The Problem of Particle-Antiparticle in Particle Theory

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Abstract

The title of this workshop is: "What comes beyond standard models?". Standard models are based on standard Poincare invariant quantum theory (SQT). Here irreducible representations (IRs) of the Poincare algebra are such that in each IR, the energies are either ≥ 0 or ≤ 0. In the first case, IRs are associated with particles and in the second case — with antiparticles, while particles for which all additive quantum numbers (electric charge, baryon and lepton quantum numbers) equal zero are called neutral. However, SQT is a special degenerate case of finite quantum theory (FQT) in the formal limit $p \to \infty$ where p is a characteristic of a ring in FQT. In FQT, one IR of the symmetry algebra describes a particle and its antiparticle simultaneously, and there are no conservation laws of additive quantum numbers. One IR in FQT splits into two standard IRs with positive and negative energies as a result of symmetry breaking in the formal limit $p \to \infty$. The construction of FQT is one of the most fundamental (if not the most fundamental) problems of particle theory.

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1 Introduction: problems with the physical interpretation of the Dirac equation

Modern fundamental particle theories (QED, QCD and electroweak theory) are based on the concept of particle-antiparticle. Historically, this concept has arisen as a consequence of the fact that the Dirac equation has solutions with positive and negative energies. The solutions with positive energies are associated with particles, and the solutions with negative energies - with corresponding antiparticles. And when the positron was found, it was treated as a great success of the Dirac equation. Another great success is that in the approximation $(v/c)^2$ the Dirac equation reproduces the fine structure of the hydrogen atom with a very high accuracy.

However, now we know that there are problems with the physical interpretation of the Dirac equation. For example, in higher order approximations, the probabilistic interpretation of non-quantized Dirac spinors is lost because the coordinate description implies that they are described by representations induced from non-unitary representations of the Lorenz algebra. Moreover, this problem exists not only for the Dirac spinors but for any functions described by relativistic covariant equations (Klein-Gordon, Dirac, Rarita-Schwinger and others). As shown by Pauli [1] in the case of fields with an integer spin there is no invariant subspace where the spectrum of the charge operator has a definite sign while in the case of fields with a half-integer spin there is no invariant subspace where the spectrum of the energy operator has a definite sign. It is also known that the description of the electron in the external field by the Dirac spinor is not accurate (e.g., it does not take into account the Lamb shift).

Another fundamental problem in the interpretation of the Dirac equation is as follows. One of the key principles of quantum theory is the principle of superposition. This principle states that if ψ_1 and ψ_2 are possible states of a physical system then $c_1\psi_1 + c_2\psi_2$, when c_1 and c_2 are complex coefficients, also is a possible state. The Dirac equation is the linear equation, and, if $\psi_1(x)$ and $\psi_2(x)$ are solutions of the equation, then $c_1\psi_1(x) + c_2\psi_2(x)$ also is a solution, in agreement with the principle of superposition. In the spirit of the Dirac equation, there should be no separate particles the electron and the positron. It should be only one particle which can be called electron-positron such that electron states are the states of this particle with positive energies, positron states are the states of this particle with negative energies and the superposition of electron and positron states should not be prohibited. However, in view of charge conservation, baryon number conservation, and lepton numbers conservations, the superposition of a particle and its antiparticle is prohibited.

Modern particle theories are based on Poincare (relativistic) symmetry. In these theories, elementary particles are described by irreducible representations (IRs) of the Poincare algebra. Such IRs have a property that energies in them can be either strictly positive or strictly negative but there are no IRs where energies have different signs. The objects described by positive-energy IRs are called particles, and objects described by negative-energy IRs are called antiparticles, and energies of both, particles and antiparticles become positive after second quantization. In this situation, there are no elementary particles which are superpositions of a particle and its antiparticle, and as explained above, this is not in the spirit of the Dirac equation.

In particle theories, only quantized Dirac spinors $\psi(x)$ are used. Here, by analogy with non-quantized spinors, x is treated as a point in Minkowski space. However, $\psi(x)$ is an operator in the Fock space for an infinite number of particles. Each particle in the Fock space can be described by its own coordinates (in the approximation when the position operator exists — see e.g., [2]). **In view of this fact, the following natural question arises: why do we need an extra coordinate x which does not have any physical meaning because it does not belong to any particle and so is not measurable?** Moreover, I can ask the following seditious question: in quantum theory, do we need Minkowski space at all?

When there are many bodies, the impression may arise that they are in some space but this is only an impression. In fact a background space-time (e.g., Minkowski space) is only a mathematical concept needed in classical theory. For illustration, consider quantum electromagnetic theory. Here we deal with electrons, positrons and photons. In the approximation when the position operator exists, each particle can be described by its own coordinates. The coordinates of the background Minkowski space do not have a physical meaning because they do not refer to any particle and therefore are not measurable. However, in classical electrodynamics we do not consider electrons, positrons and photons. Here the concepts of the electric and magnetic fields $(\mathbf{E}(x), \mathbf{B}(x))$ have the meaning of the average contribution of all particles in the point x of Minkowski space.

This situation is analogous to that in statistical physics. Here we do not consider each particle separately but describe the average contribution of all particles by temperature, pressure etc. Those quantities have a physical meaning not for each separate particle but for ensembles of many particles.

