On Intuitionistic Fuzzy Soft Ideal Topological Spaces

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Abstract: In this paper, we introduce the notion of intuitionistic fuzzy soft ideal in intuitionistic fuzzy soft set theory. Also we introduce the concept of intuitionistic fuzzy soft local function. These concepts are discussed with a view to find new intuitionistic fuzzy soft topologies from the original one. The basic structure, especially a basis for such generated intuitionistic fuzzy soft topologies also studied here. The notion of compatibility of intuitionistic fuzzy soft ideals with intuitionistic fuzzy soft topologies is introduced and some equivalent conditions concerning this topic are established here. Finally we introduce intuitionistic fuzzy soft-I-open set, intuitionistic fuzzy soft pre-I-open set, intuitionistic fuzzy soft semi-I-open set, intuitionistic fuzzy soft-α-I-open set and intuitionistic fuzzy soft-β-I-open set and discuss some of their properties.

Keywords: Intuitionistic fuzzy soft sets, intuitionistic fuzzy soft ideal, Intuitionistic fuzzy soft local function, Intuitionistic fuzzy ∗-soft topology, Compatible intuitionistic fuzzy soft ideal.

1. Introduction

The concept of a soft sets was introduced by Molodtsov in 1999 [1]. Initiated the theory of soft sets as a new mathematical tool for dealing with uncertainties. He has shown several applications of this theory in solving many practical problems in economics, engineering, social science, medical science, etc. Later other authors like Maji, Biswas and Roy [2] have further studied the theory of soft sets and used this theory to solve some decision-making problems. The concept of a fuzzy set was introduced by Zadeh in 1965 [3]. Intuitionistic fuzzy set as generalized fuzzy set is quite interesting and useful in many application areas. The concept intuitionistic fuzzy set become more meaningful, resourceful and applicable since it includes the degree of belongingness, degree of non-belongingness and the hesitation margin Atanassov in 1994 [5], 1999 [6]. In 2001 [7] and 2004 [8], Szmidt and Kacprzyk proved that intuitionistic fuzzy sets are pretty useful in situations when description of a problem by a linguistic variable given in terms of a membership function only seems too rough. Coker and Saadati in 1988 [9] defined the

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notion of intuitionistic fuzzy topology and studied the basic concept of intuitionistic fuzzy point [10]. The concept of an ideal in topological space was first introduced by Kuratowski in1966 [11] and Vaidyanathswamy in 1945 [12]. They also have defined local function in an ideal topological space. Further Hamlett and Jankovic in 1990 [13] studied the properties of an ideal topological spaces and they have introduced another operator called Ψ-operator. They have also obtained a new topology from original ideal topological space. Using the local function, they defined a Kuratowski closure operator in new topological space. In this paper, we introduce the notion of intuitionistic fuzzy soft ideal in intuitionistic fuzzy soft set theory. Also we introduce the concept of intuitionistic fuzzy soft local function. These concepts are discussed with a view to find new intuitionistic fuzzy soft topologies from the original one. The basic structure, especially a basis for such generated intuitionistic fuzzy soft topologies also studied here. The notion of compatibility of intuitionistic fuzzy soft ideals with intuitionistic fuzzy soft topologies is introduced and some equivalent conditions concerning this topic are established here. Finally we introduce intuitionistic fuzzy soft-I-open set, intuitionistic fuzzy soft pre-I-open set, intuitionistic fuzzy soft semi-I-open set, intuitionistic fuzzy soft-α-I-open set and intuitionistic fuzzy soft-β-I-open set and discuss some of their properties.

2. Preliminaries

Definition 2.1. [1] Let X be an initial universe set and E be a set of parameters. A pair (F, E) is called a soft set over X if only if F is a mapping from E into the set of all subsets of the set X, i.e., $F: E \to P(X)$, where P(X) is the power set of X.

Definition 2.2. [4] An intuitionistic fuzzy set A over the universe set X can be defined as follows

$$A =\{(x, \mu_A(x), \nu_A(x)): x \in X\}$$

Where the function $\mu_A: X \to [0, 1]$ and $\nu_A: X \to [0, 1]$ with the property $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. The values $\mu_A(x)$ and $\nu_A(x)$ represent the degree of membership and non-membership of x to A respectively.

