The SuperHyperFunction and the Neutrosophic SuperHyperFunction (revisited again)

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Abstract: In this paper, one recalls the general definition of the SuperHyperAlgebra with its SuperHyperOperations and SuperHyperAxioms [2, 6]. Then one introduces for the first time the SuperHyperTopology and especially the SuperHyperFunction and Neutrosophic SuperHyperFunction. One gives a numerical example of a Neutro-SuperHyperGroup.

Keywords: SuperHyperAlgebra; SuperHyperFunction; Neutrosophic SuperHyperFunction; SuperHyperOperations; SuperHyperAxioms; SuperHyperTopology.

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1. System of Sub-Systems of Sub-Sub-Systems and so on

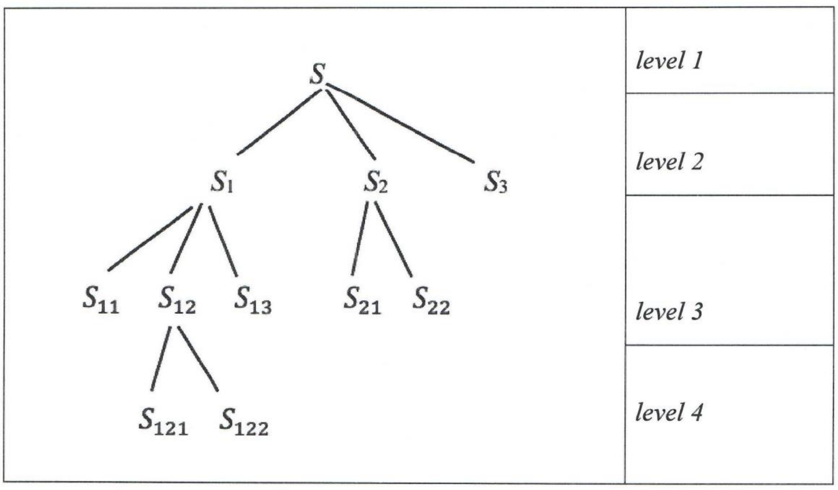
A system may be a set, space, organization, association, team, city, region, country, etc. One considers both: the static and dynamic systems.

With respect to various criteria, such as: political, religious, economic, military, educational, sportive, touristic, industrial, agricultural, etc.,

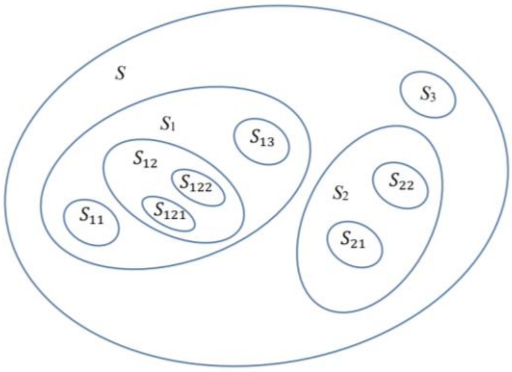
a system S is made up of several sub-systems S_1, S_2, …, S_p, for integer $p \geq 1$; then each sub-system S_i, for $i \in \{1, 2, \ldots, p\}$ is composed of many sub-sub-systems S_{i1}, S_{i2}, …, S_{ip_i}, for integer $p_i \geq 1$; then each sub-sub-system S_{ij}, for $j \in \{1, 2, \ldots, p_i\}$ is composed sub-sub-sub-systems, S_{ij1}, S_{ij2}, …, S_{ijp_j}, for integer $p_j \geq 1$; and so on.