A justification of the presence of x in quantized Dirac spinors $\psi(x)$ is that in quantum field theories (QFT) the Lagrangian density depends on the four-vector x, but this is only the integration parameter which is used in the intermediate stage. The goal of the theory is to construct the S-matrix, and when the theory is already constructed one can forget about Minkowski space because no physical quantity depends on x. This is in the spirit of the Heisenberg S-matrix program according to which in relativistic quantum theory it is possible to describe only transitions of states from the infinite past when $t \to -\infty$ to the distant future when $t \to \infty$.

The fact that the theory gives the S-matrix in the momentum representation does not mean that the coordinate description is excluded. In typical situations, the position operator in momentum representation exists not only in the nonrelativistic case but in the relativistic case as well. In the latter case, it is known, for example, as the Newton-Wigner position operator [3] or its modifications. However, as pointed out even in textbooks on quantum theory, the coordinate description of elementary particles can work only in some approximations. In particular, even in most favorable scenarios, for a massive particle with the mass m its coordinate cannot be measured with the accuracy better than the particle Compton wave length $\hbar/mc$.

2 Is Poincare symmetry the most general symmetry in particle theory?

The above discussion of the problems with Dirac spinors was based on the assumption that Poincare (relativistic) symmetry is the most general symmetry in particle theory, and Standard Model is based on this assumption. But suppose that I ask a question: why not to consider particle theory based on Galilei (nonrelativistic) symmetry? Probably, most physicists will immediately say that this question is silly because everybody knows that Poincare symmetry is more general (fundamental) than Galilei one and many facts in particle physics show that Galilei symmetry does not work here. But suppose that I am not a physicist, I do not know experimental data and I ask whether the fact that Poincare symmetry is more general than Galilei one follows only from mathematics? Is this question legitimate?

In his famous paper "Missed Opportunities" [4] Dyson explains that the fact that Poincare symmetry is more general than Galilei one follows from pure mathematical considerations. The Poincare group is more symmetric that the Galilei one: the former contains a formal parameter c (I even do not discuss its physical meaning), and the latter can be obtained from the former by a procedure called contraction when formally $c \to \infty$.

In view of this observation, I can ask whether Poincare symmetry is most general, maybe there are groups more symmetric that Poincare one such that the Poincare group can be obtained from these more symmetric groups by contraction? In his paper Dyson explains that indeed the de Sitter (dS) and anti-de Sitter (AdS) groups are more symmetric than Poincare one and the transition from the former to the latter is described by contraction when a parameter R (see below) goes to infinity. At the same time, since dS and AdS groups are semisimple, they have a maximum possible symmetry and cannot be obtained from more symmetric groups by contraction.

The paper [4] appeared in 1972, i.e., 50 years ago, and, in view of

Dyson's results, a question arises why the fundamental particle theories are still based on Poincare symmetry and not dS or AdS ones. The parameter R arises from particle theory but in the literature it is often interpreted as the radius of the universe. Probably, physicists believe that, since R is even much greater than sizes of stars, the dS and AdS symmetries can play an important role only in cosmology and there is no need to use them for describing elementary particles. I believe that this argument is not consistent because usually more general theories shed a new light on standard concepts, and my talk is a good illustration of this point.

In Sec. 3 I describe the concept of symmetry on quantum level. In Secs. 7 and 8 I consider the concept of particle-antiparticle for dS and AdS symmetries in standard quantum theory and in a quantum theory based on finite mathematics (FQT). Here I give a popular explanation why standard concepts of particle-antiparticle, electric charge and baryon number have only a limited meaning when the symmetry in FQT is broken to Poincare or standard AdS symmetries. Finally, Sec. 9 is discussion. I describe all physical quantities in units $c = \hbar = 1$.

3 Symmetry on quantum level

In the usual treatment of relativistic quantum theory, the approach to symmetry on quantum level follows. Since the Poincare group is the group of motions of Minkowski space, quantum states should be described by representations of this group. This implies that the representation generators commute according to the commutation relations of the Poincare group Lie algebra:

$$\begin{aligned} &[P^\mu, P^\nu] = 0, \quad [P^\mu, M^{\nu\rho}] = -i(\eta^{\mu\rho}P^\nu - \eta^{\mu\nu}P^\rho), \\ &[M^{\mu\nu}, M^{\rho\sigma}] = -i(\eta^{\mu\rho}M^{\nu\sigma} + \eta^{\nu\sigma}M^{\mu\rho} - \eta^{\mu\sigma}M^{\nu\rho} - \eta^{\nu\rho}M^{\mu\sigma}) \end{aligned} \quad (1)$$

where $\mu, \nu = 0, 1, 2, 3$, P^μ are the operators of the four-momentum, $M^{\mu\nu}$ are the operators of Lorentz angular momenta, and $\eta^{\mu\nu}$ is such that $\eta^{00} = -\eta^{11} = -\eta^{22} = -\eta^{33} = 1$ and $\eta^{\mu\nu} = 0$ if $\mu \neq \nu$. This approach is in the spirit of Klein's Erlangen program in mathematics.

However, as noted in Sec. 1 and discussed in detail in [2], in quantum theory, the concept of space-time background does not have a physical meaning. As argued in [2, 5], the approach should be the opposite. Each system is described by a set of linearly independent operators. By definition, the rules how they commute with each other define the symmetry algebra. In particular, *by definition*, Poincare symmetry on quantum level means that the operators commute according to Eq. (1). This definition does not involve Minkowski space at all. In particular, the fact that $\eta^{\mu\nu}$ coincides with the metric tensor in Minkowski space, does not imply that this space is involved. I am very grateful to Leonid Avksent'evich Kondratyuk for explaining me this definition during our collaboration.

