SOLVING TRIANGLES ALGEBRAICALLY

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Abstract. Quaterns are a new measure of rotation. Since they are defined in terms of rectangular coordinates, all of the analogue trigonometric functions become algebraic rather than transcendental. Rotations, angle sums and differences, vector sums, cross and dot products, etc., all become algebraic. Triangles can be solved algebraically. Computer algorithms use truncated infinite sums for the transcendental calculations of these quantities. If rotations were expressed in quaterns, these calculations would be simplified by a few orders of magnitude. This would have the potential to greatly reduce computing time. The archaic Greek letter koppa is used to represent rotations in quaterns, rather than the traditional Greek letter theta. Because calculations utilizing quaterns are algebraic, simple rotation in the first two quadrants can be done "by hand" using "pen and paper." Using the approximate methods outlined towards the end of the paper, triangles may be approximately solved with an error of less than 3% using algebra and a few simple formulas.

1. Introduction

This hypertext links to a previous paper [1] entitled Rotation without imaginary numbers, transcendental functions, or infinite sums. This previous paper defined quaterns along with their analogue trig functions. The archaic Greek character koppa ϟ is used to represent rotation in quaterns. This current paper will demonstrate the ideas presented in the previous paper by using the ϟ analogue trig and inverse-trig functions to solve triangles. First, exact definitions and solutions will be presented, providing links to calculators that use quatern rotation. Then, a triangle will be approximately solved using simple approximate quatern formulas that a person might memorize.

2. Defining rotation in quaterns

2.1. **The universal definition in all four quadrants.** Assume the coordinates (x, y) of a point that is anywhere on the cartesian plane. Assume the origin is the vertex of the angle, and that the point (x, y) is at the terminus of the angle's terminal side. Equation 1 is the function $ϟ(x, y)$ that defines the quatern measure of the positive (counter-clockwise) angle between point (x, y) and the positive x axis. The distance from the origin to the point (x, y) is r. $r = \sqrt{x^2 + y^2}$.

$$ϟ = \left(\frac{1}{\sqrt{x^2+y^2}}\right)\left(\frac{x}{|x|}y - \frac{y}{|y|}x\right) + \left|\frac{4y}{|y|} - \frac{x}{|x|}\right| - \frac{2y}{|y|} \tag{1}$$

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2.2. **Simpler version for solving triangles.** Equation 1 is simpler than it looks. Note that in quadrants I and II, it reduces to the equations in table 1, which are the only versions needed to solve triangles.

Acute rotations (Quadrant I)	**Obtuse rotations (Quadrant II)**
$ϟ = \left(\frac{y-x}{r}\right) + 1$	$ϟ = \left(\frac{-y-x}{r}\right) + 3$

Table 1. Defining rotation in quaterns with $ϟ$ in quadrants I and II

When $r = 1$, then the angle terminates on the unit circle. This means that the x coordinate is equal to the analogue $cosine$ of $ϟ$, and the y coordinate is equal to the analogue $sine$ of $ϟ$. The definitions of the the analogue $sine$ and $cosine$ functions are given in section 3. $Y_{ϟ}(ϟ)$ symbolizes the analogue $sine$ function, and $X_{ϟ}(ϟ)$ symbolizes the analogue $cosine$ function. On the unit circle, the point (x, y) is therefore equivalent to $(X_{ϟ}(ϟ), Y_{ϟ}(ϟ))$ These facts will help to define quatern addition and subtraction, which are explained in subsection 4.

3. Exact analogue trigonometric functions for the first two quadrants

Table 2 lists the $ϟ$ analogue $Y_{ϟ}(ϟ)$ ($sine$) and $X_{ϟ}(ϟ)$ ($cosine$) functions for use with acute or obtuse angles respectively. The online calculator is entitled Triangles [2].