2. Example 1 and Application of Systems made up of Sub-Sub-Sub-Systems (four levels)

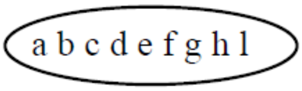
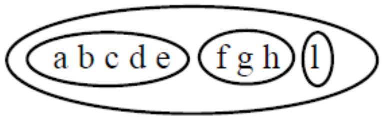
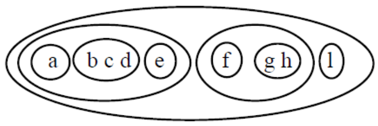
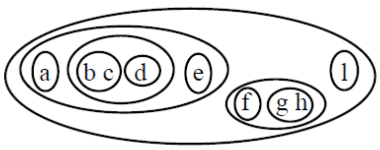
i) Using a *Tree-Graph Representation*, one has:



ii) Using a *Geometric Representation*, one has:



iii) Using an *Algebraic Representation* through pairs of braces { }, one has:

$P^0(S) \stackrel{\text{def}}{=} S = \{a, b, c, d, e, f, g, h, l\}$ 1 level of pairs of braces	 1level of closed curves	*level 1*
$P^1(S) \stackrel{\text{def}}{=} P(S) \ni \{\{a, b, c, d, e\}, \{f, g, h\}, \{l\}\}$ 2 levels of pairs of braces i.e. a pair of braces { } inside, another pair of braces { }, or { … { … } … }	 2 levels of closed curves	*level 2*
$P^2(S) \stackrel{\text{def}}{=} P(P(S))$ $\ni \{\{\{a\}, \{b, c, d\}, \{e\}\}, \{\{f\}, \{g, h\}\}, \{l\}\}$ 3 levels of pairs of braces	 3 levels of closed curves	*level 3*
$P^3(S) \stackrel{\text{def}}{=} P(P^2(S))$ $\ni \{\{\{a\}, \{b, c\}, \{d\}, \{e\}\}, \{\{f\}, \{g, h\}\}, \{l\}\}$ 4 levels of pairs of braces	 4 levels of closed curves	*level 4*

where the symbol "∋" means "contain(s)", it is the opposite of the symbol "∈" (belong(s) to), for example $M \ni x$ means the set M contains the element x, which is equivalent to $x \in M$.

Industrial Application.

Let's assume an auto-repair corporation called MotorX Inc. resides in the United States (system S). MotorX has three branches, one in each of the states: California, Arizona, and New Mexico (sub-systems S_1, S_2, and S_3 respectively).

In California, MotorX has branches in three cities: San Francisco, Los Angeles, and San Diego (sub-sub-systems S_{11}, S_{12}, and S_{13} respectively), and in Arizona in two cities: Phoenix and Tucson (sub-sub-systems S_{21}, and S_{22} respectively).

In the city of Los Angeles, MotorX has branches in two of the city's districts or neighborhoods, such as Fairfax and Northridge (sub-sub-sub-systems S_{121}, and S_{122} respectively).

2.1 Remark 1

The pairs of braces { } make a difference on the structure of a set. For example, let's see the distinction between the sets A and B, defined as bellow:

$A = \{a, b, c, d\}$ represents a system (organization) made up of four elements,

while $B = \{\{a, b\}, \{c, d\}\}$ represents a system (organization) made up of two sub-systems and each sub-system made up of two elements. Therefore B has a richer structure, it is a refinement of A.

3. Definition of n^{th}-Power of a Set

The n^{th}-Power of a Set (2016) was introduced by Smarandache in the following way:

$P^n(S)$, as the n^{th}-PowerSet of the Set S, for integer $n \geq 1$, is recursively defined as:

$P^2(S) = P(P(S))$, $P^3(S) = P(P^2(S)) = P\left(P(P(S))\right)$, …,

$P^n(S) = P(P^{n-1}(S))$, where $P^0(S) \stackrel{\text{def}}{=} S$, and $P^1(S) \stackrel{\text{def}}{=} P(S)$. (For any subset A, we identify {A} with A.)

The n^{th}-Power of a Set better reflects our complex reality, since a set S (that may represent a group, a society, a country, a continent, etc.) of elements (such as: people, objects, and in general aany items) is organized onto subsets $P(S)$, which on their turns are also organized onto subsets of subsets, and so on. That is our world.