By analogy with the definition of Poincare symmetry on quantum level, the definition of dS symmetry on quantum level should not involve the fact that the dS group is the group of motions of dS space. Instead, *the definition* is that the operators M^{ab} ($a, b = 0, 1, 2, 3, 4$, $M^{ab} = -M^{ba}$) describing the system under consideration satisfy the commutation relations *of the dS Lie algebra,* i.e.,

$$[M^{ab}, M^{cd}] = -i(\eta^{ac}M^{bd} + \eta^{bd}M^{ac} - \eta^{ad}M^{bc} - \eta^{bc}M^{ad}) \tag{2}$$

where η^{ab} is such that $\eta^{00} = -\eta^{11} = -\eta^{22} = -\eta^{33} = -\eta^{44} = 1$ and $\eta^{ab} = 0$ if $a \neq b$. The *definition* of AdS symmetry on quantum level is given by the same equations but $\eta^{44} = 1$.

The procedure of contraction from dS and AdS symmetries to Poincare one is defined as follows. If we *define* the operators P^ν as $P^\nu = M^{\nu 4}/R$ where R is a parameter with the dimension *length* then in the formal limit when $R \to \infty$, $M^{\nu 4} \to \infty$ but the quantities P^ν are finite, Eqs. (2) become Eqs. (1). This procedure is the same for the dS and AdS symmetries.

The above contraction is analogous to the contraction from Poincare symmetry to Galilei one, where the parameter of contraction is c. On quantum level, R and c are only the parameters describing the relations between Lie algebras of higher and lower symmetries. On classical level, the physical meaning of c is well-known, while R is the radius of the dS or AdS space. A detailed discussion of the both contractions is described in a vast literature, in particular, in [2] where it has been proposed the following

Definition: *Let theory A contain a finite nonzero parameter and theory B be obtained from theory A in the formal limit when the parameter goes to zero or infinity. Suppose that, with any desired accuracy, theory A can reproduce any result of theory B by choosing a value of the parameter. On the contrary, when the limit is already taken, one cannot return back to theory A, and theory B cannot reproduce all results of theory A. Then theory A is more general (fundamental) than theory B and theory B is a special degenerate case of theory A.*

As proved in [2], dS and AdS symmetries are more general (fundamental) than Poincare symmetry. The latter is a special degenerate case of the former in the formal limit $R \to \infty$. As noted above, in contrast to Dyson's approach based on Lie groups, our approach is based on Lie algebras. Then, as proved in [2], classical theory is a special degenerate case of quantum one in the formal limit $\hbar \to 0$, and nonrelativistic theory (NT) is a special degenerate case of relativistic one (RT) in the formal limit $c \to \infty$. In the literature the above facts are explained from physical considerations but, as shown in [2] they can be proved mathematically by using properties of Lie algebras. In particular, since, from mathematical point of view, de Sitter symmetry is more general (fundamental) than Poincare one, there should exist physical phenomena which can be explained by de Sitter symmetries but cannot be explained by Poincare symmetry. Below I will discuss such phenomena.

4 Problems with describing nature by classical mathematics

Standard quantum theory (SQT) is based on classical mathematics involving limits, infinitesimals, continuity etc. Mathematical education at physics departments develops a belief that classical mathematics is the most fundamental mathematics, while, for example, discrete and finite mathematics is something inferior what is used only in special applications. And many mathematicians have a similar belief.

Historically it happened so because more than 300 years ago Newton and Leibniz proposed the calculus of infinitesimals, and, since that time, a titanic work has been done on foundation of classical mathematics. This problem has not been solved till the present time, but for most physicists and many mathematicians the most important thing is not whether a rigorous foundation exists but that in many cases standard mathematics works with a very high accuracy.

The idea of infinitesimals was in the spirit of existed experience that any macroscopic object can be divided into arbitrarily large number of arbitrarily small parts, and, even in the 19th century, people did not know about atoms and elementary particles. But now we know that when we reach the level of atoms and elementary particles, standard division loses its usual meaning and in nature there are no arbitrarily small parts and no continuity.

For example, typical energies of electrons in modern accelerators are millions of times greater than the electron rest energy, and such electrons experience many collisions with different particles. If it were possible to break the electron into parts, then it would have been noticed long ago.

Another example is that if we draw a line on a sheet of paper and look at this line by a microscope then we will see that the line is strongly discontinuous because it consists of atoms. That is why standard geometry (the concepts of continuous lines and surfaces) can work well only in the approximation when sizes of atoms are neglected, standard macroscopic theory can work well only in this approximation etc.

Of course, when we consider water in the ocean and describe it by differential equations of hydrodynamics, this works well but this is only an approximation since water consists of atoms. However, it seems unnatural that even quantum theory is based on continuous mathematics. Even the name "quantum theory" reflects a belief that nature is quantized, i.e., discrete, and this name has arisen because in quantum theory some quantities have discrete spectrum (i.e., the spectrum of the angular momentum operator, the energy spectrum of the hydrogen atom etc.). But this discrete spectrum has appeared in the framework of classical mathematics.

