MONGE'S THEOREM AND DEPTH PERCEPTION

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In this brief note, we propose a novel mechanism that might contribute to the visual ability of depth perception. We suggest a mathematical approach based on the Monge's theorem which allows the perception of the object's distance from the observer's eye and its three-dimensional features.

Depth perception (DP) is the visual ability to perceive the distance of an object and its three-dimensional structure. DP arises from a variety of depth cues that are typically classified into:

1) Binocular cues. They are based on the receipt of sensory information in three dimensions from both eyes. They include:
    a) retinal disparity, which exploits parallax and vergence, and
    b) stereopsis, or retinal (binocular) disparity, or binocular parallax.

1) Monocular cues. They provide depth information when viewing a scene with one eye. Monocular cues include relative size (distant objects subtend smaller visual angles than near objects), texture gradient, occlusion, perspective, contrast differences (e.g., texture gradient, Lighting and shading), motion parallax, context-dependent interpretation of the size, accommodation.

Here we propose a novel mechanism that might be able to give rise to depth perception. Our approach is based on a theorem from geometry, the Monge's theorem (MT). MT states that for any three nonoverlapping circles of distinct radii in the two-dimensional analytical plane equipped with the Euclidean metric, none of which is completely inside one of the others, the intersection points of each of the three pairs of external tangent lines are collinear (Ermis and Gelisgen, 2021). See **Figure A**.

MT implies that the projections arising from three separated objects (i.e., two- or three-dimensional circles) give rise to a line that is external to the three objects. MT has been operationally used to calculate the correct camera position for the wearing examination of the hob's cutting edge, by calculating the inner points of the Monge cuboid and their parallel shifting (defined by the bijective Monge projections) bordered by a surface (*Balajti Z, Dudás I. 2017. The Monge theorem and its application in engineering practice. Int J Adv Manuf Technol 91, 739–749. https://doi.org/10.1007/s00170-016-9763-1*). Further, MT is still valid even if the plane is equipped with non-Euclidean metrics (Ermis and Gelisgen, 2021).

Here we suggest to locate the line external to the three objects in the eye lens (**Figure B**). This line is produced by three sensed objects located in the environment. Due to MT, the projections of the three objects on the eye lens are slightly far apart one each other. The line is subsequently projected from the eye lens to the retina, giving rise to retinal objects that are slightly far apart one each other. This simple ocular mechanism, that does not involve higher mental faculties, might be the mathematical device that contributes to our three-dimensional perception of the world (**Figures C-D**).

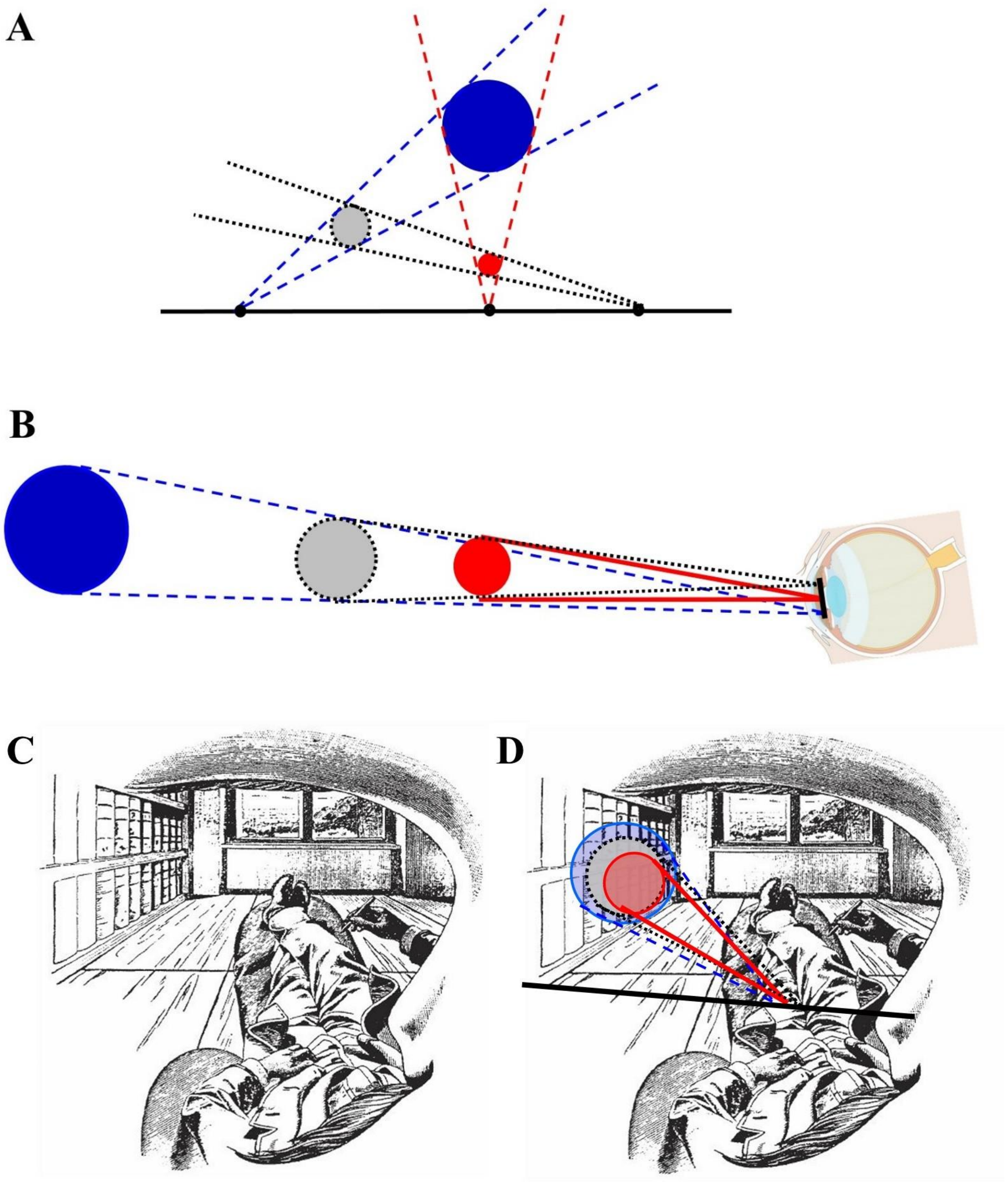

Figure. The Monge's theorem (MT) and its application to the evaluation of depth perception. **Figure A**: pictorial rendering of the Monge's theorem. **Figure B**: MT used to investigate the physiological mechanism of depth perception. The line on the eye lens corresponds to the line joining the projections from the three objects embedded in the environment. **Figures C-D**: an example of MT's depth perception drawn from the "classical" Mach's picture.