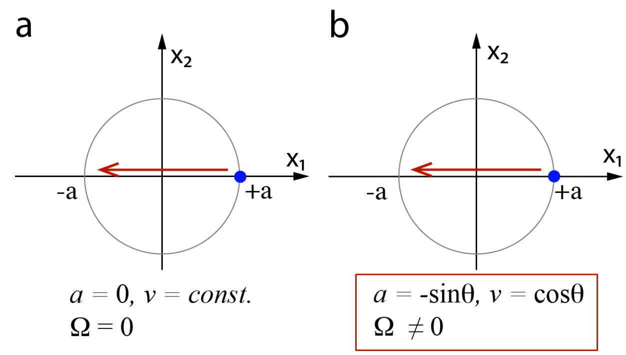

Fig. 1. Point mass observed in the laboratory coordinate system. The blue dots move from $+a$ to $-a$ with different accelerations. (**a**) As the point mass passes through the origin of the coordinate axes in uniform linear motion ($a = 0$), the angular momentum, Ω, is zero. (**b**) According to Thomas's study, the angular momentum does not have a zero value when the point mass passes through the origin of the coordinate axes in accelerated motion.

Pauli did not reject quantum mechanics based on the classical manner. We shall go back in time to 1925 and re-produce spin images based on classical quantum theory. The behaviour of an electron travelling between two points can be described by a simple sinusoidal function, as shown by the results in Eq. (V.5) in the Appendix. That is, the central kinetic energy of the virtual photon with simple harmonic oscillation for an electron can be described by a simple sinusoidal function.

Herein, the image of a spinning top with precession in uniform circular motion has been discarded. We abandon the diagram (**a**) shown on Fig. 1 and seek a new spin image within the diagram (**b**). Instead, the harmonic oscillator has been placed on the coordinate axis and its angular acceleration has been considered. The electron is not assumed to be in uniformly accelerated motion, but to have an acceleration represented by a sinusoidal function, giving a completely new spin picture that has never been seen before.

II. THE ACCELERATION OF THE ELECTRON COULD NOT BE CONSTANT

A. Review the Thomas precession

This study does not make the assumption of constant acceleration ($a : acceraration = f : fource$) in Thomas theory. The acceleration of the electrons can be changing. The electron does not travel in an uniform linear motion but with an intrinsic velocity, which could be expressed by a sinusoidal function. If the velocity $\boldsymbol{v}$ is expressed by $\boldsymbol{v} = \cos\theta$, the acceleration is expressed by its derivative, $\boldsymbol{a} = -\sin\theta$. In this study, at the beginning, we reviewed Thomas's work and substituted $\boldsymbol{a} = -\sin\theta$ instead of constant value ($\boldsymbol{a} = \boldsymbol{f}$) into the Thomas precession.

The discussion begins with the background of the association of spin with precessional motion. In relativity, if the electron is in uniform linear motion, the coordinate system describing the electron's motion can be calculated by Lorentz transformation. However, if the electron is in an accelerated motion, it is calculated that the axis of the coordinate system describing this electron rotates when observed from the laboratory system. Thomas wrote in his paper that the axes of a coordinate system with an origin and translating with the electrons are observed in a laboratory system to rotate with the following angular velocity as in Eq. (II.1),

$$\boldsymbol{\Omega} = \frac{1}{2c^2}[\boldsymbol{a} \times \boldsymbol{v}], \tag{II.1}$$

where $\boldsymbol{a}$ is the acceleration of the electron and $\boldsymbol{v}$ is the velocity of the electron. Note that in Eq. (II.1), the approximation ($\beta = 1 - v^2/c^2 \risingdotseq 1$) is set in Lorenz transformation. Equation (II.1) can also be applied to the general case where the particles are not in uniform circular motion. As the particles are in uniform circular motion, the following equation is obtained,

$$\Omega = -\frac{1}{2}\frac{v^2}{c^2}\omega_{\text{const}}. \tag{II.2}$$

The spin image in precession that we now recall comes from Eq. (II.2). The angular velocity Ω obtained is a constant proportional to ω_{const}. In this study, however, we will not consider the issue using Eq. (II.2), but rather equation (II.1).