I asked physicists and mathematicians whether, in their opinion, the indivisibility of the electron shows that in nature there are no infinitesimals, and standard division does not work always. Some mathematicians say that sooner or later the electron will be divided. On the other hand, as a rule, physicists agree that the electron is indivisible and in nature there are no infinitesimals. They say that, for example, dx/dt should be understood as $\Delta x/\Delta t$ where Δx and Δt are small but not infinitesimal. I ask them: but you work with dx/dt, not $\Delta x/\Delta t$. They reply that since mathematics with derivatives works well then there is no need to philosophize and develop something else (and they are not familiar with finite mathematics).

One of the key problems of modern quantum theory is the problem of infinities: the theory gives divergent expressions for the S-matrix in perturbation theory. In renormalized theories, the divergencies are eliminated by the renormalization procedure where finite observable quantities are formally expressed as products of singularities. Although this procedure is not well substantiated mathematically, in some cases it results in excellent agreement with experiment. Probably the most famous case is that the results for the electron and muon magnetic moments obtained at the end of the 40th agree with experiment at least with the accuracy of eight decimal digits (see, however, a discussion in [6]). In view of this and other successes of quantum theory, most physicists believe that agreement with the data is much more important than the rigorous mathematical substantiation.

At the same time, in nonrenormalized theories, infinities cannot be eliminated by the renormalization procedure, and this a great obstacle for constructing quantum gravity based on quantum field theory (QFT). As the famous physicist and the Nobel Prize laureate Steven Weinberg writes in his book [7]: "*Disappointingly this problem appeared with even greater severity in the early days of quantum theory, and although greatly ameliorated by subsequent improvements in the theory, it remains with us to the present day*". The title of Weinberg's paper [8] is "Living with infinities".

In view of efforts to describe discrete nature by continuous mathematics, my friend told me the following joke: "A group of monkeys is ordered to reach the Moon. For solving this problem each monkey climbs a tree. The monkey who has reached the highest point believes that he has made the greatest progress and is closer to the goal than the other monkeys". Is it reasonable to treat this joke as a hint on some aspects of the modern science? Indeed, people invented continuity and infinitesimals which do not exist in nature, created problems for themselves and now apply titanic efforts for solving those problems.

The founders of quantum theory and scientists who essentially contributed to it were highly educated. But they used only classical mathematics, and even now finite mathematics is not a part of standard education for physicists. The development of quantum theory has shown that the theory contains anomalies and divergences. Most physicists considering those problems, worked in the framework of classical mathematics and did not acknowledge that they arise just because this mathematics was used.

5 Quantum theory based on finite mathematics

Several well-known physicists, including the Nobel Prize laureates Gross, Nambu and Schwinger, discussed approaches when quantum theory involves finite mathematics. While classical mathematics starts from the ring of integers $Z = (-\infty, ... - 1, 0, 1, ...\infty)$, finite mathematics rejects infinities from the beginning. It starts from the ring $R_p = (0, 1, 2, ...p - 1)$ where addition, subtraction and multiplication are performed as usual but modulo p, and p is called the characteristic of the ring. In number theory, p is the usual notation for the characteristic and this has nothing to do with the fact that in particle theory the notation p is used for denoting a particle four-momentum.

Since the operations in R_p are modulo p, then, if p is odd, one can say that R_p contains the numbers $(-(p-1)/2, ... - 1, 0, 1, ...(p-1)/2)$. Then, if elements of Z are depicted as integer points on the x axis of the xy plane, the elements of R_p can be depicted as points of the circle in Figure 1 and analogously if p is even.

The analogy between R_p and the circle follows from the following observations. If we take an element of R_p and successively add 1 to it, then after p steps we will return to the original element because addition in R_p is modulo p. This is analogous to the fact that if we are moving along the circle in the same direction then, sooner or later, we will arrive to the initial point.

Figure 1 is natural from the following historical analogy. For many years people believed that the Earth was flat and infinite, and only after a long period of time they realized that it was finite and curved. It is difficult

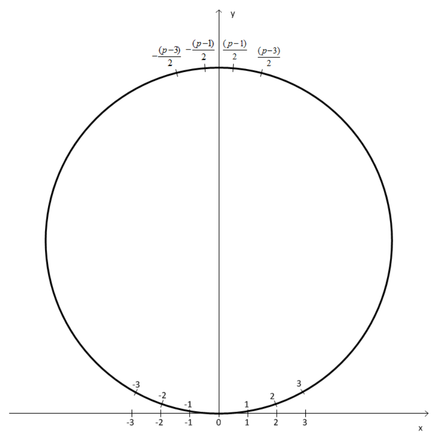

Figure 1: Relation between R_p and Z

to notice the curvature when we deal only with distances much less than the radius of the curvature. Analogously, when we deal with numbers the modulus of which is much less than p, the results are the same in Z and R_p, i.e., we do not notice the "curvature" of R_p. This "curvature" is manifested only when we deal with numbers the modulus of which is comparable to p.

As proved in my book [2], as follows from **Definition**, classical mathematics (involving the concepts of limits, infinitesimals, continuity etc.) is a special degenerate case of finite mathematics in the formal limit when the characteristic p of the ring or field in the latter goes to infinity. Therefore standard dS and AdS symmetries over the field of complex numbers can be generalized to dS and AdS symmetries over a finite ring or field of characteristic p.

We use the abbreviation FQT (finite quantum theory) to denote quantum theory over the ring or field of characteristic p. Since mathematically FQT is more general (fundamental) than SQT, there are physical phenomena which can be explain only by FQT but cannot be explained by SQT. An example of such a phenomenon is discussed in Sec. 8, for other examples —see [2].

