

The Limited Amount of Work that can be Generated in the Universe, and the Ironic use of Inexhaustible Energy Transfer from Ideal Temperature Reservoirs to Develop the Principle of Entropy Increase

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Abstract

We consider entropy based on its original conceptualization for heat engines. With this historical perspective, we devise a statement of entropy that could be inferred by mechanical engineering students: *Entropy generation reduces the amount of work than can be generated in the universe.* There is an irony that Clausius developed the Principle of Entropy Increase, which points to a limited amount of work that can be generated in the universe, from a cycle that utilized temperature reservoirs with a seemingly inexhaustible supply of energy capable of producing work.

I. INTRODUCTION

In this paper, we consider entropy based on its historical development for heat engines during the nineteenth century.¹ Carnot² and Clausius were interested in maximizing work and efficiency. They considered heat engines that worked with idealized hot and cold temperature reservoirs. First, we will briefly review how they provided the foundations for entropy and the universe. Later, we will show how logical deductions show an irony for entropy.

II. BACKGROUND REVIEW

A maximally efficient heat engine is reversible, such as the Carnot cycle. During the Carnot cycle, the maximum possible net-work output W is obtained for the cycle operating between a hot temperature reservoir at temperature T_h and a cold temperature reservoir at temperature T_c . The maximal thermal efficiency is given by the Carnot efficiency, $1 - \frac{T_c}{T_h}$. Specifically, the Carnot cycle extracts heat energy Q_h isothermally from the hot temperature reservoir at temperature T_h and rejects heat energy Q_c isothermally to the cold temperature reservoir at temperature T_c . For the adiabatic processes between the temperature reservoirs, no heat energy is exchanged (there is no heat energy transfer through a finite temperature difference). It is well-known that the relation $Q_h/T_h = Q_c/T_c$ was established by this cycle, which motivated Clausius to define the state variable entropy, $dS = \frac{\partial Q}{T}$.

The Carnot cycle transfers the maximum possible amount of heat to work. Some heat energy must always be transferred from the heat source to the heat sink in order to generate this maximum amount of work. Yet, since a reversible cycle generates the maximum possible amount of work when transferring heat energy

from the hot temperature reservoir to the cold temperature reservoir, then no entropy is generated.

When entropy generation has occurred for a heat engine, then energy, which could have otherwise been transferred to work by a maximally efficient cycle, has been transferred to a cold temperature reservoir that is closer to absolute 0. Since heat energy transfer only occurs spontaneously from hotter-to-colder, then the amount of work that can be generated in the universe has decreased.

Entropy is a measure of the unavailability of energy for doing work.³

For all reversible processes, the change in entropy of the universe, $\Delta S_{universe}$ does not change:

$$\Delta S_{universe} = 0 \quad \text{For reversible processes only}^4 \quad (1)$$

When two finite objects at different temperatures are allowed to interact thermally, heat energy transfer occurs spontaneously from the hotter object to the colder object until thermal equilibrium exists between the objects; they obtain the same temperature. In this case, heat transfer occurs without generating any work. This process involves the maximum generation of entropy.

For all irreversible processes, the change in entropy of the universe, $\Delta S_{universe}$ increases:

$$\Delta S_{universe} > 0 \quad \text{For all natural (that is, irreversible) processes}^4 \quad (2)$$

Combining equations 1 and 2, we have the Principle of Entropy Increase (PEI), which is a form of the Second Law of Thermodynamics:

$$\Delta S_{universe} \geq 0 \quad (3)$$

Entropy generation only occurs when there is heat energy transfer from hotter-to-colder without the accompanying maximal work output possible. This loss in the potential of energy to do work, or “lost work”, is prevalent in the literature,^{5,6,7,8} as are discussions of “exergy”.

II. Discussion

It is important to note that this lost work capability discussed in the previous section must refer to the amount of work than can be generated in the entire universe.

Key Point 1:

Entropy generation reduces the amount of work than can be generated in the universe.

This is reminiscent of the words of Sir William Thomson (Lord Kelvin):⁹

The second great law of thermodynamics involves a certain principle of *irreversible action in Nature*. It is thus shown that, although mechanical energy is *indestructible*, there is a universal tendency to its dissipation, which produces gradual augmentation and diffusion of heat, cessation of motion, and exhaustion of potential energy through the material universe. The result would inevitably be a state of universal rest and death, if the universe were finite and left to obey existing laws...

It is also important to note that idealized temperature reservoirs were used during the development of the Carnot cycle and entropy. The hot source temperature reservoir remained at temperature T_h during energy extraction, and the cold sink temperature reservoir remained at T_c during energy absorption. This allowed Carnot and Clausius to perform much simpler calculations than if the source or sink temperatures changed as energy was extracted or absorbed, respectively. That is, it becomes mathematically simpler (*i.e.*, less specific to the circumstances and more general) to use idealized temperature reservoirs that do not change temperature when heat energy is added or removed. Of course, we do not see these idealized temperature reservoirs in our observable universe.

