The impossibility of the long-distance quantum correlation in an example

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In this brief report we point out that the example in the famous paper of D. Bohm and Y.Aharonov in 1957 may not realize the long-distance quantum correlation proposed by Einstein, Rosen and Podolsky in 1935. The reason is presented briefly.

Introduction In the famous paper [1] D. Bohm and Y. Aharonov have designed an illustrative example to analyze the paradox proposed by Einstein, Rosen and Podolsky in 1935 [2]. This example concerns a molecule with zero total spin, consisting of two atoms, each with a spin of one-half. The wave function of the total spin can be written as

$$\psi_0 = \frac{1}{\sqrt{2}}(\mid +\rangle_1 \mid -\rangle_2 - \mid -\rangle_1 \mid +\rangle_2) \tag{1}$$

where $\mid +\rangle_1$ refers to the state in which atom 1 has spin $+\hbar/2$, etc. The Authors then supposed two atoms being separated by a method that does not affect the total spin. After the atoms being separated sufficient far away so that they no longer interact, if one measures the first particle with resulted spin $-\hbar/2$, then one can conclude the other particle's spin is $+\hbar/2$, since the total spin is zero. This idea was straightforwardly taken over to photon-pair produced by annihilation of a positron-electron pair, which was thought to be easily tested experimentally.

Analysis Here we put our analysis on this example. Let's begin with definitions in any textbook of quantum mechanics, so without references henceforth. Let $\hat{S}_1$ and $\hat{S}_2$ be operators of the spins of particle 1 and particle 2 respectively. The sum of the two operators is

$$\hat{S} = \hat{S}_1 + \hat{S}_2 \ . \tag{2}$$

It is well know that the total spin $\hat{S}$ has eigen values of 0 and 1. The above equation (1) is exactly the eigenfunction with eigenvalue 0. Without losing generality, assuming the measurement of the last paragraph is along the z-axis, i.e. $\hat{S}_{1z}$ or $\hat{S}_{2z}$, so as the aforementioned state $\mid +\rangle_1$ is very the eigen state of $\hat{S}_{1z}$. Let's now perform the measurement the Authors proposed, for example, of the spin of the first particle. According to standard quantum mechanics, that's to measure the eigenvalues of the operator $\hat{S}_{1z}$. In the past six decades discussions, the following commutation relation has never been mentioned,

$$\begin{aligned}
[\hat{S}_{1z}, \hat{S}^2] &= [\hat{S}_{1z}, (\hat{S}_1 + \hat{S}_2)^2] = [\hat{S}_{1z}, 2\hat{S}_1 \cdot \hat{S}_2] \\
&= [\hat{S}_{1z}, 2\hat{S}_{1x}\hat{S}_{2x}] + [\hat{S}_{1z}, 2\hat{S}_{1y}\hat{S}_{2y}] \\
&= 2i\hbar\hat{S}_{1y}\hat{S}_{2x} - 2i\hbar\hat{S}_{1x}\hat{S}_{2y} \\
&= -2i\hbar(\hat{S}_1 \times \hat{S}_2)_z \neq 0 \cdot
\end{aligned} \tag{3}$$

According to quantum measurement principle, the above equation means while one measures the operator $\hat{S}_{1z}$, the operator $\hat{S}^2$ must be disturbed and its eigen values no longer conserved. Therefore, after the measurement, whatever the spin of particle 1 is spin up or down, one cannot tell the spin of particle 2 since the eigen state of $\hat{S}^2$ would not be ψ_0 any longer. In short, in this example the above commutation relation denies the existence of long distance correlation of spin eigenvalues.

Conclusion and Discussions Though the Authors discussed the conservation of angular-momentum in the text around their equation 2, it is just on many-body effect, related to fluctuations of different components' value of angular-momentum etc. The effect of the final measurement actually ruining the initial state were not involved in the paper. The effect is indirectly due to the uncertainty relation caused by the above commutation relation (3).

In summary, we conclude that such kinds of prototypes cannot be the paradigms realizing the long distance quantum correlations as proposed.

[1] D. Bohm and Y. Aharonov Phys. Rev. 108, 1070(1957).
[2] A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. 47, 777 (1935).