6 Particles and antiparticles in Poincare invariant theories

As noted in Sec. 1, solutions of the Dirac equation with positive energies are associated with particles and solution with negative energies — with antiparticles. It has been noted that there are problems with the interpretation of the non-quantized Dirac spinor $\psi(x)$ and for the quantized Dirac spinor the problem is that the quantity x does not have the physical meaning. Elementary particles in Poincare invariant theory are described by IRs of the Poincare algebra by selfadjoint operators. Therefore a problem arises whether the concept of particle-antiparticle can be defined proceeding only from such IRs without mentioning the nonphysical parameter x.

Let p^ν be the four-momentum of a particle in Poincare invariant theory. Define $p^2 = p^\nu p_\nu$, where a sum over repeated indices is assumed. Then for usual particles $p^2 \geq 0$ while for tachyons $p^2 < 0$. The existence of tachyons is a problem, and we will consider only usual particles. Then the mass of the particle can be defined as a nonnegative number m such that $m^2 = p^2$.

The energy E of a particle with the momentum $\mathbf{p}$ and mass m equals $\pm(m^2+\mathbf{p}^2)^{1/2}$. The choice of the sign of the square root is only the matter of convention but not the matter of principle. Depending on this sign, there are IRs where energies can be only either positive or negative while the probability to have zero energy is zero.

When we consider a system consisting of particles and antiparticles then the energy sign of both, particles and antiparticles should be the same. Indeed, consider, for example a system of two particles with the same mass m and let the momenta $\mathbf{p}_1$ and $\mathbf{p}_2$ be such that the total momentum $\mathbf{p}_1+\mathbf{p}_2$ equals zero. Then, if the energy of particle 1 is positive, and the energy of particle 2 is negative then the total four-momentum of the system would be zero what contradicts experimental data. By convention, the energy sign of all particles and antiparticles in question is chosen to be positive. For this purpose, the procedure of second quantization is defined such that after the second quantization the energies of antiparticles become positive. Then the mass of any particle is the minimum value of its energy in the case when the momentum equals zero.

Suppose now that we have two particles such that particle 1 has the mass m_1, spin s_1 and is characterized by some additional quantum numbers (e.g., electric charge, baryon quantum number etc.), and particle 2 has the mass m_2, spin $s_2 = s_1$ and all additional quantum numbers characterizing particle 2 equal the corresponding additional quantum numbers for particle 1 with the opposite sign. A question arises when particle 2 can be treated as an antiparticle for particle 1. Is it necessary that m_1 should be exactly equal m_2 or they can slightly differ each other? In particular, can we guarantee that the mass of the positron exactly equals the mass of the electron, the mass of the proton exactly equals the mass of the antiproton etc.?

If particle 2 (for some reasons) is treated as an antiparticle for particle 1, and the particles are considered only on the level of IRs, then the relation between m_1 and m_2 is fully arbitrary. However, in QFT, $m_1 = m_2$ because IRs for a particle and its antiparticle are combined together in the framework of a local field. For example, the Dirac spinor combines together two IRs for the electron and positron. However, as noted in Sec. 1, this procedure encounters the following problems:

- The quantity x in quantized fields $\psi(x)$ does not have a physical meaning.
- There is no probabilistic interpretation of $\psi(x)$ because it is described by a non-unitary representation of the Poincare algebra.
- Although $\psi(x)$ satisfies a linear equation, a superposition of solutions with positive and negative energies is prohibited.

A usual statement in the literature is that in QFT the fact that $m_1 = m_2$ follows from the CPT theorem which is a consequence of locality since *we construct* local covariant fields from a particle and its antiparticle with equal masses. However, as noted in Sec. 1, since on quantum level there are problems with the physical interpretation of covariant fields and the quantity x, the very meaning of locality on quantum level is problematic.

Also, a question arises what happens if locality is only an approximation: in that case the equality of masses is exact or approximate? Consider a simple model when electromagnetic and weak interactions are absent. Then the fact that the proton and the neutron have equal masses has nothing to do with locality; it is only a consequence of the fact that the proton and the neutron belong to the same isotopic multiplet. In other words, they are simply different states of the same object—the nucleon.

Since the concept of locality is not formulated in terms of selfadjoint operators, this concept does not have a clear physical meaning, and this fact has been pointed out even in known textbooks (see e.g. [9]). Therefore, QFT does not give a physical proof that $m_1 = m_2$. Note also that in Poincare invariant quantum theories there can exist elementary particles for which all additional quantum numbers are zero. Such particles are called neutral because they coincide with their antiparticles.

In Secs. 7 and 8 I consider how the concept of particle-antiparticle in treated for dS and AdS invariant theories, respectively.

7 Particles and antiparticles in dS invariant theories

The descriptions of elementary particles in the dS and AdS cases are considerably different. In the former case all the operators $M^{\nu 4}$ ($\nu = 0, 1, 2, 3$) are on equal footing. Therefore, M^{04} can be treated as the Poincare analog of the energy only in the approximation when R is rather large. In the general case, the sign of M^{04} cannot be used for the classification of IRs.

In his book [11] Mensky describes the implementation of dS IRs when the representation space is the three-dimensional unit sphere in the four-dimensional space. In this implementation, there exist one-to-one relations between the northern hemisphere and the upper Lorentz hyperboloid with positive Poincare energies and between the southern hemisphere and the lower Lorentz hyperboloid with negative Poincare energies, while points on the equator correspond to infinite Poincare energies. However, the operators of IRs are not singular in the vicinity of the equator and, since the equator has measure zero, the properties of wave functions on the equator are not important.

