The first and second laws of thermodynamics are logically equivalent

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In the September 2022 issue of *The Physics Teacher* (*TPT*),[1] Richard Kaufman and Harvey Leff showed the interdependence of the first and second laws of thermodynamics. Here, we go further and use the facts that the first law implies the second law, and that the second law implies the first law. This two-way implication establishes the logical equivalence of the first and second laws. Although the laws are logically equivalent (when one is true, then the other must be true), this does not mean that they are the same. The equivalence provides for a deeper and richer understanding of the laws of thermodynamics, as discussed in a section on pedagogy.

Background

History has borne out that the laws of thermodynamics are universally valid; we might wonder if there are any connections between them. A review of the literature has shown some instances where people have looked for linkages between the laws of thermodynamics, specifically the zeroth and second laws.[2,3,4,5] Linkages between the first and second laws were found in the recently published *TPT* paper[1], "Interdependence of the First and Second Laws of Thermodynamics." That paper was an extension of the *TPT* paper[6] "What if energy flowed from hot to cold? Counterfactual thought experiments". (Also, see appendix)

The present paper is an extension of those papers. Points from those papers are provided next:

> **First law of thermodynamics (1st):** The first law of thermodynamics is a generalized form of conservation of energy that is adopted for thermodynamic processes and is based on the existence of equilibrium states. The first law applies to processes that connect initial and final equilibrium states *i* and *f*, and conserves energy.
>
> **Clausius statement of the second law (2nd):** Heat energy transfer can only occur spontaneously from hotter-to-colder regions.

The papers[1,6] show that if heat energy flowed spontaneously in the other direction (*i.e.,* colder-to-hotter regions), then thermal equilibrium would not result. In that case, temperature variations within an object would not smooth out, and would not lead to a uniform temperature necessary for the equilibrium conditions of thermodynamics.

We illustrate this[1] in a similar way here. Figure 1 shows a thin metal rod in between a hot plate and an ice cube. With heat energy flow from colder-to-hotter, then any distribution of temperatures in the metal rod causes further colder-to-hotter flow between adjacent layers of the metal. The resulting white and dark bands shown symbolize cold and hot regions and are of undetermined thickness. No uniform temperature is reached for the rod, and in fact, the hot plate and ice cube would also form these indeterminate bands. We do not see this occur in our universe.

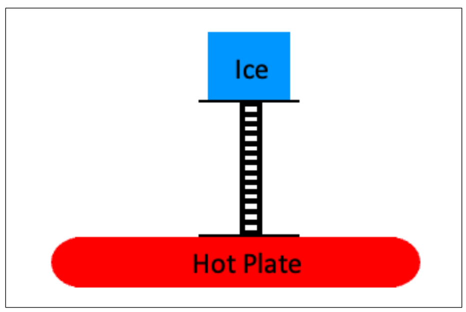

Figure 1: Depiction of colder-to-hotter energy through a metal rod. The white and dark bands are symbolic regions of cold and hot regions of ambiguous thickness, etc…

The main results about the universal laws stated in the paper, "Interdependence of the First and Second Laws of Thermodynamics", by Kaufman and Leff[1] are quoted here:

Main result 1: "The first law of thermodynamics requires the Clausius statement to assure equilibration to the initial and final states *i* and *f*."

Main result 2: "The Clausius statement requires energy conservation from the first law to assure that all heat processes conserve energy throughout their duration."

Logic and the equivalence

The present paper uses simple concepts and terminology from logic. "Two propositions or statements are said to be equivalent when the truth of one implies the truth of the other."[7] In other words, when "*if then*" statements can be shown in both directions, then the statements are equivalent. Using X and Y to connote propositions: (a) *Implies* is a logical if/then relationship. The statement, "*if X is true, then Y is true*", means "X implies Y", *i.e.*, $X \Longrightarrow Y$. (b) *Equivalence* requires a two-way implication, *i.e.,* both $X \Longrightarrow Y$ and $Y \Longrightarrow X$, so that "*X and Y are equivalent*", *i.e.,* $X \Leftrightarrow Y$.

The main results 1 and 2 from Kaufman and Leff, which were quoted above, are implications about the universal laws. Main result 1 states that, "The first law of thermodynamics requires the Clausius statement to assure equilibration to the initial and final states *i* and *f*." So, if the first law is true, then the second law must be true; the first law implies the second law. Main result 2 states that, "The Clausius statement requires energy conservation from the first law to assure that all heat processes conserve energy throughout their duration." So, if the second law is true then the first law must be true; the second law implies the first law. Therefore, we have the following:

The first law implies that the second law: $1^{st} \Longrightarrow 2^{nd}$

The second law implies that the first law: $2^{nd} \Longrightarrow 1^{st}$

These two-way *implications* establish that: **The first and second laws are logically *equivalent.***

This means that if one of these laws is true, then so is the other. In mathematical notation, the equivalence is shown in Fig. 2:

$$\boxed{1^{st} \Leftrightarrow 2^{nd}}$$

Fig. 2: The 1^{st} and 2^{nd} laws of thermodynamics are equivalent.

Pedagogical perspectives

In this section, we elaborate on important points for 1) the teaching of the thermodynamics, 2) the equivalence of the first and second laws of thermodynamics, and 3) a deeper understanding of thermodynamics overall.

We begin by reiterating a point made by Kaufman and Leff[8] that students should understand about the first law and equilibrium.

> Students might be surprised that the first law requires the second law, because the first law is often mistakenly viewed to be *only* a statement of energy conservation. Commonly overlooked is the need for the Clausius statement to guarantee that initial and final equilibrium states can be reached. Such surprises during the learning process can lead to more intense thought, and perhaps provide a spark that leads to enhanced student understanding.