Acute angles	**Obtuse angles (Quadrant II)**
$Y_{Aϟ}(ϟ) = \frac{1}{2}\left(\sqrt{-ϟ^2 + 2ϟ + 1} + ϟ - 1\right)$	$Y_{Oϟ}(ϟ) = \frac{1}{2}\left(\sqrt{-ϟ^2 + 6ϟ - 7} - ϟ + 3\right)$
$X_{Aϟ}(ϟ) = \frac{1}{2}\left(\sqrt{-ϟ^2 + 2ϟ + 1} - ϟ + 1\right)$	$X_{Oϟ}(ϟ) = \frac{1}{2}\left(-\sqrt{-ϟ^2 + 6ϟ - 7} - ϟ + 3\right)$

Table 2. Analogue trig functions

Table 3 lists the $ϟ$ analogue $ϟ(Y_{ϟ})$ ($arcsine$) and $ϟ(X_{ϟ})$ ($arccosine$) functions to find acute or obtuse angles. In this notation $Y_{ϟ}$, for example, would mean "trigonometric $sine$ ratio", and $X_{ϟ}$ would mean "trigonometric $cosine$ ratio".

There are universal versions of these analogue trigonometric functions and inverse functions. The universal version of the $sine$ and $cosine$ functions are available at the Desmos site called Rotation in Quaterns [3]. For inverse trig functions there are two options. First, if one has x and y coordinates, one can simply use the definition of $ϟ$ given in equation 1 to find any quatern measure. If one is solving a triangle without knowing any coordinates, then one can instead use the $sine$ and $cosine$ ratios as inputs for equation 1 by just letting r $\left(\text{or, in other words } \sqrt{x^2 + y^2}\right)$ be equal to 1. The output will be the quatern rotation, $ϟ$. Doing this in equation 1 works in all four quadrants. Table 3 is simply the adaptation of equation 1 to just the first two quadrants. One can enter these

Acute angles	**Obtuse angles (Quadrant II)**
$\text{Ϟ}(Y_{A\text{ϟ}}) = Y_{\text{ϟ}} - \sqrt{1 - Y_{\text{ϟ}}^2} + 1$	$\text{Ϟ}_O(Y_{O\text{ϟ}}) = \sqrt{1 - Y_{\text{ϟ}}^2} - Y_{\text{ϟ}} + 3$
$\text{Ϟ}(X_{A\text{ϟ}}) = \sqrt{1 - X_{\text{ϟ}}^2} - X_{\text{ϟ}} + 1$	$\text{Ϟ}_O(X_{O\text{ϟ}}) = -\sqrt{1 - X_{\text{ϟ}}^2} - X_{\text{ϟ}} + 3$

TABLE 3. Analogue inverse-trig functions

$sine$ and $cosine$ ratios into the graphing calculator mentioned above in order to do these calculations quickly and easily.

4. Adding and subtracting rotations measured in quaterns

Angles in degrees or radians can be simply added or subtracted. **Note that Quaterns can be added just like traditional angle measure if one is satisfied with an approximate result.** The algebraic equations below demonstrate how to add or subtract quaterns exactly. They are based upon the analogue trigonometric angle addition and subtraction formulas for $sine$ and $cosine$ of Ϟ. The symbol for quatern addition will be $\ddagger$, and quatern subtraction will be indicated with $\rightharpoondown$. Let us first consider the sum: $\text{Ϟ}_3 = \text{Ϟ}_1 \ddagger \text{Ϟ}_2$. Equation 2 is used to find the x_3 coordinate for Ϟ_3 on the unit circle. The equations in this section are valid in all four quadrants:

$$x_3 = X_{\text{ϟ}}(\text{Ϟ}_3) = X_{\text{ϟ}}(\text{Ϟ}_1)X_{\text{ϟ}}(\text{Ϟ}_2) - Y_{\text{ϟ}}(\text{Ϟ}_1)Y_{\text{ϟ}}(\text{Ϟ}_2) \tag{2}$$

Then, equation 3 is used to find the y_3 coordinate for Ϟ_3 on the unit circle:

$$y_3 = Y_{\text{ϟ}}(\text{Ϟ}_3) = Y_{\text{ϟ}}(\text{Ϟ}_1)X_{\text{ϟ}}(\text{Ϟ}_2) + X_{\text{ϟ}}(\text{Ϟ}_1)Y_{\text{ϟ}}(\text{Ϟ}_2) \tag{3}$$

