LARGER TYPES OF INFINITIES AND ITS IMPACT ON SOCIETY

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ABSTRACT

In this paper, we consider an extended real number denoting "larger types of infinity" through elements of a polynomial in a way that justify concepts as infinity plus one, or multiplication of infinities. Then, we equip the set with sums and multiplication such that it forms a ring, and finally, define its order relations.

1 Introduction

The goal is to create a system in which $1 < \infty < \infty + 1$. This could potentially solve many social problems of "say the biggest number" kind of discussions. The first assumption is to extend the real number with $\{\infty, \infty + 1, \infty + 2\}$ with $\forall x \in \mathbb{R} : x < \infty < \infty + 1 < \ldots$, but this leaves "non-integer infinity" such as 0.25∞ out.

Our next step is to consider then numbers on the form $a + b\infty$ where $a, b \in \mathbb{R}$. This kind of number has the same form of complex numbers, and it's closed under addition. The problem arises when we consider multiplication: in complex numbers, $i \cdot i = -1$, so it's closed under multiplication, but what is $\infty \cdot \infty$?

2 Solving with infinite powers of infinity

An **Infinite-extended real number system** $\mathbb{M}$ is a system composed of elements on the form

$$a_0 + a_1\infty + a_2\infty^2 + a_3\infty^3 + \ldots + a_D\infty^D, \qquad a_k \in \mathbb{R}, \infty^k \in \underline{\overline{\infty}} \tag{1}$$

Where D is the biggest infinity, and $\underline{\overline{\infty}}$ is the set of all infinities:

$$\underline{\overline{\infty}} = \bigcup_{k=1}^{\infty} \{\infty^k\} \tag{2}$$

We define multiplication of infinities following the usual rules of exponentiation:

$$\infty^a \cdot \infty^b = \infty^b \cdot \infty^a := \infty^{a+b} \tag{3}$$

Additions and subtractions are defined as:

$$m \pm n = \sum_{k=0}^{D} (m_k \pm n_k)\infty^k, \qquad m, n \in \mathbb{M} \tag{4}$$

Where ∞^0 is a shorthand that means $a_0\infty^0 = a_0$. This should not be confused with the indeterminate form ∞^0. The product $m \cdot n, m, n \in \mathbb{M}$ can be obtained following the distributive property and the product/sum defined earlier. We can clearly see that $\mathbb{M}$ is a polynomial ring $K[\{\infty\}]$ (where the powers of ∞ are well-defined). In this sense, the usual arithmetic operations are well-defined, allowing people to make the most needed statements like "$\infty + 1$" or "∞^2".

3 Order relation

Now that we know that it form a ring, it's time to define the most important thing, the order relation. Assume $m, n \in \mathbb{M}$. Define a_k to be the coefficient of ∞^k in the polynomial expansion of m, and b_k to be the coefficient of ∞^k in the polynomial expansion of n. Then, the following rule apply:

$$m < n \iff a_\mu < b_\mu, a_\mu \neq b_\mu, \forall \nu > \mu, a_\nu = b_\nu \tag{5}$$

Remark. *From this definition, it's clear that:*

$$1,000,000 < \infty < \infty + 1 < \infty^2 < \dots \tag{6}$$

Achieving what we wanted.

4 Infinite series

From this definition, we can check an "order relation limit". Let m_k and n_k be sequences of numbers in $\mathbb{M}$. If there exist a number λ such that, for all $s > \lambda$, $m_s < n_s$, then we say $m <_{\lim} n$.

We can check some sequences to see what they result:

Example 1.

$$m_s = \sum_{k=0}^{s} 1\infty^k \tag{7}$$

$$n_s = \sum_{k=0}^{s} 2\infty^k \tag{8}$$

For every s, if we set $\mu = s$, μ follows (5). This means that, for any s, $m_s < n_s$, hence, $m <_{\lim} n$. In this case, the smallest λ was 0.

Example 2.

$$m_s = \sum_{k=0}^{s} \frac{-1}{2^k}\infty^k \tag{9}$$

$$n_s = \sum_{k=0}^{s} \frac{1}{2^k}\infty^k \tag{10}$$

We apply the same logic of example 1, it follows that $m <_{\lim} n$. In this case, the limit of the sequences a_k and b_k was both 0, but what matter to us is not the limit of the values, but the limit of order relations.

Example 3.

$$m_s = \sum_{k=0}^{s} (-1)^k\infty^k \tag{11}$$

$$n_s = \sum_{k=0}^{s} 2\infty^k \tag{12}$$

We can see here that the limit of the sequence really doesn't matter, only the fact that $m_s < n_s$ everytime, hence, $m <_{\lim} n$.

Example 4.

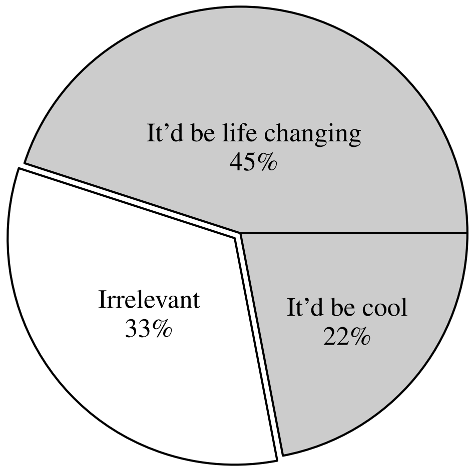
$$m_s = \sum_{k=0}^{s} -(-1)^k\infty^k \tag{13}$$

$$n_s = \sum_{k=0}^{s} (-1)^k\infty^k \tag{14}$$

This illustrate why we can't have well-defined orders for all sequences. The order relation alternates between $m_s < n_s$ and $n_s < m_s$. The limit of the order is then undefined.

5 Impact on society

One may wonder the purpose of all this. The team of experts[1] responsible for this paper made a research on how would people feel if they could scientifically say $\infty + 1 > \infty$:



There was also the option "It would ruin my life" but no one choose that. The pie chart shows that 67% (two thirds) of the population would be positively impacted by this paper.

Children will be the most benefited. Based on research[2], the most common problem related to the biggest number game is when a kid says "well, actually, infinity is not a number, and also you can't add 1 to it and make it bigger". It's now possible for the wiser kid rebut the propositions with a scientific background and a actual study, resulting in more interesting fights between children.

As stated by a specialist[3] in the area:

> "The $\infty + 1 > \infty$ is an extremely important knowledge nowadays, since we live in the information era and any information can end up enriching us more as a person, a concept so simple and easy to understand, but that may not be accessible to everyone."

6 Conclusion

Overall, we found that it is mathematically possible to construct a system where $\infty + 1 > \infty$ and even for infinite series. This could potentially solve many world conflicts, i leave for the scientific community to expand this concept.

Fields medal when?

References

[1] Me and my friend vinicius.

[2] I remember having this discussion when i was a kid.

[3] Vinicius again.