Since the number of states in dS IRs is twice as big as the number of states in IRs of the Poincare algebras, one might think that each IR of the dS algebra describes a particle and its antiparticle simultaneously. However, a detailed analysis in [2] shows that states described by dS IRs cannot be characterized as particles or antiparticles in the usual meaning.

For example, let us call states with the support of their wave functions on the northern hemisphere as particles and states with the support on the southern hemisphere as their antiparticles. Then states which are superpositions of a particle and its antiparticle obviously belong to the representation space under consideration, i.e., they are not prohibited.

As noted in Sec. 1, in the spirit of the Dirac equation, there should be no separate particles the electron and the positron. It should be only one particle which can be called electron-positron such that electron states are the states of this particle with positive energies, positron states are the states of this particle with negative energies and, as follows from the principle of superposition in quantum theory, the superposition of electron and positron states should not be prohibited. *However, since in standard particle theory, charge conservation is treated as more fundamental than the principle of superposition, the superposition of a particle and its antiparticle is prohibited.*

However, we see that in the dS case the situation is in the spirit of the Dirac equation: there are no independent particles and antiparticles, there are only objects described by IRs of the dS algebra, and, if states of each object with positive energies are called particle states and states with negative energies — antiparticle states, superpositions of such states are not prohibited. *Therefore, in the dS case, the principle of superposition is stronger than the electric charge conservation.* Note that the law of electric charge conservation comes from classical physics. The existing experimental data confirms that this law takes place. However, a problem arises whether those data describe all possible situations. We discuss this problem below.

As noted in Sec. 3, dS symmetry is more general than Poincare one, and the latter can be treated as a special degenerate case of the former in the formal limit $R \to \infty$. This means that, with any desired accuracy, any phenomenon described in the framework of Poincare symmetry can be also described in the framework of dS symmetry if R is chosen to be sufficiently large, but there also exist phenomena for explanation of which it is important that R is finite and not infinitely large (see [2]).

As shown in [2, 10], dS symmetry is broken in the formal limit $R \to \infty$ because one IR of the dS algebra splits into two IRs of the Poincare algebra with positive and negative energies and with equal masses. Therefore, the fact that the masses of particles and their corresponding antiparticles are equal to each other, can be explained as a consequence of the fact that observable properties of elementary particles can be described not by exact Poincare symmetry but by dS symmetry with a very large (but finite) value of R. In contrast to QFT, for combining a particle and its antiparticle into one object, there in no need to assume locality and involve local field functions because a particle and its antiparticle already belong to the same IR of the dS algebra (compare with the above remark about the isotopic symmetry in the proton-neutron system).

The fact that dS symmetry is higher than Poincare one is clear even from the fact that, in the framework of the latter symmetry, it is not possible to describe states which are superpositions of states on the upper and lower hemispheres. Therefore, breaking the IR into two independent IRs defined on the northern and southern hemispheres obviously breaks the initial symmetry of the problem. This fact is in agreement with the Dyson observation (mentioned above) that dS group is more symmetric than Poincare one.

When $R \to \infty$, standard concepts of particle-antiparticle, electric charge and baryon and lepton quantum numbers are restored, i.e., in this limit, superpositions of particle and antiparticle become prohibited because now a particle and its antiparticle belong to different IRs. Therefore, those concepts arise as a result of symmetry breaking, i.e., they are not universal.

8 Particles and antiparticles in AdS invariant theories

In theories where the symmetry algebra is the AdS algebra, the structure of IRs is known (see e.g., [2, 12]). The operator M^{04} is the AdS analog of the energy operator. Let W be the Casimir operator $W = \frac{1}{2}\sum M^{ab}M_{ab}$ where a sum over repeated indices is assumed. As follows from the Schur lemma, the operator W has only one eigenvalue in every IR. By analogy with Poincare invariant theory, we will not consider AdS tachyons and then one can define the AdS mass μ such that $\mu \geq 0$ and μ^2 is the eigenvalue of the operator W.

As noted in Sec. 3, the procedure of contraction from the AdS algebra to the Poincare one involves the definition of P^ν such that $M^{\nu 4} = RP^\nu$. This relation has a physical meaning only if R is rather large. In that case the AdS mass μ and the Poincare mass m are related as $\mu = Rm$, and the relation between the AdS and Poincare energies is analogous. Since AdS symmetry is more general (fundamental) then Poincare one then μ is more general (fundamental) than m. In contrast to the Poincare masses and energies, the AdS masses and energies are dimensionless. From cosmological considerations (see e.g., [2]), the value of R is usually accepted to be of the order of $10^{26}m$. Then the AdS masses of the electron, the Earth and the Sun are of the order of 10^{39}, 10^{93} and 10^{99}, respectively. The fact that even the AdS mass of the electron is so large might be an indication that the electron is not a true elementary particle. In addition, the present accepted upper level for the photon mass is $10^{-17}ev$. This value seems to be an extremely tiny quantity. However, the corresponding AdS mass is of the order of 10^{16}, and so, even the mass which is treated as extremely small in Poincare invariant theory might be very large in AdS invariant theory.