Then the coordinates of point (x_3, y_3) (which is on the unit circle, making $r = 1$) are plugged into equation 1 to yield Ϟ_3, as shown in equation 4

$$\text{Ϟ}_3 = \text{Ϟ}_1 \ddagger \text{Ϟ}_2 = \text{Ϟ}(x_3, y_3) \tag{4}$$

Subtraction follows a similar pattern using analogue trigonometric angle subtraction. $\text{Ϟ}_4 = \text{Ϟ}_1 \rightharpoondown \text{Ϟ}_2$. Equation 5 is used to find the x_4 coordinate for Ϟ_4 on the unit circle:

$$x_4 = X_{\text{ϟ}}(\text{Ϟ}_4) = X_{\text{ϟ}}(\text{Ϟ}_1)X_{\text{ϟ}}(\text{Ϟ}_2) + Y_{\text{ϟ}}(\text{Ϟ}_1)Y_{\text{ϟ}}(\text{Ϟ}_2) \tag{5}$$

Equation 6 is used to find the y_4 coordinate for Ϟ_4 on the unit circle:

$$y_4 = Y_{\text{ϟ}}(\text{Ϟ}_4) = Y_{\text{ϟ}}(\text{Ϟ}_1)X_{\text{ϟ}}(\text{Ϟ}_2) - X_{\text{ϟ}}(\text{Ϟ}_1)Y_{\text{ϟ}}(\text{Ϟ}_2) \tag{6}$$

Then the coordinates of point (x_4, y_4) (which is on the unit circle, making $r = 1$) are plugged into equation 1 to yield Ϟ_4, as shown in equation 7

$$\text{Ϟ}_4 = \text{Ϟ}_1 \rightharpoondown \text{Ϟ}_2 = \text{Ϟ}(x_4, y_4) \tag{7}$$

These calculations can be done very easily at the Desmos graphing calculator site entitled: Add or subtract quaterns [4].

5. Solving an ambiguous case triangle exactly

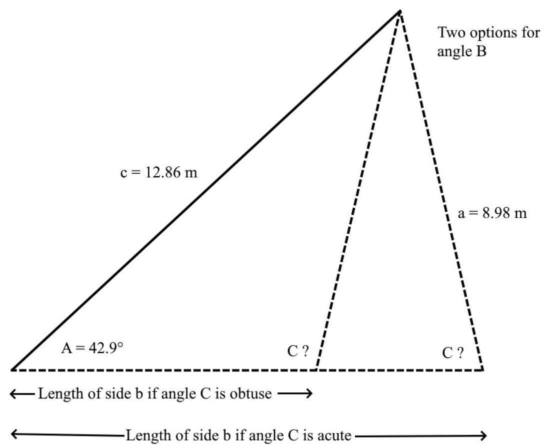

Figure 1. Ambiguous case triangle

Solving triangles has a long history. Van Brummelen [5] recounts the history of trigonometry in his book *Heavenly mathematics: The forgotten art of spherical trigonometry*. This page on the website Math is fun [6] summarizes the standard way trigonometry is used to solve triangles. This section presents a new approach to an old problem.

Figure 1 presents an ambiguous case triangle – Angle-Side-Side. The first step would be to exactly convert angle $A = 42.9°$ to quaterns. The graphing calculator site From Theta to Koppa [7] will convert degrees into quaterns. The accuracy of the conversion is dependent upon the value of k entered at this site. For our purposes a value of 1,000 for K will be more than sufficient. Using this site we find that angle $A = A = 42.9° \approx .9482$ quaterns. Now we can use the acute version of the $sine$ function from table 2 to compute $Y_ϟ(A)$ in equation

$$Y_ϟ(A) = \frac{1}{2}\left(\sqrt{-(.9482)^2 + 2(.9482) + 1} + (.9482) - 1\right) = .6807 \tag{8}$$

Note that all of the trigonometric functions and inverse functions for the first two quadrants may be performed easily at the site entitled triangles [2]. The next step is to use the quatern analogue law of sines in the traditional way. Doing this, we set up equation 9.

