Field Theory of Temperature

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April 30, 2021

Abstract

Motivated by the well-known contradiction of special relativity and the heat equation, a wave equation for temperature scalar field is presetned. Applying the proposed equation to the Bekenstein-Hawking entropy, a wave equation is derived for temperature fluctuations of a Schwarzschild blackhole.

Contents

1 Incompatibility of Special Relativity and Fourier's theory of Heat Conduction

It is well known that the Fourier equation is incompatible with the theory of relativity[1] for at least one reason: it admits infinite speed of propagation of heat signals within the continuum field. For example, consider a pulse of heat at the origin; then according to Fourier equation, it is felt (i.e. temperature changes) at any distant point, instantaneously. The speed of information propagation is faster than the speed of light in vacuum, which is inadmissible within the framework of relativity. Therefore there is little doubt that the heat equation needs to be modified so that

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it will be a wave equation. There have been attempts of *Hyperbolic heat conduction*[2][3][4]

$$\frac{1}{C^2}\frac{\partial^2 T}{\partial t^2}+\frac{1}{\alpha}\frac{\partial T}{\partial t}=\nabla^2 T$$

but they are still unsatisfactory in that they contain both first and second derivatives, thus still not expressible in a special relativistic-invariant form. If we recall the well-known analogy of Fourier law (heat conduction)

$$\mathbf{q}=-k\nabla T,$$

and Ohm law

$$\mathbf{J}=\sigma\mathbf{E},$$

we can immediately see that **temperature is the potential function of the field of heat**; furthermore, we find that any attempt of hyperbolic heat conduction is misguided because such attempts are just like trying to modify Maxwell's equations to include speed of electromagnetic waves in media after observing that

$$\frac{\partial \mathbf{B}}{\partial t}=\frac{1}{\mu_0\sigma}\nabla^2\mathbf{B}$$

is not a wave equation!

2 Preliminary: Construction of classical field theories

In order to find a wave equation for T field, we must first try to see how a field theory is constructed.
Field theories essentially rely on a wave equation; the only characteristic of a wave is its speed of propagation and by Einstein's principle of constancy of velocity of light, all fields in vacuum propagate at the speed of light. The mathematical implementation of field theories is best done via the formalism of four-potentials, and this formalism is even applicable to Newtonian gravity resulting in $\Box\phi=-4\pi G\rho$. In this formalism one adds the relevant material source of fields (e.g. mass density for gravity, charge density for electrostatics) to the source term J^μ and by applying Maxwell equation $\Box A^\mu=-J^\mu$ arrives at the desired field equation. For example in case of gravity,

$$J^0=c\sqrt{4\pi\epsilon_0 G}\rho$$

and

$$A^0=\frac{\phi}{c\sqrt{4\pi\epsilon_0 G}}.$$

Needless to say that one must always pay attention to the dimensions so that the relevant potential is multiplied by proper constants to match the dimension of electric current density and magnetic vector potential. For gravity, one can find those constants by dimensional analysis, except for 4π and this indicates that the correct constant of gravitoelectromagnetism is $\eta_0=\sqrt{4\pi\epsilon_0 G}$, therefore we will use η_0 in all of our dimensional analysis considerations.

3 Wave Equation for Temperature Scalar Field

3.1 Fourier and Ohm law analogy

In light of the discussion of previous section, our task is simple: Add the temperature scalar field to the four-potential. A dimensional analysis using all the known fundamental constants of physics shows that the right constant whose multiplication with T will make for T possible to sit in the four-potential of electromagnetism is

$$\frac{k_B}{\sqrt{4\pi\epsilon_0\hbar c^3}};$$

Thus,

$$A^0 := \frac{k_B}{\sqrt{4\pi\epsilon_0\hbar c^3}} T(\mathbf{x}, t),$$

using the 'recipe' mentioned in [5] the source term would be

$$J^0 := \frac{\sqrt{4\pi\epsilon_0\hbar c^3}}{k_B}\sigma,$$

a dimensional analysis would lead to

$$[\sigma] = \text{entropy per volume}$$

which means in this view, enotrpy is the source of temperature scalar field. To find the field equation we simply apply Maxwell equation $\Box A^\mu = -J^\mu$, which yields

$$\boxed{\Box T = -\frac{4\pi\hbar c}{k_B^2}\sigma} \tag{1}$$

3.2 Temperature fluctuations of event horizon

Bekenstein-Hawking formula for entropy of a Schwarzschild blackhole being[6][7],

$$S = \frac{k_B A}{4l_P^2},$$

to make application of (1) possible, we need to divide the Bekenstein-Hawking formula by the volume of the blackhole; as

$$A = 4\pi r_s^2,$$

and

$$V = \frac{4}{3}\pi r_s^3,$$

we have

$$\sigma = \frac{S}{V} = \frac{k_B(4\pi r^2)}{4l_P^2 V} = \frac{3}{4}\frac{k_B}{r_s l_P^2},$$

and

$$r_s = \frac{2GM}{c^2} \Longrightarrow \sigma = \frac{3k_B c^2}{8GMl_P^2}. \tag{2}$$

Inserting (2) in (1) yields the temperature fluctuations of the event horizon of a Schwarzshild blackhole

$$\Box T = -\frac{3\pi c^6}{2Mk_B G^2}. \tag{3}$$

4 Entropy of single particles

Via the principle of interaction-property correspondence[5], one can conclude that single particles possess entropy. If that is the case, rest-energy of a *particle of entropy*, is given by

$$\boxed{E = T_P S = \sqrt{\frac{\hbar c^5}{G k_B^2}} S_0} \tag{4}$$

In special circumstances where conservation of energy allows for mass to be converted to entropy, one can write

$$\sqrt{\frac{\hbar c^5}{G k_B^2}} S_0 = m_0 c^2; \tag{5}$$

in such circumstances (5) would yield the entropy of an electron, for example, to be

$$S_{0,\text{electron}} \approx 5.7 \times 10^{-46} \text{ Joules per Kelvins.}$$

5 Afterword

In [5] we presented a field equation for entropy, based on taking

$$k_B T = mc^2$$

seriously. There is an issue which signals for incompleteness of this analogy: If this analogy is correct then k_B must be the maximum entropy possible in universe, but Boltzmann constant is extremely small[1] and having a maximum entropy in the universe seems to face problems with the second law of thermodynamics.
Another problem for the field theory of entropy is that fluctuations of entropy are extremely weak and hence the theory might remain irrefutable in any time in our lifetimes.
In this paper we took

$$\mathbf{q} = -k\nabla T \quad \text{V.S.} \quad \mathbf{J} = \sigma \mathbf{E}$$

[1] One can use $S = k_B \log W$ to infer that since $\log W$ is always a positive number, k_B is in fact the minimum entropy possible in the universe; but such reasoning cannot be ontologically original in that scheme for $S = k_B \log W$ can only be an effective formula. More importantly, this conclusion seems to be wrong in view of (5), as the entropy of electron is much smaller than k_B.

seriously and found a field equation for the temperature scalar field. This theory seems more easy to refute and the analogy resulting from

$$\sqrt{\frac{\hbar c^5}{G k_B^2}} S = mc^2$$

is perfect, for both c^2 and $\sqrt{\hbar c^5/G k_B^2}$ are maximums in their realm.

If any of these theories turn out to be right, radical changes of our understanding of statistical mechanics will be required.

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