B. Assuming a simple harmonic oscillation instead of uniform circular motion

This section is the innovative part of this study. The quantisation of the orbital angular momentum into units of $\hbar$ reflects the nature of space, which returns to its original state after one rotation. According to the relationship between angular momentum and magnetic moment, if the angular momentum is halved to $\hbar/2$, the magnetic moment should also be $\mu_e/2$. However, the magnetic moment of the spin angular momentum is equal to μ_e, even though the angular momentum is $\hbar/2$. This means that spin rotation can generate magnetic fields twice as efficiently as orbital rotation and responds to magnetic fields with twice the sensitivity. This property could not be explained by theories based on circular currents observed in three-dimensional space.

Consider this discrepancy from the perspective of the Thomas precession. Equation (V.5) forms an important basis for this paper. The traveling of the virtual photon, γ^*, is represented by a sinusoidal function (cf. Eq. (V.5) and see yellow line on Fig. 3). The study was described as the 0-Sphere electron model. In this electron model, the thermal potential energy (TPE) of the electron is a set of radiation and absorption, which describes the motion of the electron; the TPE changes partly kinetic energy, which drives the photon. The motion of the photon could be represented by a very simple sinusoidal function in this research model. First, we let the two values as follows;

$$\begin{aligned} (Verocity) &: v_{\gamma^*} = \cos\omega t, \\ (Acceraration) &: a_{\gamma^*} = -\sin\omega t. \end{aligned} \tag{II.3}$$

Substitute Eq. (II.3) into Eq. (II.1) then,

$$\begin{aligned} \boldsymbol{\Omega} &= \frac{1}{2c^2}[\boldsymbol{a}_{\gamma^*} \times \boldsymbol{v}_{\gamma^*}] \\ &= \frac{1}{2c^2}[-\sin\omega t \times \cos\omega t] \\ &= \frac{1}{2c^2} \cdot \left(-\frac{1}{2}\sin 2\omega t\right). \end{aligned} \tag{II.4}$$

The above discussion yields an extremely important result. Namely, when the outer product of cosine and sine is calculated, $-\sin 2\omega t$ appears. Equation (II.4) is the basis for obtaining a doubled angular velocity cycle. It was found that the displacement, velocity and period of a single oscillation have a cycle of omega t, whereas the angular velocity has a cycle of $2\omega t$. One wave period of single oscillation is determined by the angular velocity. The angular velocity with Thomas precession has a period of half the displacement.

The results of the study of the above equation provide a basis for the quantisation of the spin angular momentum to a value half the Planck constant.

III. LORENTZ CONTRACTION CAUSING THE ANOMALOUS MAGNETIC MOMENT

A. Associating the anomalous magnetic moment with Lorentz contraction

This section describes the anomalous magnetic moment of electrons as an application. Einstein made the following point in a paper published in 1912 [4]. Namely, in the rotational coordinate system the ratio of circumference to diameter differs from that in Euclidean geometry. For example, imagine a bicycle wheel circumference spinning at close to the speed of light. In the direction along the rim sidewall, the Lorentz transformation causes a length contraction, whereas no Lorentz contraction occurs in the direction of the tangential spokes from the periphery towards the centre.

In this section, the goal is to use the Lorentz transformation of rotationality to consider that one rotation of space, 2π, becomes shorter than 2π when affected by Lorentz contraction as shown in Fig 2. Then, since one rotation affected by Lorentz contraction becomes shorter than 2π, the difference is interpreted as anomalous magnetic moment in this study.

The results of the previous discussions showed that if an electron in accelerated motion is significantly slower than the speed of light ($\beta = 1 - v^2/c^2 \fallingdotseq 1$), one period is halved from 2π to 1π on the basis of Eq. (II.4). This provided a basis for generating a magnetic field twice as efficiently as orbital rotation.