Definition 2.3. [4] Let $A =\{(x, \mu_A(x), \nu_A(x)): x\in X\}$ and $B =\{(x, \mu_B(x), \nu_B(x)): x\in X\}$ be intuitionistic fuzzy sets of X. Then

i) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$, for all $x\in X$.

ii) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.

iii) $A^c = \{(x, \nu_A(x), \mu_A(x)): x \in X\}$.

iv) $A \cap B = \{(x, \min\{\mu_A(x), \mu_B(x)\}, \max\{\nu_A(x), \nu_B(x)\}): x \in X\}$.

v) $A \cup B = \{(x, \max\{\mu_A(x), \mu_B(x)\}, \min\{\nu_A(x), \nu_B(x)\}): x \in X\}$.

Definition 2.4. [4] An intuitionistic fuzzy set A over the universe set X defined as $A=\{(x, 0, 1): x \in X\}$ is said to be intuitionistic fuzzy null set and is denoted by $\tilde{0}$.

Definition 2.5. [4] An intuitionistic fuzzy set A over the universe set X defined as $A = \{(x, 1, 0): x \in X\}$ is said to be intuitionistic fuzzy absolute set and is denoted by $\tilde{1}$.

Definition 2.6. [14] Let X be an initial universe set and E be the set of parameters. Let P(X) denote the collection of all intuitionistic fuzzy subsets of X. Let $A \tilde{\subseteq} E$. A pair (F, A) or (F_A) is called an intuitionistic fuzzy soft set over X where F is a mapping given by $F: A \rightarrow P(X)$.

In general, for every $e \in A$, F(e) is an intuitionistic fuzzy set of X and it is called intuitionistic fuzzy value set of parameter e. Clearly, F(e) can be written as an intuitionistic fuzzy set such that $F(e) = \{(x, \mu_A(x), \nu_A(x)): x \in X:\}$.

Definition 2.7. [14] Let F_A and G_B be two intuitionistic fuzzy soft sets over X. we say that F_A is an intuitionistic fuzzy soft subset of G_B and write $F_A \tilde{\subseteq} G_B$ if

(i) $A \subseteq B$.
(ii) For all $e \in A$, $F(e) \subseteq G(e)$.

Definition 2.8. [14] Let F_A and G_B be two intuitionistic fuzzy soft sets over X. Then $F_A = G_B$ if and only if $F_A \tilde{\subseteq} G_B$ and $G_B \tilde{\subseteq} F_A$.

Definition 2.9. [14] Let F_A and G_B be two intuitionistic fuzzy soft sets over X. Then

Union $F_A \tilde{\cup} G_B = H_C$ where $C = A \cup B$ and for all $e \in C$

$$H(e) = \begin{cases} F(e) & \text{if } e \in A - B \\ G(e) & \text{if } e \in B - A \\ F(e) \cup G(e) & \text{if } e \in A \cap B \end{cases}$$

Intersection $F_A \tilde{\cap} G_B = H_C$ where $C = A \cap B$ and for all $e \in C$, $H(e) = F(e) \cap G(e)$.

Definition 2.10. [14] Let F_A be an intuitionistic fuzzy soft sets over X. Then the complement F_A is denoted by F_A^c and is defined by $(F, A)^c = (F^c, A)$, where $F^c : A \to P(X)$ is a mapping given by $F^c(e) = (F(e))^c$ for all $e \in A$. Thus if $F(e) = \{(x, \mu_{F(e)}(x), \nu_{F(e)}(x)): x \in X\}$, then for all $e \in A$, $F^c(e) = (F(e))^c = \{(x, \nu_{F(e)}(x), \mu_{F(e)}(x)): x \in X\}$.

Definition 2.11. [14] An intuitionistic fuzzy soft set F_A over X is said to be null intuitionistic fuzzy soft set and is denoted by $\widetilde{\varphi}$, if and only if for all $e \in A$, $F(e) = \{(x, 0, 1): x \in X\}$, where 0 is the membership function of the null fuzzy set over X and 1 is the membership function of the absolute intuitionistic fuzzy set over X.