In the AdS case there are IRs with positive and negative energies, and they belong to the discrete series [2, 12]. Therefore, by analogy with standard particle theory, one can define particles and antiparticles. Consider first the construction of positive energy IRs. We start from "the rest state" where the AdS energy equals the AdS mass μ_1. Then we obtain the states with the AdS energies $\mu_1, \mu_1 + 1, \mu_1 + 2, ...\infty$ (see Figure 2). Analogously, if μ_2 is the AdS mass of the antiparticle, we start from the state where the energy equals $-\mu_2$ and obtain the states with the AdS energies $-\mu_2, -\mu_2 - 1, -\mu_2 - 2, ... - \infty$. (see Figure 2) Therefore, the situation is pretty much analogous to that in Poincare invariant theories, and there is no way to conclude whether the mass of a particle equals the mass of the corresponding antiparticle.

In view of the results in this and preceding sections, we conclude that the descriptions of elementary particles in the cases of dS and AdS symmetries are considerably different. In the dS case, one IR describes particle and antiparticle states simultaneously and their superpositions are not prohibited, i.e. the principle of superposition is more fundamental than the conservation of electric charge and other additive quantum numbers. On the other hand, in the AdS case, the situation is analogous to that in Poincare invariant theories; in particular the electric charge conservation is more fundamental than the principle of superposition.

So, a question arises which of those possibilities in SQT is more physical. However, as discussed in [2], FQT is more general (fundamental) than SQT, in FQT it is also possible to define the concepts of dS and AdS symmetries and here the dS and AdS cases are physically equivalent. Below we will consider a direct generalization of the AdS symmetry from SQT to FQT.

The description of the energy spectrum in standard IRs of the AdS algebra has been given above. We will now explain why in FQT the spectrum is different, and in FQT the situation is similar to that in standard dS case but not standard AdS one because IRs in FQT contain both, positive and negative energies. Let us note first that, while in SQT the quantity μ can be an arbitrary real number, in FQT μ is an element of R_p. As noted above, if p is odd then R_p contains the elements $-(p-1)/2, ... -1, 0, 1, ...(p-1)/2$ (see Figure 1) and the case when p is even is analogous. For definiteness, we consider the case when p is odd.

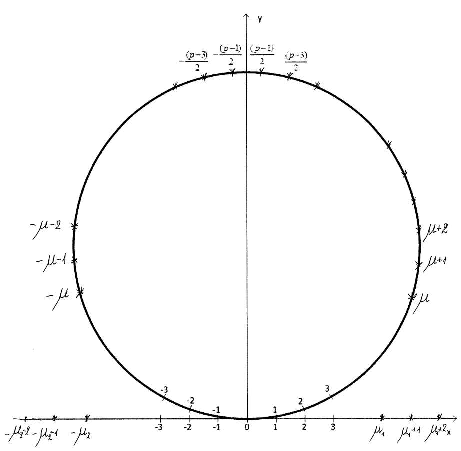

Figure 2: Spectrum of Energies of Elementary Particle

By analogy with the construction of positive energy IRs in SQT, in FQT we start the construction from "the rest state", where the AdS energy is positive and equals μ. Then we act on this state by raising operators and gradually get states with higher and higher energies, i.e., $\mu+1, \mu+2, ...$ However, now we are moving not along the straight line but along the circle in Figure 1 and, in contrast to the situation in SQT, we cannot obtain infinitely large numbers. When we reach the state with the energy $(p-1)/2$, the next state has the energy $(p-1)/2+1 = (p+1)/2$ and, since the operations are modulo p, this value also can be denoted as $-(p-1)/2$

i.e., it may be called negative. When this procedure is continued, one gets the energies $-(p-1)/2+1 = -(p-3)/2, -(p-3)/2+1 = -(p-5)/2, ...$ and, as shown in [2], the procedure finishes when the energy $-\mu$ is reached (see Figure 2).

Therefore, in contrast to the situation in SQT, in FQT IRs are finite-dimensional (and even finite since the ring R_p and its complex extension $R_p + iR_p$ are finite). By analogy with the dS case in SQT, one can say that the states with the energies $\mu, \mu+1, \mu+2, ...$ refer to a particle and states with the energies $... -\mu-2, -\mu-1, -\mu$ — to an antiparticle. Therefore, in FQT the mass of a particle automatically equals the mass of the corresponding antiparticle. This is an example when FQT can solve a problem which standard quantum AdS theory cannot. By analogy with the situation in the dS case, for combining a particle and its antiparticle together, there is no need to involve additional coordinate fields because a particle and its antiparticle are already combined in the same IR.

Then, since states which are superpositions of particles and antiparticles belong to the representation space, we conclude by analogy with the situation in Sec. 7, that in FQT there are no superselection rules which prohibit superpositions of states with opposite electric charges, baryon quantum numbers etc. Moreover, the representation operators of the enveloping algebra can perform transformations *particle* $\leftrightarrow$ *antiparticle*.

As shown in Ref. [2], in the formal limit $p \to \infty$, one IR in FQT splits into two standard IRs of the AdS algebra with positive and negative energies. This result seems natural from Figure 2 since the spectrum of positive energies becomes $\mu, \mu+1, \mu+2, ...\infty$ and the spectrum of negative energies becomes $-\infty, ... -\mu-2, -\mu-1, -\mu$ by analogy with the spectrum in SQT (see Figure 2). Therefore, in this limit the concept of particle-antiparticle and the superselection rules have the usual meaning. In turn, in situations when one can define the quantity R such that the contraction to the Poincare algebra works with a high accuracy, one can describe particles and antiparticles in the framework of Poincare symmetry.