Thomas studied parallel infinitesimal displacement of coordinate axes. He concluded that parallel displacement of axes means that at any instant the axis at that instant is parallel to the axis after an infinitesimal amount of time. Thomas used the Lorentz transformation to make this calculation. When an object is in uniform linear motion, a coordinate transformation can be performed using the special Lorentz transformation, which is non-rotational. On the other hand, the Lorentz transformation that Thomas verified for angular momentum was a rotational transformation.

Under the assumption that the acceleration of electrons is sufficiently slow compared to the speed of light, the Thomas precession would have created a picture of the piece rotating. It has been assumed that electrons in an atom move much slower than the speed of light and are not affected by Lorentz contraction.

We would like to consider this assumption. This could mean that the oscillation period of the electron is so fast that the Lorentz contraction cannot be ignored. The purpose of this study is to reconsider this assumption. In the 0-Sphere electron model, an electron travels at two points. At these two spatially distant points, thermal Potential Energy (TPE) radiates and absorbs respectively. These points are spatially discrete. In the author's paper [5], this distance was assumed to be the Compton wavelength for a free electron (cf. Fig. 3). Even if the free electron moves in one direction, this model can describe its movement. And importantly, in the 0-Sphere electron model, electrons do not move in a uniform linear motion. Its motion was assumed to be traveling with acceleration expressed as a sinusoidal function.

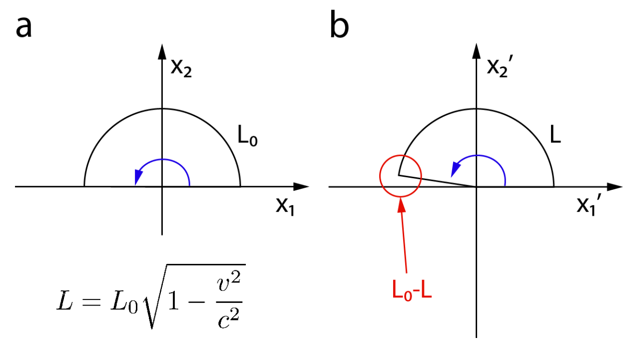

Fig. 2. Presence of rotational Lorentz contraction. Presence of rotational Lorentz contraction. To be precise, rotation should be regarded as the motion of a point through the origin, as shown in Fig. 1. However here it is shown as a circumference for visual clarity. (**a**) Lorenz contraction was applied to rotational coordinates. If the electrons are travelling significantly slower than the speed of light, the rotational Lorentz contraction can be neglected. ($\beta = 1 - v^2/c^2 \fallingdotseq 1$) (**b**) Lorentz contraction cannot be ignored when the speed of electrons travelling approaches the speed of light. Therefore, the length of the π circumference shrinks. This contraction is considered to be the cause of the anomalous magnetic moment.

According to the 0-Sphere model, the electron is traveling discretely in space. The model claimed that during its movement, the TPE is converted to kinetic energy, which is transferred by the virtual photon. Therefore, an important consequence of the application of this study is as follows. That is, the oscillation of the electron described by the model is represented by a sinusoidal function, and the transfer is below the speed of light. Even if it is intuitively possible, assuming that the hypothetical photon moved at the speed of light at the highest speed of the sinusoidal function, it is clear that the overall speed of the electron moving spatially from point $+a$ to point $-a$ (cf. Fig. 3) is on average less than the speed of light, since the acceleration varies.

In this section, we calculated the average speed of electrons moving spatially from point $+a$ to point $-a$ based on the rotational Lorentz transformation from the values of the anomalous magnetic moment obtained in our experiments. The result was about 0.048 times the speed of light. The calculation is described in detail below.

B. Average velocity of electron micro-oscillation

The difference is the anomalous magnetic moment, denoted a and defined as

$$a = \frac{g-2}{2}. \tag{III.1}$$

As can be seen from the fact that this defining equation is divided by 2, we should consider the fraction of the circumference of 1π that is shortened by Lorentz contraction, not the circumference of 2π per circumference.