In the classical HyperOperation and Classical HyperStructures, the empty set $\emptyset$ does not belong to the power set, or $P_*(S) = P(S) \setminus \{\emptyset\}$.

However, in the real world we encounter many situations when a HyperOperation $\circ$ is:

- *indeterminate*, for example $a \circ b = \emptyset$ (unknown, or undefined);
- or *partially indeterminate*, for example $a \circ b = \{[0.2, 0.3], \emptyset\}$.

In our everyday life, there are many more operations and laws that have some degrees of indeterminacy (vagueness, unclearness, unknowingness, contradiction, etc.), than those that are totally determinate.

That is why is 2016 we have extended the classical HyperOperation to the Neutrosophic HyperOperation, by taking the whole power $P(S)$ (that includes the empty-set $\emptyset$ as well), instead $P_*(S)$ (that does not include the empty-set $\emptyset$), as follows.

3.1 Remark 2

Throughout this paper the definitions, theorems, remarks, examples and applications work for both classical-type and Neutrosophic-type SuperHyper-Algebra and SuperHyper Function.

3.2 Theorem 1

Let S be a discrete finite set of 2 or more elements, and $n \geq 1$ an integer.

Then: $P^0(S) \subset P^1(S) \subset P^2(S) \subset \cdots \subset P^{n-1}(S) \subset P^n(S)$.

Proof

For a discrete finite set $S = \{a_1, a_2, \dots, a_m\}$ for integer $m \geq 2$ one has:

$P^0(S) \equiv S = \{a_1, a_2, \dots, a_m\}$.

$P^1(S) = P(S) = \{a_1, a_2, \dots, a_m; \{a_1, a_2\}, \{a_1, a_2, a_3\}, \dots, \{a_1, a_2, \dots, a_m\}\}$,

and cardinal of P(S) is $Card(P(S)) = C_m^1 + C_m^2 + \cdots + C_m^n = 2^m - 1$,

where C_m^i, $1 \leq i \leq m$, means combinations of m elements taken in groups of i elements.

It is clear that $P^0(S) \subset P^1(S)$.

In general, one computes the set of $P^{k+1}(S)$ by taking the set of the previous $P^k(S) = \{\alpha_1, \alpha_2, \dots, \alpha_r\}$, where $r = Card(P^k(S))$, and making all possible combination of its r elements; but, at the beginning, when one takes the elements only by one, we get just $P^k(S)$, afterwards one takes the elements in group of two, then in groups of three, and so on, and finally all r elements together as a single group.

4. Definition of SuperHyperOperations

We recall our 2016 concepts of SuperHyperOperation, SuperHyperAxiom, SuperHyperAlgebra, and their corresponding Neutrosophic SuperHyperOperation Neutrosophic SuperHyperAxiom and Neutrosophic SuperHyperAlgebra [2].

Let $P_*^n(H)$ be the n[th]-powerset of the set H such that none of $P(H)$, $P^2(H)$, …, $P^n(H)$ contain the empty set ϕ.

Also, let $P_n(H)$ be the n[th]-powerset of the set H such that at least one of the $P(H)$, $P^2(H)$, …, $P^n(H)$ contain the empty set ϕ. For any subset A, we identify {A} with A.

The SuperHyperOperations are operations whose codomain is either $P_*^n(H)$ and in this case one has **classical-type SuperHyperOperations**, or $P^n(H)$ and in this case one has **Neutrosophic SuperHyperOperations**, for integer $n \geq 2$.

4.1 Classical-type m-ary SuperHyperOperation {or more accurate denomination (m, n)-SuperHyperOperation}

Let U be a universe of discourse and a non-empty set H, $H \subset U$. Then:

$$\circ_{(m,n)}^{*} : H^m \to P_*^n(H)$$

where the integers $m, n \geq 1$,

$$H^m = \underbrace{H \times H \times \ldots \times H}_{m\ times},$$

and $P_*^n(H)$ is the n[th]-powerset of the set H that includes the empty-set.