5.1. **The acute option.**

$$Y_ϟ(C) = \frac{cY_ϟ(A)}{a} = \frac{(12.86)(.6807)}{8.98} = .9748 \tag{9}$$

Figure 1 tells us that there are two different options for angle C. One of them is acute, and one of them is obtuse. We will begin with the acute option. We select the acute $arcsine$ option from table 3. This calculation (and any other inverse-trig calculation) may also be done at the site mentioned above, and it is shown in equation

$$\Koppa(Y_{\koppa})(.9748) = (.9748) - \sqrt{1-(.9748)^2} + 1 = 1.752 \tag{10}$$

We must now use the rules of quatern angle addition demonstrated in equations 2 and 3 to find the sum of angle A and the acute option for angle C. This is shown in equations 11 and 12.

$$x_{\ddagger} = X_{\koppa}(\Koppa_{\ddagger}) = X_{\koppa}(.9482)X_{\koppa}(1.752) - Y_{\koppa}(.9482)Y_{\koppa}(1.752) = -.5004 \tag{11}$$

Then, equation 3 is used to find the y_3 coordinate for $\Koppa_3$ on the unit circle:

$$y_{\ddagger} = Y_{\koppa}(\Koppa_{\ddagger}) = Y_{\koppa}(.9482)X_{\koppa}(1.752) + X_{\koppa}(.9482)Y_{\koppa}(1.752) = .8658 \tag{12}$$

Using these values in equation 1 yields a value of 2.635 quaterns. Now for the exception that proves the rule. If angles are added according to quatern angle addition rules, then the angles of every triangle in the plane must add up to 4 quaterns, which is the exact equivalent of $180°$. Since we have found that the sum of angles A and C is 2.635, might we assume that in this case $4 - 2.635 = 4 \rightharpoondown 2.635 = 1.365$? In this case the answer is yes. If one has found the sum of two angles in a triangle through the proper rules of quatern angle addition, then it logically follows that the remaining angle must be $4 - (A \ddagger C) = 4 \rightharpoondown (A \ddagger C)$. So angle $B = 1.365$. Now that we know all three angles, we find the remaining side by setting up the analogue law of sines in equation 13. Note that we can use the site mentioned previously to find the quantity $Y_{\koppa}(B) = .8656$.

$$b = \frac{aY_{\koppa}(B)}{Y_{\koppa}(A)} = \frac{(8.98)(.8656)}{.6807} = 11.42 \tag{13}$$

The solution to the acute option is angle $B = 1.365$, angle $C = 1.752$, and side $b = 11.42$. We can use the calculator site From Koppa to Theta [8] to find that angle $B = 59.96°$ and angle $C = 77.12°$ in traditional measure. This is also the result if the acute option is solved using traditional trigonometry.

6. The obtuse option

We begin with the knowledge that the measure of angle A in quaterns is $.9482$, and $S_{\koppa}(A) = .6807$. We also know that $Y_{\koppa}(C) = .9749$, because it will be the same whether angle C is acute or obtuse. This time, we will use the obtuse analogue $arcsine$ function from table 3 to find angle C. This is done in equation 14 . Note that all calculations can be done at the websites mentioned previously.

$$\Koppa_O(Y_{\koppa}) = \sqrt{1-(.9749)^2} - (.9749) + 3 = 2.248 \tag{14}$$

We now use the quatern addition rule illustrated in equations 11 and 12 to find the quantity $A \ddagger C$. This is done in equations 15 and 16.

$$x_{\ddagger} = X_{ϟ}(ɀ_B) = X_{ϟ}(.9482)X_{ϟ}(2.248) - Y_{ϟ}(.9482)Y_{ϟ}(2.248) = -.8269 \tag{15}$$

$$y_{\ddagger} = Y_{ϟ}(ɀ_B) = Y_{ϟ}(.9482)X_{ϟ}(2.248) + X_{ϟ}(.9482)Y_{ϟ}(2.248) = .5624 \tag{16}$$

These values are then used in equation 1 to find that $A \ddagger C = 3.264$. $\therefore B = 4 - (3.264) = .736$ Duplicating similar work done previously, we find that $Y_{ϟ}(B) = Y_{ϟ}(.736) = .5627 \therefore$

$$b = \frac{aY_{ϟ}(B)}{Y_{ϟ}(A)} = \frac{(8.98)(.5627)}{.6807} = 7.42 \tag{17}$$

The solution to the obtuse option is angle $B = .736$, angle $C = 2.248$, and side $b = 7.42$. Using the same calculator site, we find that angle $B = 34.24°$ and angle $C = 102.9°$ in traditional measure. This is also the result if the obtuse option is solved using traditional trigonometry. The next section will show abbreviated, approximate methods to solve a triangle algebraically using simple formulas that might be memorized.

