Medial Triangle and Irrational Numbers

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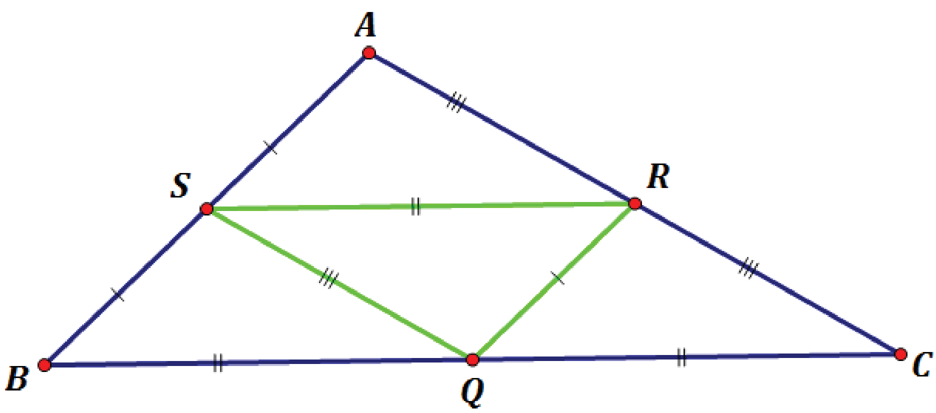
Abstract

If the co-ordinates of a triangle is rational, then there is at least one irrational number present that will satisfy the equation of sides of its medial triangle.

Proof- A rational number can be defined as number which cannot be expressed as the ratio of two integers, or we can say it is a decimal number having non-recurring infinite decimal expansion.

We can also define rational number as, those numbers which have fixed distance from two other rational numbers, $y = \frac{(y-1)+(y+1)}{2}$ here y is a rational number having fixed distance from (y-1) and (y-2). So, from this we can deduce a irrational numbers are those which doesn't have fixed distance from rational numbers.

Now, coming to the proof, if all the (x,y) coordinates of 3 sides of a triangle is rational then the co-ordinate of mid-points (S,R,Q) of the sides will also rational as it has fixed distance from 2 rational numbers. consider the figure given below.



Now, (S,R) have rational co-ordinates then, as we know there exists at least one irrational numbers between 2 rational numbers so, There will be at least one irrational number that will satisfy the equation of line (S,R) in this case. Hence proved, similarly we can also prove this for rest of the sides of medial triangle.

One, more strong observation is that we can also say every line in XY plane will satisfy at least one irrational number as we can make $y = mx + c$ as a side of medial triangle of rational co-ordinate and prove it.