This SuperHyperOperation is a m-ary operation defined from the set H to the n[th]-powerset of the set H.

4.2 Neutrosophic m-ary SuperHyperOperation {or more accurate denomination Neutrosophic (m, n)-SuperHyperOperation}

Let U be a universe of discourse and a non-empty set H, $H \subset U$. Then:

$$\circ_{(m,n)} : H^m \to P^n(H)$$

where the integers $m, n \geq 1$; $P^n(H)$ - the n-th powerset of the set H that includes the empty-set.

5. SuperHyperAxiom

A **classical-type SuperHyperAxiom** or more accurately a ***(m, n)*-SuperHyperAxiom** is an axiom based on classical-type SuperHyperOperations.

Similarly, a **Neutrosophic SuperHyperAxiom** {or Neutrosphic (m, n)-SuperHyperAxiom} is an axiom based on Neutrosophic SuperHyperOperations.

There are:

- **Strong SuperHyperAxioms**, when the left-hand side is equal to the right-hand side as in non-hyper axioms,
- and **Week SuperHyperAxioms**, when the intersection between the left-hand side and the right-hand side is non-empty.

For examples, one has:

- **Strong SuperHyperAssociativity**, for any $x_1, x_2, \ldots, x_m, y_1, y_2, \ldots, y_{m-1} \in H$, one has

  $$\circ_{(m,n)}(\circ_{(m,n)}(x_1, x_2, \ldots, x_m), y_1, y_2, \ldots, y_{m-1}) = \circ_{(m,n)}(x_1, \circ_{(m,n)}(x_2, x_3, \ldots, x_m, y_1), y_2, y_3, \ldots, y_{m-1}) = \ldots$$
  $$\ldots = \circ_{(m,n)}(x_1, x_2, x_3, \ldots, x_{m-1}, \circ_{(m,n)}(x_m, y_1, y_2 \ldots, y_{m-1})).$$

  and **Week SuperHyperAssociativity**, for any $x_1, x_2, \ldots, x_m, y_1, y_2, \ldots, y_{m-1} \in H$ one has

  $$\circ_{(m,n)}(\circ_{(m,n)}(x_1, x_2, \ldots, x_m), y_1, y_2, \ldots, y_{m-1}) \cap \circ_{(m,n)}(x_1, \circ_{(m,n)}(x_2, x_3, \ldots, x_m, y_1), y_2, y_3, \ldots, y_{m-1}) \cap \ldots$$
  $$\ldots \cap \circ_{(m,n)}(x_1, x_2, x_3, \ldots, x_{m-1}, \circ_{(m,n)}(x_m, y_1, y_2 \ldots, y_{m-1})) \neq \phi.$$

6. SuperHyperAlgebra and SuperHyperStructure

A **SuperHyperAlgebra** or more accurately ***(m-n)*-SuperHyperAlgebra** is an algebra dealing with SuperHyperOperations and SuperHyperAxioms.

Again, a **Neutrosophic SuperHyperAlgebra** {or Neutrosphic (m, n)-SuperHyperAlgebra} is an algebra dealing with Neutrosophic SuperHyperOperations and Neutrosophic SuperHyperOperations.

In general, we have **SuperHyperStructures** {or *(m-n)*-SuperHyperStructures}, and corresponding **Neutrosophic SuperHyperStructures**.

For example, there are SuperHyperGrupoid, SuperHyperSemigroup, SuperHyperGroup, SuperHyperRing, SuperHyperVectorSpace, etc.

7. Distinction between SuperHyperAlgebra vs. Neutrosophic SuperHyperAlgebra

i. If none of the power sets $P^k(H)$, $1 \le k \le n$, do not include the empty set ϕ, then one has a classical-type SuperHyperAlgebra;

ii. If at least one power set, $P^k(H)$, $1 \le k \le n$, includes the empty set ϕ, then one has a Neutrosophic SuperHyperAlgebra.