7. Approximately solving a triangle

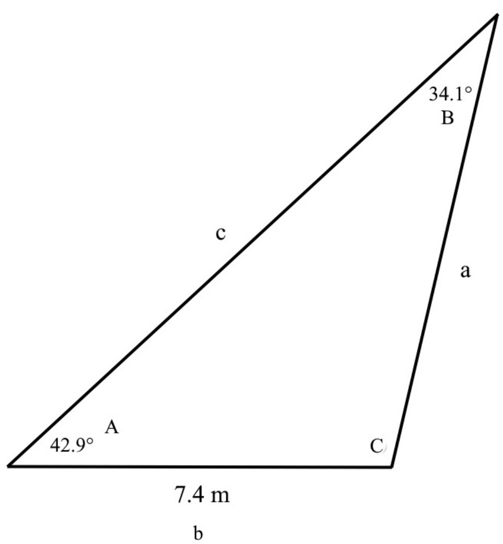

Figure 2. Triangle example 1

7.1. **Given triangle.**

7.2. **Approx. conversions between traditional and quatern angle measure.** The formulas in this subsection allow approximate conversions between traditional angle measure and quaterns. These conversions are all accurate to within 3%.

The formulas in table 4 will be used to approximately convert back and forth between degrees and quaterns. These same approximations can also be done at the site Approx convert to quaterns [9]:

$\theta \leq \frac{1}{2}^{\circ}$	$\frac{1}{2}^{\circ} < \theta < 30^{\circ}$	$30^{\circ} \leq \theta \leq 180^{\circ}$
$\text{ϟ} \approx (.0175)\theta$	$\text{ϟ} \approx (.00012)\theta^2 + (.0175)\theta$	$\text{ϟ} \approx \left(\frac{1}{45}\right)\theta$
$\text{ϟ} \leq .01$	$.01 < \text{ϟ} < \frac{2}{3}$	$\frac{2}{3} \leq \text{ϟ} \leq 4$
$\theta \approx \left(\frac{180}{\pi}\right)\text{ϟ}$	$\theta \approx \left(\frac{180}{\pi}\right)\left(-\frac{1}{5}\text{ϟ}^2 + .95\text{ϟ}\right)$	$\theta \approx 45\text{ϟ}$

Table 4. Approximate conversions for degrees and quaterns

The formulas in table 5 will be used to approximately convert back and forth between radians and quaterns.

$\theta \leq (.01)$	$.01 < \theta < \frac{\pi}{6}$	$\frac{\pi}{6} \leq \theta \leq \pi$
$\text{ϟ} \approx \theta$	$\text{ϟ} \approx \left(\frac{2}{5}\right)\theta^2 + \theta$	$\text{ϟ} \approx \left(\frac{4}{\pi}\right)\theta$
$\text{ϟ} \leq .01$	$.01 < \text{ϟ} < \frac{2}{3}$	$\frac{2}{3} \leq \text{ϟ} \leq 4$
$\theta \approx \text{ϟ}$	$\theta \approx \left(-\frac{1}{5}\text{ϟ}^2 + .95\text{ϟ}\right)$	$\theta \approx \left(\frac{\pi}{4}\right)\text{ϟ}$

Table 5. Approximate conversions for radians and quaterns

7.3. **Approximate solution of the given triangle.**

7.3.1. *Converting the given angles into quaterns.* We will use 4 significant figures in our calculations, and then round our final answers to 3 significant figures.

$$\angle A = 42.9^{\circ} \quad \frac{42.9^{\circ}}{45^{\circ}} \approx .9533 \text{ quaterns} \tag{18}$$

$$\angle B = 34.1^{\circ} \quad \frac{34.1^{\circ}}{45^{\circ}} \approx .7578 \text{ quaterns} \tag{19}$$

Strictly speaking, quaterns can not be added if one is looking for exact solutions. But if one is satisfied with an approximate solution to a triangle, the angles may be added and they should sum to approximately 4 quaterns, which is exactly $180°$. This means that angle $C \approx 4-(A+B) = 4-(.9533+.7578) = 2.289$ quaterns.