This is important to keep in mind for the following pedagogical outline.

Thermodynamic laws are universal laws. People seek to understand how the universe works and have prized universal truths, such as thermodynamic laws, that are true everywhere and for all times. Observations that have limited applicability, scope, duration, or experimental evidence do not rise to the level of such "laws." An experiment in which a law holds part of the time would not be universal. Accordingly, students should understand that thermodynamics laws are universal in their application.

Thermodynamics is about equilibrium. Thermodynamic equilibrium exists when mechanical, chemical, and thermal equilibrium occurs throughout a system. Equilibrium states, such as temperature and pressure are required in thermodynamics, as indicated earlier for the first law. "Thermodynamics does not attempt to deal with any problem involving the rate at which a process takes place."[9]

Consider a mixer that stirs a liquid in a sealed container, like James Prescott Joule's experiments. Before the mixer is turned on, the temperature of the liquid is measured with a thermometer. Then the mixer stirs the liquid for a period of time. While the mixer is running, the liquid is not at a homogeneous temperature. We must wait until after the mixer is turned off for equilibrium to be established in order to measure the final temperature of the liquid. This demonstrates how equilibrium states are required in thermodynamics.

Usage of logic and equivalence. When two statements imply each another, they are equivalent. This is well-established and irrefutable by mathematical logic and is shown in truth tables. It is important for students to understand what equivalence means and what it does not. Equivalent statements mean that when one statement is true, then the other must be true. Similarly, when one statement is false, then the other statement is false. Equivalent statements cannot have one statement be true and the other false. A key point is that when statements are equivalent, it does not mean that they are *equal* (*i.e.,* they are not the same thing). This last point is worthy of repeating to students.

Mathematical logic is the basis for many conclusions that are made in the sciences. It is clear that physics can rely on logic for conclusions - such as "equivalence".

Equivalence and its use in physics. Equivalence is used in physics, beyond what has already been shown in this paper. For example, in addition to the Clausius statement (CS) of the second law, there is the Kelvin-Planck statement (KPS) of the second law, as discussed in the book by Zemansky and Dittman:[10]

> It is impossible to construct an engine that, operating in a cycle, will produce no effect other than the extraction of heat [energy] from a reservoir and the performance of an equivalent amount of work.

The book uses the same logic for equivalence to show an equivalence of the KPS and CS:[11]

> Two propositions or statements are said to be equivalent when the truth of one implies the truth of the second, and the truth of the second implies the truth of the first.

The proof is known from many sources, including many physics and thermodynamics textbooks. By using a hypothesized engine and refrigerator, it is demonstrated that $CS \Longrightarrow KPS$, and $KPS \Longrightarrow CS$, so the Clausius and Kelvin-Planck statements of the second law are equivalent, $KPS \Longleftrightarrow CS$.[12,13] This is another example of the use of equivalence in physics, so it is important that students understand the concept.

Experiments and the thermodynamic laws. This paper shows that the first and second laws of thermodynamics are equivalent. Since they are universal laws, they must both apply everywhere, if either one of them is true. In other words, we could not have a universe where the first law were true, and the second law were false (or vice versa). If a student does an experiment where the first law is shown to be true, then this automatically shows that the second law is true (and vice versa). Students may not have realized that they are showing that both laws are true whenever they show that one is true.

Consilience to relate concepts. Consilience is a term that has been used for the linking of ideas that are thought to be from different disciplines. When very different ideas are brought together, they are said to have a high consilience. Although this paper sticks to the discipline of thermodynamics, we see that there is a high consilience between the first and second laws, which were previously viewed to be non-equivalent. While there was really no doubt that both laws were true in our universe, the equivalence shows that they must be true together (and not true independent of each other). It is hoped that this deeper connection between the laws provides for a deeper understanding of them.

Conclusion

There is much that can be taught, learned, and appreciated about the laws of thermodynamics – especially in the course of showing that the first and second laws are equivalent.

Appendix

There were no other linkages found between the first and second laws in the literature before the original submission of this paper to the *TPT*. At the time of a revision to this paper, a paper[14] was found that was submitted in the same month as this one (August 2022), but published online in October 2022. That paper suggests a linkage between the laws, especially that a violation of the second law would violate the first law, as indicated in part of the abstract:

> In this work, we bring to light evidences [sic] to prove the absolute validity of the second law as a fundamental law of physics. For this purpose, we propose a short revisit of the history of the discovery of the second law in order to highlight the connection between the second law and the first law of energy conservation. We then demonstrate that the perpetual motion machine of the second kind also violate the first law of

> thermodynamics, albeit indirectly, contrary to the common belief. This result confirms the second law is an inviolable fundamental law of physics, just like the law of energy conservation. Denying one of these conjoined twin laws is to deny the other. Any presumed violation of the second law, even a probabilistic one, inevitably violates the laws of energy and mass conservation, and undermines all fundamental laws of physics and chemistry.

The paper discusses entropy, statistical entropy, Maxwell's demon, thermodynamic laws, perpetual motion devices, etc. In the conclusion section, the paper states:

> We have then proved that, the perpetual motion machine of the second kind, regarded most textbooks […] as a process violating the second law but not the first law, also necessarily violate the first law of energy conservation, albeit indirectly. This result confirms the unbreakable link between the two laws.

The paper appears to state one of the implications stated in the present paper, namely that the second law implies the first law. Although it does refer to a linkage between the laws, it does not explicitly state a two-way implication - or state that the laws are equivalent, which is stated in the present paper. We do not believe that the papers contradict each other, and, in fact, they both support any stated (or inferred) implication.

Availability and requirements: Data sharing not applicable to this article as no datasets were generated or analyzed during the current study.

Competing interests: The author declares that he has no competing interests.

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