8. Example 2 of SuperHyperGroup

The below $(P^2(S), \#)$ Table represents a Commutative Neutro-*SuperHyperGroup*.

$\#$	$\{a\}$	$\{b\}$	$\{a,b\}$	$\{\{a\},\{a,b\}\}$	$\{\{b\},\{a,b\}\}$	$\{\{a\},\{b\},\{a,b\}\}$
$\{a\}$	$\{a\}$	$\{b\}$	$\{a,b\}$	$\{\{a\},\{a,b\}\}$	$\{\{b\},\{a,b\}\}$	$\{\{a\},\{b\},\{a,b\}\}$
$\{b\}$	$\{b\}$	$\{a\}$	$\{\{a\},\{a,b\}\}$	$\{\{b\},\{a,b\}\}$	$\{\{a\},\{b\},\{a,b\}\}$	$\{a,b\}$
$\{a,b\}$	$\{a,b\}$	$\{\{a\},\{a,b\}\}$	$\{a\}$	$\{b\}$	$\{a\}$	$\{\{b\},\{a,b\}\}$
$\{\{a\},\{a,b\}\}$	$\{\{a\},\{a,b\}\}$	$\{\{b\},\{a,b\}\}$	$\{b\}$	$\{a\}$	$\{a,b\}$	$\{\{a\},\{a,b\}\}$
$\{\{b\},\{a,b\}\}$	$\{\{b\},\{a,b\}\}$	$\{\{a\},\{b\},\{a,b\}\}$	$\{a\}$	$\{a,b\}$	{a}	$\{b\}$
$\{\{a\},\{b\},\{a,b\}\}$	$\{\{a\},\{b\},\{a,b\}\}$	$\{a,b\}$	$\{\{b\},\{a,b\}\}$	$\{\{a\},\{a,b\}\}$	$\{b\}$	$\{a\}$

The *SuperHyperLaw* $\#$ is clearly well-defined, according to the above Table. This law is commutative since Table's matrix is symmetric with respect to the main diagonal.

$\{a\}$ is the *SuperHyperNeutral*.

And the *SuperHyperInverse* of an element $x \in P^2(S)$ is itself: $x^{-1} = x$.

8.1 Theorem 2

The above algebraic structure is a Commutative Neutro-SuperHyperGroup.

Proof

The axiom of associativity, with respect to the law #, is a NeutroAxiom, since:

there are three elements *{a}*, *{b}*, and *{a, b}* from the set $P^2(S)$

such that: *{a} # ({b} # {a, b}) = {a} # {{a}, {a, b}} = {{a}, {a, b}}*

and *({a} # {b}) # {a, b} = {b} # {a, b} = {{a}, {a, b}}*,

therefore, it has some degree of truth (*T > 0*);

and there are three elements *{a, b}*, *{{a}, {a, b}}*, and *{{b}, {a, b}}* from the set $P^2(S)$ such that:

({a, b} #{{a}, {a, b}}) # {{b}, {a, b}} = {b} # {{b}, {a, b}} = {{a}, {b}, {a, b}}

and {a, b} # ({{a}, {a, b}} # {{b}, {a,b}}) = {a, b} # {a, b} = {a}, which is different from {{a}, {b}, {a, b}}, therefore, it has some degree of falsehood (F > 0). While the other four axioms (well-defined, commutativity, unit element, and inverse element are classical (100% true, or T = 1, degree of indeterminacy I = 0, and F = 0).

9. SuperHyperTopology and Neutrosophic SuperHyperTopology

A topology defined on a SuperHyperAlgebra $(P_*^n(S), \#)$, for integer $n \geq 2$, is called a SuperHyperTopology, and it is formed from SuperHyperSubsets. Similarly for Neutrosophic SuperHyper of $P_*^n(S)$ Topology, where $P_*^n(S)$ is replaced by $P_n(S)$, that includes the empty-set as well.