7.3.2. *Using the analogue Law of Sines to solve for side A.*

$$a = \frac{bY_{ϟ}(A)}{Y_{ϟ}(B)} \tag{20}$$

$$Y_{ϟ}(A) = \frac{1}{2}\left(\sqrt{-(.9533)^2 + 2(.9533) + 1} + (.9533) - 1(.9533)\right) = .6834 \tag{21}$$

$$Y_{ϟ}(B) = \frac{1}{2}\left(\sqrt{-(.7578)^2 + 2(.7578) + 1} + (.7578) - 1(.7578)\right) = .5756 \tag{22}$$

Therefore the approximation for side a would be:

$$a = \frac{bY_{ϟ}(A)}{Y_{ϟ}(B)} = \frac{(7.4)(.6834)}{.5756} = 8.786 \tag{23}$$

Rounding our result for side a to three significant figures gives a length of 8.79. The true value rounded to three significant figures is 8.98. This is an error of approximately 2%.

7.3.3. *Using the quatern analogue Law of Cosines to solve for side C.* The quatern analogue Law of Cosines (for an obtuse angle) is:

$$c = \sqrt{a^2 + b^2 - 2abX_{Oϟ}} \tag{24}$$

$$X_{Oϟ}(2.289) = \frac{1}{2}\left(-\sqrt{-(2.289)^2 + 6(2.289) - 7} - (2.289) + 3\right) = -.2557 \tag{25}$$

$$c = \sqrt{8.786^2 + 7.4^2 - 2(8.786)(7.4)(-.2557)} = 12.85 \tag{26}$$

The true value for length c is 12.86. This is an error less than one part in a thousand.

8. Conclusion: The potential uses for quatern measure

8.1. **Graphics performance.** Programmers continue to seek better graphics performance, as shown by Rohlf [10]. Three dimensional rendering uses vectors and the rotation of vectors; calculating results depends upon trigonometry. Technology typically relies upon truncated infinite Taylor series [11]. Programmers are always open to new algorithms that might represent these functions more efficiently, as in Kumar [12].

Quaterns represent rotations and their related trig functions in an algebraic manner. Graphics representations can be entirely done in quaterns, rapidly giving algebraic exact answers, where the only truncation would be rounding. This would be very different, and much simpler than traditional approaches like the one mentioned by Cook [13]. Programs requiring large quantities of calculations involving position and rotation might be orders of magnitude faster using quaterns rather than traditional angle measure.

8.2. **physics simulations at the quantum level.** A circle drawn by a compass upon a piece of paper is not actually an ideal circle. If one were able to zoom in under a microscope, and then under an electron microscope, and then still further (in theory), one would discover that at the quantum level, everything is discrete. One might guess that at the smallest possible scale, even space and time are quantized. Lindemann and Weierstrass proved [14] that π was transcendental. This means that there is something unreal about a circle in the sense that the circumference of any real circle will forever be an approximation. Because traditional angle measure relies upon a transcendental relationship with the circle, there is something unreal about it.

This suggests that simulations of quantum processes might better be represented by quaterns, because it is a description of rotation that would represent discrete rational relationships very well.

8.3. **pencil and paper calculations.** It is true that a person might memorize the first few terms of appropriate Taylor series to obtain "pencil and paper" simple memorized formulas for the trig functions and their inverses.

But the quatern equivalents are intuitive, related to x and y coordinates, and their exact versions are potentially memorizable as well. Especially in the first quadrant.

$ϟ = \left(\frac{y-x}{r}\right) + 1$ is simple.

Please see the references to find the full written-out URL of each of the websites mentioned in this paper. The author provides a video demonstration of solving the ambiguous case triangle [15].

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