The current experimental value and uncertainty is,

$$a_{\mathrm{e}}^{\mathrm{exp}} = 0.001\,159\,652\,180\,59(13). \tag{III.2}$$

Let L_0 be the length of a bar in the coordinate system moving with the electrons and L be the length of the bar when the moving electrons are viewed from the laboratory system, the following relationship holds between the two. Lorentz contraction is expressed by the following equation,

$$L = L_0\sqrt{1-\frac{v^2}{c^2}}. \tag{III.3}$$

According to Eq. (II.4), the angular momentum moving with acceleration $\cos\theta$ was expressed by $\sin 2\theta$. This is a strong evidence that spin rotation can generate a magnetic field twice as efficiently as orbital rotation. This was due to the change from θ to 2θ.

In other words, the interpretation was that instead of having to rotate 360 degrees in space to generate a magnetic field, one half of that, 180 degrees, could be used to generate a magnetic field. In this study, we can consider that the anomalous magnetic moment generates the magnetic field at an angle even less than 180 degrees. That is, we reinterpret the 180-degree angle as a rotational Lorentz contraction that can generate a magnetic field at an angle shorter than 180 degrees (Fig. 2).

According to the above view, the equation since expresses the relationship between Lorentz contraction and anomalous magnetic moment,

$$\frac{L}{L_0} = \frac{1}{1+a_{\mathrm{e}}^{\mathrm{exp}}}. \tag{III.4}$$

Furthermore, from the following relationship,

$$\frac{L}{L_0} = \sqrt{1-\frac{v^2}{c^2}}. \tag{III.5}$$

From these two equations, we obtained,

$$\sqrt{1-\frac{v^2}{c^2}} = \frac{1}{1+a_{\mathrm{e}}^{\mathrm{exp}}}. \tag{III.6}$$

Substituting the anomalous magnetic moment obtained experimentally for $a_{\mathrm{e}}^{\mathrm{exp}}$, $\beta^2 = (v/c)^2$ is obtained,

$$\beta^2 = (\frac{v_{\gamma^*}}{c})^2 = 0.00231822854 \tag{III.7}$$

$$v_{\gamma^*}^{\mathrm{electron}} \fallingdotseq 0.00481428668 \times c. \tag{III.8}$$

With this beta value, the average speed was calculated to be approximately 14425 km/s. For reference, we can compare the values of the muon with the results of Eq. (III.8). Applying the same procedure to the anomalous magnetic moment of the muon, we obtain $v_{\gamma^*}^{\mathrm{muon}} \fallingdotseq 0.0482714975c$.

Combining the beta implications of the above equation with the 0-Sphere electron model yields the following consequence. This means that the energy of the electron is moving from point $+a$ to point $-a$ with an average 0.048 times the speed of light. Applying this result, the wavelength of this electron is also extended from Compton's wavelength by the factor of 0.048. The modified frequency is calculated as follows,

$$\begin{aligned}\nu_{\mathrm{electron}} &= \frac{\beta c}{\lambda_{\mathrm{compton}}}\\ &= 0.048 \times 299792458 \div 2.42631 \times 10^{-12}\\ &= 5.9308 \times 10^{18}(\mathrm{Hz}).\end{aligned} \tag{III.9}$$

The above frequency could be equivalent to that of X-rays. Originally, the frequency derived from Compton wavelengths was 1.24×10^{20}. The modified electron frequency is 5.9308×10^{18} when the anomalous magnetic moment is calculated from a consideration that relies on the Lorentz contraction of rotationality in this paper.

IV. CONCLUSION

We discarded the image of the electron spinning on its own axis and offered the view that spin occurs when it moves back and forth between two points as an simple harmonic oscillator. Whereas spin has traditionally been thought of as a uniform circular motion, in this study it is replaced by a simple harmonic motion. When a point of mass passes through the origin and moves between two points, no angular momentum is generated if the motion is uniformly linear. However, when an electron moves back and forth between two points, this assumption is negated and accelerated motion occurs between the two points.