Even from the fact that in standard quantum theory, there are no superpositions of states belonging to a particle and its antiparticle, it is clear that symmetry described by one IR in FQT is higher than symmetry described by two IRs obtained from one IR in FQT in the formal limit $p \to \infty$. Therefore standard concepts of particle-antiparticle and superselection rules arise as a result of symmetry breaking, i.e., they are not universal.

9 Discussion

As explained in Sec. 6, in quantum theory based on Poincare symmetry, the concept of particle-antiparticle arises because IRs have the property that energies in them can be either positive or negative, and there are no IRs where energies have different signs. Then IRs with positive energies are associated with particles and IRs with negative energies — with antiparticles, and superpositions of particles and antiparticles are prohibited because they belong to different IRs. As shown in Sec. 8, in SQT based on AdS symmetry, the situation is analogous.

On the other hand, as shown in Secs. 7 and 8, in SQT based on dS symmetry and in FQT, IRs contain states with both, positive and negative energies. If states with positive energies are called particle states and states with negative energies — antiparticle states then their superpositions are not prohibited because they belong to the same IR. The principle of superposition is a fundamental principle of quantum theory but in SQT based on Poincare and AdS symmetries, superpositions of particles and antiparticles are prohibited because they contradict the electric charge conservation, baryon number conservation etc. Therefore, in those cases, e.g., the electric charge conservation is treated as more fundamental than the principle of superposition but in SQT based on dS symmetry and in FQT the situation is the opposite.

One might think that for this reason the latter theories are not physical but in fact they are more physical than the former theories. The matter is that, as explained in Secs. 7 and 8:

- Standard Poincare invariant theory arises as a result of symmetry breaking at $R \to \infty$ in dS invariant quantum theory because in this limit one IR in the latter splits into two IRs in the former.

- Standard Poincare and AdS invariant theories arise as a result of symmetry breaking at $p \to \infty$ in FQT because in this limit one IR in the latter splits into two IRs in the former.

Then experimentally the electric charge conservation, baryon number conservation etc. are observed with a very high accuracy as a consequence of the fact that at the present stage of the universe the quantities R and p are extremely high and then standard quantum theory based on Poincare symmetry works with a very high accuracy. However, there are reasons to think [2] that at early stages of the universe those quantities were much less than now . That is why at those stages the conservation of the electric charge and baryon quantum number did not take place. As argued in [13], this is the reason of the baryon asymmetry of the universe.

The present fundamental particle theories are based on Poincare invariant QFT, and, as noted in Sec. 6, for solving the problem why a particle and its antiparticle have equal masses, those theories involve local quantized field $\psi(x)$ where x does not belong to any particle and is simply a parameter arising from the second quantization of a non-quantized field. So, the physical meaning of x is not clear. Although QFT has many successes, it also has problems because, as noted, for example, in the textbook [9], $\psi(x)$ is an operatorial distribution, and the product of distributions at the same point is not a well defined mathematical operation.

As explained in Secs. 7 and 8, in quantum theories based on dS symmetry and FQT, the masses of a particle and the corresponding antiparticle are automatically equal, and this is achieved without introducing local quantized fields. However, as noted above, in those theories the concepts of particle-antiparticle and additive quantum numbers differ from standard ones because one IR combines together a particle and its antiparticle. The construction of such theories is one of the most fundamental (if not the most fundamental) problems of particle theory.

References

[1] W. Pauli: The connection between spin and statistics, Phys. Rev. **58**, 716–722 (1940).

[2] F. Lev: Finite mathematics as the foundation of classical mathematics and quantum theory. With application to gravity and particle theory. ISBN 978-3-030-61101-9. Springer, https://www.springer.com/us/book/9783030611002 (2020).

[3] T.D. Newton and E.P. Wigner: Localized States for Elementary Systems, Rev. Mod. Phys. **21**, 400–405 (1949).

[4] F.G. Dyson: Missed Opportunities. Bull. Amer. Math. Soc., **78**, 635–652 (1972).

[5] F.M. Lev: de Sitter Symmetry and Quantum Theory, Phys. Rev. D**85**, 065003 (2012).

[6] O. Consa: Something is wrong in the state of QED, arXiv preprint (2021), https://arxiv.org/abs/2110.02078.

[7] S. Weinberg: The Quantum Theory of Fields, Vol. I, Cambridge University Press, Cambridge, UK, 1999.

[8] S. Weinberg: Living with Infinities, arXiv preprint (2009), https://arxiv.org/abs/0903.0568.

[9] N.N. Bogolubov, A.A. Logunov, A.I. Oksak and I.T. Todorov: General Principles of Quantum Field Theory, Nauka: Moscow (1987).

[10] F.M. Lev: Could Only Fermions Be Elementary? J. Phys. A: Mathematical and Theoretical, **A37**, 3285-3304 (2004).

[11] M.B. Mensky: Method of Induced Representations. Space-time and Concept of Particles. Moscow: Nauka (1976)

[12] N.T. Evans: Discrete Series for the Universal Covering Group of the 3+2 de Sitter Group, J. Math. Phys. **8**, 170-184 (1967).

[13] F.M. Lev: The Concept of Particle-Antiparticle and the Baryon Asymmetry of the Universe, Physics of Particles and Nuclei Letters, **18**, 729-737 (2021).