The use of idealized temperature reservoirs might only be thought of as a useful approximation, especially when the energy requirements of the cycle are much less than the energy available in a source or sink. If so, then any energy extracted or absorbed by a temperature reservoir does not significantly impact the reservoir's temperature. However, this means that the thermodynamics becomes decoupled from the size requirements of a cycle. In other words, *any* cycle which operates between the temperature reservoirs, no matter how large, would have no impact on the reservoir temperatures. Moreover, a cycle which is repeated again and again would have no impact on a reservoir's temperatures. We do not see this in our universe.

For the sake of argument, let us assume that the arguments put forth by Clausius and Carnot hold for idealized temperature reservoirs and their constant temperatures. With idealized temperature reservoirs, entropy generation could not reduce the amount of work that can be generated in the universe because temperature reservoirs can always supply an infinite amount of heat energy from which work could be generated. Therefore, the use of temperature reservoirs indicate that the amount of work generated in the universe could *never* be reduced. This is at odds with Key Point 1: *Entropy generation reduces the amount of work that can be generated in the universe.*

Key Point 2:

There is an irony that Clausius developed the Principle of Entropy Increase, which points to a limited amount of work that can be generated in the universe, from a cycle that utilized temperature reservoirs with a seemingly inexhaustible supply of energy capable of producing work.

The Principle of Entropy Increase (PEI), which is based on the Clausius Inequality that Clausius came up with in 1855, is one form of the second law of thermodynamics. The PEI has an inherent assumption; it assumes that the universe does not have an infinite amount of energy that could be converted to work.

Consider the three possibilities for the universe (including the universe beyond what we can observe):

- 1) The universe has a finite amount of energy that can be converted to work. Here the PEI is true.
- 2) The universe has an infinite amount of energy, but this energy can all be converted to a lower form, leading to the “heat death” of the universe. We have not seen this “infinite amount of energy” in our observable universe.
- 3) The universe has an infinite amount of energy, where infinite means an inexhaustible supply – that does not result in a “heat death” of the universe. Although we have not seen this in our observable universe, it would render the “unavailability of energy for doing work” as a meaningless statement. In other words, entropy of the universe could never increase, and the term entropy would be rendered meaningless for the entire universe (the numeric values for the entropy change of an “irreversible” process would result in an inconsistent equation). Here the PEI would be false.

Bullet 1 shows that:

If the Principle of Entropy Increase is true, then the universe is only capable of producing a finite amount of work.

The contrapositive of the last statement is:

If the universe is capable of producing an infinite amount of work, then the principle of entropy increase is false.

Bullet 2 is ambiguous about any limit to the amount of work that can be produced in the universe. Although the eventual “heat death” of the universe suggests that the universe cannot produce any more work at some point.

Bullet 3 shows that:

If the Principle of Entropy Increase is false, then the universe is capable of producing an infinite amount of work.

The last bullet shows that if the universe can produce an infinite amount of work, then the Second Law of Thermodynamics is false. But a recent paper¹⁰ showed that the first and second law of thermodynamics are interdependent. So, if the second

law is false, then the First Law of Thermodynamics would also be false, which we have also not seen in our observable universe.

III. Conclusion

This brief paper demonstrates the irony that the Principle of Entropy Increase, which indicates a limited amount of work that can be generated in the universe, was developed from a cycle that utilized idealized temperature reservoirs with an inexhaustible supply of energy.

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- ¹ In more modern parlance, entropy is also described as a measure of energy spreading, among other formulations.
 - ² Sadi Carnot, *Reflections on the Motive Power of Fire*, edited by E. Mendoza and translated by R. H. Thurston (Dover, New York, 1960), pp. 1–59.
 - ³ Alvin Hudson and Rex Nelson, *University Physics* (Saunders College, a subsidiary of Holt, Rinehart and Winston, New York, 1990), 2nd ed. P. 548
 - ⁴ Ibid P. p.543
 - ⁵ Çengel Yunus A., and Michael A. Boles. *Thermodynamics an Engineering Approach*. McGraw Hill, 1994, 2nd ed. P.316
 - ⁶ Sussman, Martin Victor. *Elementary General Thermodynamics*. Addison-Wesley, 1972, P.159
 - ⁷ Sears, Francis Weston. *An Introduction to Thermodynamics, the Kinetic Theory of Gases and Statistical Mechanics*. Addison-Wesley Publishing Company, 1959, 2nd ed. P.142
 - ⁸ Marcella, Thomas V. “Entropy Production and the Second Law of Thermodynamics: An Introduction to Second Law Analysis.” *American Journal of Physics*, vol. 60, no. 10, 1992, pp. 888–895., <https://doi.org/10.1119/1.17138>
 - ⁹ Thomson, William (Lord Kelvin). “On the Age of the Sun’s Heat.” *Macmillan’s Magazine*, vol 5, 1862, pp. 388-393. Reprint in: *Popular Lectures and Addresses*, vol 1, 2nd ed., pp 356-375, from https://zapatopi.net/kelvin/papers/on_the_age_of_the_suns_heat.html#fn1
 - ¹⁰ R. Kaufman & H. S. Leff, “Interdependence of the first and second laws of thermodynamics,” *Phys. Teach.* **60**, 501-503, (Sept. 2022) DOI: 10.1119/5.0074493. Available Open Access: <https://doi.org/10.1119/5.0074493>