Two important results were achieved in this work. The first was the withdrawal of the classical basis for the motion of the piece. This precessional motion is a picture that results from the assumption that the electrons are in uniformly accelerated motion. In this paper the perspective of uniformly accelerated motion is reviewed. Instead, the electron is an oscillator, and the velocity and acceleration, described by trigonometric functions, are adapted to Thomas's theory. As a result, a factor of 1/2 was calculated in the Eq. (II.4). This means that the magnetism generated by rotation in space is half as efficient.

Another issue raised was whether the anomalous magnetic moment of the electron could be caused by a rotational Lorentz contraction. Calculations based on the experimentally measured anomalous magnetic moment showed that the oscillations of the electrons repeatedly travel at an average speed of about four percent of the speed of light.

The electron that is in accelerated motion between two points was obtained as a consequence of the 0-Sphere model in the paper by the author [5]. To put it bluntly, the model would allow an electron to behave like an inchworm. Its footprints are discrete as the inchworm moves.

There, when thermal energy was transferred between two points by radiation and absorption, the kinetic energy could be represented by a simple sinusoidal function. The electron model obeyed the law of conservation of energy. The centroid of the kinetic energy of the electron moving between the two points, or the energy gradient formed by the thermal energy of radiation and absorption, could be represented as the reciprocating motion of a simple harmonic oscillator. In this behaviour, the electrons have spin even though they do not move in a uniform circular motion.

[1] L. H. Thomas. The Motion of the Spinning Electron : *Nature*, **117** (1926), 514; *Philosoph Mag.,* 3 (1927), 1. https://doi.org/10.1038/117514a0

[2] G. E. Uhlenbeck, S. A. Goudsmit. Spinning Electrons and the Structure of Spectra : *Naturwissenschaften*, **13** (1925), 953; *Nature*, **117** (1926), 264. https://doi.org/10.1038/117264a0

[3] W. Pauli. Exclusion principle and quantum mechanics Nobel Lecture, December 13, (1946) Wolfgang Pauli Nobel Prize lecture 1945

[4] A. Einstein, Theorie des statischen Gravitationsfeldes, *Annalen der Physik* 38 (1912) 443. Theory of the Static Gravitational Field

[5] S. Hanamura. A Model of an Electron Including Two Perfect Black Bodies, viXra:1811.0312 (2018)

V. APPENDIX

A. An electron's structure in this study

In the 0-Sphere electron model, an electron's structure is assumed as follows. First, consider there is a tiny thermal source in the center. This thermal spot, named bare electron or an spinor in this study, can be moved by radiation, however, it stops time and fixes it in the center of the electron. Next, consider a real photon that surrounds the bare electron. This real photon has an electromagnetic interaction with the bare electron.

The concept of virtual photons has not changed since mentioned on paper [5]. The photons surrounding the two thermal sources exchanging energy with each other are real photons. Because the photon is connected to the thermal spot by the electromagnetic force, this photon does not emit energy to the external system and cannot be observed. In this paper, one electron is regarded as a closed system in thermodynamics, and this paper is not expanded to the interaction with other electrons.

From this viewpoint, this real photon may be called a virtual photon. However, the virtual photons used in the past are particles that are temporarily generated during an interaction, and the meaning of the virtual photons in this paper is very different in that they do not satisfy the energy conservation law.