Definition 2.12. [14] An intuitionistic fuzzy soft set F_A over X is said to be an absolute intuitionistic fuzzy soft set and is denoted by $\widetilde{1}$ if and only if for all $e \in A$, $F(e) = \{(x, 1, 0): x \in X\}$.

Definition 2.13. [15] Let F_A and G_B be two intuitionistic fuzzy soft sets over X. We define the difference of F_A and G_B as the intuitionistic fuzzy soft H_C written as $F_A - G_B = H_C$ where $C = A \cap B$ and for all $e \in C$, $x \in X$, $\mu_{H(e)}(x) = \min(\mu_{F(e)}(x), \nu_{G(e)}(x))$ and $\nu_{H(e)}(x) = \max(\nu_{F(e)}(x), \mu_{G(e)}(x))$.

Definition 2.14. [16] An intuitionistic fuzzy soft topology τ on (X, E) is a family of intuitionistic fuzzy soft sets over (X, E) satisfying the following properties

(i) $\widetilde{\varphi}, \widetilde{1} \,\widetilde{\in}\, \tau$.
(ii) if $F_A, G_B \,\widetilde{\in}\, \tau$, then $F_A \,\widetilde{\cap}\, G_B \,\widetilde{\in}\, \tau$.
(iii) if $F_{A_\alpha} \,\widetilde{\in}\, \tau$ for all $\alpha \,\widetilde{\in}\, \Delta$ an index set, then $\cup_{\alpha \in \Delta} F_{A_\alpha} \,\widetilde{\in}\, \tau$.

The triple (X, τ, E) is called an intuitionistic fuzzy soft topological space. If $F_A \,\widetilde{\in}\, \tau$, then the intuitionistic fuzzy soft sets F_A is said to be intuitionistic fuzzy soft open set. An intuitionistic fuzzy soft set F_A over X is said to be an intuitionistic fuzzy soft closed set in X, if its complement F_A^c belongs to τ.

Definition 2.15. [17] Let (X, τ, E) be an intuitionistic fuzzy soft topological space and let F_A be an intuitionistic fuzzy soft set over (X, E). The intuitionistic fuzzy soft closure of F_A is defined as the intersection of all intuitionistic fuzzy soft closed sets which contained F_A and is denoted by

$cl(F_A)$ we write

$$cl(F_A) = \widetilde{\cap}\{G_B : G_B \text{ is intuitionistic fuzzy soft closed and } F_A \widetilde{\subseteq} G_B\}.$$

Definition 2.16. [17] Let (X, τ, E) be an intuitionistic fuzzy soft topological space and let F_A be an intuitionistic fuzzy soft set over X. The intuitionistic fuzzy soft interior of F_A is defined as the union of all intuitionistic fuzzy soft open sets content in F_A and is denoted by $int(F_A)$. We write

$$int(F_A) = \widetilde{\cup}\{G_B : G_B \text{ is intuitionistic fuzzy soft open and } G_B \widetilde{\subseteq} F_A\}.$$

Definition 2.17. [17] Let (X, τ, E) be an intuitionistic fuzzy soft topological space and let F_A be an intuitionistic fuzzy soft set over X. An intuitionistic fuzzy soft point e_F is said to be in an intuitionistic fuzzy soft set G_A, denoted by $e_F \widetilde{\in} G_A$ if for the element $e \in A$, $F(e) \leq G(e)$.

Definition 2.18. [17] Let (X, τ, E) be an intuitionistic fuzzy soft topological space. An intuitionistic fuzzy soft set F_A is a neighborhood of an intuitionistic fuzzy soft set G_B if and only if there exists an intuitionistic fuzzy soft open set $Q_C \widetilde{\in} \tau$ such that $F_A \widetilde{\subseteq} Q_C \widetilde{\subseteq} G_B$.

