

# The exterior connections

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## Abstract

We define the notion of a exterior connection which is a connection for the exterior forms.

## 1 The Koszul connections

Let  $M$  be a manifold and  $E$ , a vector fiber bundle over  $M$ . The Koszul connections are operators acting on sections such that:

$$\nabla_X(f.s) = Xf.s + f.\nabla_X(s)$$

$f$  is a smooth function and  $s$  is a section of the fiber bundle  $E$ .

## 2 The exterior connections

### 2.1 Definition

An exterior fiber bundle is a vector fiber bundle which is a modulus for the exterior algebra. The exterior connections  $\nabla$  are such that:

$$(\alpha \wedge \beta).s = \alpha.(\beta.s)$$

$$\nabla(\alpha.s) = d\alpha.s + (-1)^{\deg(\alpha)}\alpha.\nabla(s)$$

with  $\alpha, \beta \in \Lambda^*(TM)$  and  $s$  a section of  $E$ .

### 2.2 Curvature

The curvature of the connection is  $R = \nabla \circ \nabla$  and is linear, such that:

$$R(\alpha.s) = \alpha.R(s)$$

### 2.3 Characteristic classes

The characteristic classes are defined in the cohomology:

$$c_k = tr(R^k)$$

## References

- [BG] R.Bishop, S.Goldberg, "Tensor Analysis on Manifolds", Dover, New-York, 2014.