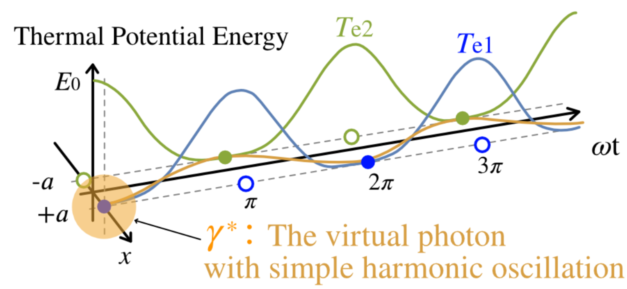

Fig. 3. Behavior of the virtual photon as a spatial simple harmonic oscillator while the two bare electrons behave as emitters and absorbers. Since the equation of $T_{\rm e1} + T_{\rm e2} + \gamma^*_{\rm Kinetic.E} = E_0$, the sum of the thermal potential energy of the two spinors and the kinetic energy of the virtual photon is constant. The energy conservation law is preserved. See paper [5] for details.

B. Thermal energy gradient caused by two spinors

The Appendix quotes from paper [5] on how the energy gradient arises from two spinors. To maintain the law of conservation of energy, we take two bare electron as thermal potential energies. These two electrons act as both emitters and absorbers in turn. To meet the requirements for simultaneous emission and absorption, assign $T_{\rm e1}$ and $T_{\rm e2}$ as follows;

$$\begin{aligned} (Oscillator\ 1) : T_{\rm e1} &= E_0 \cos^4\left(\frac{\omega t}{2}\right), \\ (Oscillator\ 2) : T_{\rm e2} &= E_0 \sin^4\left(\frac{\omega t}{2}\right). \end{aligned} \tag{V.1}$$

Set the two electrons as paired oscillators with $T_{\rm e1} = E_0 \cos^4 \omega t/2$ and $T_{\rm e2} = E_0 \sin^4 \omega t/2$. The temperature gradient between the two bare electrons is calculated as,

$$\text{grad } T_{\rm e} = \text{grad } (T_{\rm e2} - T_{\rm e1}) \ . \tag{V.2}$$

Since the values of thermal energy at both thermal spots vary with time, the temperature gradient changes with time. Let the previous ωt is θ,

$$\begin{aligned} \text{grad } T_{\rm e1} &= \frac{d}{d\theta}\left(E_0 \cos^4\left(\frac{\theta}{2}\right)\right) \\ &= -2E_0 \cos^3\left(\frac{\theta}{2}\right) \sin\left(\frac{\theta}{2}\right) \ . \end{aligned} \tag{V.3}$$

$$\begin{aligned} \text{grad } T_{\rm e2} &= \frac{d}{d\theta}\left(E_0 \sin^4\left(\frac{\theta}{2}\right)\right) \\ &= 2E_0 \cos\left(\frac{\theta}{2}\right) \sin^3\left(\frac{\theta}{2}\right) \ . \end{aligned} \tag{V.4}$$

grad $T_{\rm e1}$ and grad $T_{\rm e2}$ include only time derivative terms; their space derivatives are zero, because the bare electrons do not change in position with time. That is,

$$\begin{aligned} \text{grad } (T_{\rm e2} - T_{\rm e1}) &= 2E_0 \cos\left(\frac{\theta}{2}\right) \sin^3\left(\frac{\theta}{2}\right) \\ &\quad + 2E_0 \cos^3\left(\frac{\theta}{2}\right) \sin\left(\frac{\theta}{2}\right) \\ &= 2E_0 \cos\left(\frac{\theta}{2}\right) \sin\left(\frac{\theta}{2}\right) \\ &= E_0 \sin\theta \ . \end{aligned} \tag{V.5}$$

Equation (V.5) shows that the temperature gradient between grad $T_{\rm e1}$ and grad $T_{\rm e2}$ produces a force $\mathbf{F}$. The force drives the velocity of the virtual photon along with simple harmonic motion. On the basis of the above assumption, the virtual photon swing back and force spatially between the two bare electrons.

Interaction between thermal and kinetic energy is essential in the 0–sphere electron model, because the interaction between the two kinds of energy, i.e., the thermal potential energy of the spinors and the kinetic energy of the virtual photon, drives the virtual photon along with the harmonic oscillator. See yellow line on Fig. 3.