Orthogonality of Two Lines and Division by Zero Calculus

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September 10, 2021

Abstract: In this paper, we will give a pleasant representation of the orthogonality of two lines by means of the division by zero calculus. For two lines with gradients m and M, they are orthogonal if $mM = -1$. Our common sense will be so stated. However, note that for the typical case of x, y axes, the statement is not valid. Even for the high school students, the new result may be pleasant with surprising new results and ideas.

David Hilbert:

The art of doing mathematics consists in finding that special case which contains all the germs of generality.

Oliver Heaviside:

Mathematics is an experimental science, and definitions do not come first, but later on.

Key Words: Division by zero, division by zero calculus, orthogonality, isolated singular point, $1/0 = 0/0 = z/0 = \tan\frac{\pi}{2} = 0$.

2010 AMS Mathematics Subject Classification: 34A24, 41A30, 41A27, 51N20, 00A05, 00A09, 42B20, 30E20.

1 Introduction and the result

In this paper, we will give a pleasant representation of the orthogonality of two lines by means of the division by zero calculus. For two lines with gradients m and M, they are orthogonal if

$$mM = -1.$$

Our common sense will be so stated. However, note that for the typical case of x, y axes, the statement is not valid. Even for the high school students, the new result may be pleasant with surprising new results and ideas.

Proposition: *Two non-parallel lines with gradients m and M are orthogonal if and only if*

$$M = -\frac{1}{m}.$$

Indeed, if $m \neq 0$, the result is well known and trivial. In order to consider the case $m = 0$, note that in a general line equation

$$ax + by + c = 0,$$

the gradient m is given by

$$m = -\frac{a}{b}$$

and the fraction has its mean always by the concept of the division by zero calculus. For the division by zero calculus, see the papers in the references.

In particular, note that $m = 0$ if and only if $a = 0$ or $b = 0$.

By the division by zero calculus, since for $m = 0$,

$$M = -\frac{1}{0} = 0,$$

we thus obtain the desired result by the condition in the proposition.

2 Remarks

The result

$$\tan \frac{\pi}{2} = 0$$

is the typical and important result of the division by zero calculus. This property was interpretated as a natural one by the Proposition. The result will show a new surprising concept and give a great impact to analysis and geometry as we see from the references in the Bibliography.

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