10. Definition of classical-type Unary HyperFunction (f_H)

Let S be a non-empty set included in a universe of discourse U.

$$f_H: S \to P_*(S)$$

11. Definition of classical-type m-ary HyperFunction (f_H^m)

$$f_H^m: S^m \to P_*(S)$$

where m is an integer ≥ 2, and $P_*(S)$ is the classical powerset of S.

12. Definition of Unary SuperHyperFunction (f_H)

We now introduce for the first time the concept of **SuperHyperFunction** (f_{SH}).
$f_{SH}: S \to P_*^n(S)$, for integer $n \geq 1$, where $P^n(S)$ is the n-th powerset of the set S.

13. Definition of m-ary SuperHyperFunction (f_{SH}^m)

$f_{SH}^m: S^m \to P_*^n((S)$, for integer $m \geq 2$.

14. General Definition of SuperHyperFunction

$f_{SH}^{SH}: P_*^r(S) \to P_*^n(S)$, for integers $r, n \geq 0$.
$f_{SH}: S \to P^n(S)$
$f_{SH}^m: S^m \longrightarrow P^n(S)$
$f_{SH}^{SH}: P_*^r(S) \to P^n(S)$

15. Example 3 and Application of SuperHyperFunctions

Let $S = \{a, b\}$ be a discrete set. The first and second powersets of the set S are:

$$P(S) = \{\{a\}, \{b\}, \{a, b\}\}$$

$$P^2(S) = \left\{\begin{matrix} \{a\}, \{b\}, \{a, b\} \\ \{\{a\}, \{a, b\}\}, \{\{b\}, \{a, b\}\} \\ \{\{a\}, \{b\}, \{a, b\}\} \end{matrix}\right\}$$

Let's define the SuperHyperFunction f_{SH} as follows:
$f_{SH}: S \to P^2(S)$
$f_{SH}(x)$ = the system (organization) or { } set that x best belongs to
$f_{SH}(a) = \{a, b\}$
$f_{SH}(b) = \{\{b\}, \{a, b\}\}$

For example, the system $\{b\}$ means that person b is a strong personality and himself alone makes a system.

16. Example 4 and Application of Neutrosophic SuperHyperFunctions

$S = [0, 5]$, a continuous set.

$P_o(S) = \{A, A \text{ is a subset}, A \subseteq [0,5]\}$

$P_o^2(S) = \{A_1, \{A_1, A_2\}, \{A_1, A_2, A_3\}, \dots\}$,

where all A_k are subsets of $[0, 5]$ with index $k \in [0,5]$, therefore one has an uncountable infinite set of subsets of [0, 5].

$f_{SH}: [0, 5] \to P_o^2([0, 5])$

$$f_{SH}(x) = \{[x-1, x] \cap [0, 5], [x+1, x+2] \cap [0, 5]\}$$

For example:

$$f_{SH}(2) = \{[1, 2], [3, 4]\}.$$

$$f_{SH}(3.4) = \{[2.4, 3.4], [4.4, 5]\}$$

since $[4.4, 5.4] \cap [0, 5] = [4.4, 5]$.

$$f_{SH}(0) = \{[0, 0], [1, 2]\} = \{0, [1, 2]\}$$

since $[-1, 0] \cap [0, 5] = [0, 0] = 0$.

$$f_{SH}(5) = \{[4,5], \emptyset\}$$

since $[6, 7] \cap [0, 5] = \emptyset$.

Conclusion

In this paper we recalled the concepts of SuperHyperAlgebra and Neutrosophic HyperSuperAlgebra, and presented an example of Neutro-SuperHyperGroup. Then, for the first time we introduced and gave examples of SuperHyperFunction and Neutrosophic SuperHyperFunction.

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