A Note on Invo-Regular Rings

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ABSTRACT

In this paper we provide an important and significant observation on a result related to invo-regular rings [1].

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Introduction

Let R is a unital and associative ring. A ring R is called invo-regular if for each $a \in R$ there exists $b \in Inv(R)$ such that $a = aba$ [1-3]. Here $Inv(R)$ is the set of all involutions. One may note that an element b of R satisfying $b^2 = 1$ is called an involution [1-3].

The proposition 2.5 of [1] states that a ring R is invo-regular iff $R \cong R_1 \times R_2$, here R_1 is an invo-regular ring of characteristic two and R_2 is an invo-regular ring of characteristic three.

In this note we improve this result by providing a suitable counterexample. In the next section we provide a counterexample for this result.

2. An Important Observation

$$\text{Let } R = \left\{ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 2 & 2 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 2 \\ 0 & 2 \end{pmatrix}, \begin{pmatrix} 0 & 2 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 2 \end{pmatrix} \right\}.$$

Then R is a commutative ring of characteristic three under addition and multiplication of matrices modulo three. We note that R is an invo-regular ring. Now we have the following cases.

Case I: $R \cong R \times \{0\}$. One may note that R is not a ring of characteristic two.

Case II: $R \cong \{0\} \times R$. It is clear that $\{0\}$ is not a ring of characteristic two.

Case III: $R \cong R_1 \times R_2$. Here $R_1 = Z_3 = R_2$. We note that the characteristic of $R_1 = Z_3$ is not two.

Thus we see that in the above example the characteristic of R_1 can never be two even though R is an invo-regular ring. Hence the above example serves as a counterexample for the above result of [1].

References

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