Definition 2.19. [17] Let (X, τ, E) be an intuitionistic fuzzy soft topological space. An intuitionistic fuzzy soft set F_A is a neighborhood of the intuitionistic fuzzy soft point $e_F \widetilde{\in} F_A$ if there exists an intuitionistic fuzzy soft open set $Q_C \widetilde{\in} \tau$ such that $e_F \widetilde{\in} Q_C \widetilde{\subseteq} F_A$.

3. Intuitionistic fuzzy soft ideal topological spaces

Definition 3.1. A non-empty collection of intuitionistic fuzzy soft sets $\widetilde{I}$ of a soft set X is called an intuitionistic fuzzy soft ideal if the following properties are satisfies.

(i) If $F_A \widetilde{\in} \widetilde{I}$ and $G_B \widetilde{\in} \widetilde{I}$ then $F_A \widetilde{\cup} G_B \widetilde{\in} \widetilde{I}$.

(ii) If $F_A \widetilde{\in} \widetilde{I}$ and $G_B \widetilde{\subseteq} F_A$ then $G_B \widetilde{\in} \widetilde{I}$.

Example 3.1. Let (X, τ, E) be an intuitionistic fuzzy soft topological space. Then

1. If $\widetilde{I} = \{\widetilde{\varphi}\}$. Then $\widetilde{I}$ is an intuitionistic fuzzy soft ideal on (X, E).

2. If $\widetilde{I} = \{\widetilde{1}\}$. Then $\widetilde{I}$ is an intuitionistic fuzzy soft ideal on (X, E).

3. If $\widetilde{I} = \{\widetilde{\varphi}, F_A, G_B\}$, where $F_A = \{F(e_1) = \{(a, 0.7, 0.2), (b, 0.9, 0.1)\}, F(e_2) = \{(a, 0.3, 0.6), (b, 0.6, 0.2)\}\}$ and $G_B = \{G(e_1) = \{(a, 0.6, 0.3), (b, 0.7, 0.2)\}, G(e_2) = \{(a, 0.2, 0.7), (b, 0.4, 0.5)\}\}$. Then $\widetilde{I}$ is an intuitionistic fuzzy soft ideal on (X, E).

Definition 3.2. Let (X, τ, E) be an intuitionistic fuzzy soft topological space and let $\widetilde{I}$ be an intuitionistic soft ideal on (X, E). Then the triplet (X, τ, $\widetilde{I}$) is called intuitionistic fuzzy soft ideal topological space.

Definition 3.3. Let (X, τ, E) be an intuitionistic fuzzy soft topological space and let $\widetilde{I}$ be an intuitionistic soft ideal on (X, E). The intuitionistic fuzzy soft local function F_A with respect to τ and $\widetilde{I}$ is denoted by $F_A^*(\tau, \widetilde{I})$ or F_A^* and defined as

$$F_A^* = \widetilde{\cup}\{e_F \widetilde{\in} X : F_A \widetilde{\cap} Q_C \widetilde{\notin} \widetilde{I} \text{ for every } Q_C \widetilde{\in} \tau\}.$$

Theorem 3.1. Let (X, τ, E) be an intuitionistic fuzzy soft topological space and let $\widetilde{I}$ and $\widetilde{J}$ be any two intuitionistic soft ideal on (X, E). Let F_A and G_B are two intuitionistic fuzzy soft set. Then

(i) $\widetilde{\varphi}^* = \widetilde{\varphi}$.

(ii) $F_A \widetilde{\subseteq} G_B \Rightarrow F_A^* \widetilde{\subseteq} G_B^*$.

(iii) $(F_A \widetilde{\cup} G_B)^* = F_A^* \widetilde{\cup} G_B^*$.

(iv) $(F_A \widetilde{\cap} G_B)^* \widetilde{\subseteq} F_A^* \widetilde{\cap} G_B^*$.

(v) $\widetilde{I} \widetilde{\subseteq} \widetilde{J} \Rightarrow F_A^*(\widetilde{J}) \widetilde{\subseteq} F_A^*(\widetilde{I})$.

(vi) $F_A^* \widetilde{\subseteq} cl(F_A)$.

(vii) F_A^* is an intuitionistic fuzzy closed soft set.

Proof.

(i) Obvious from Definition 3.3.

(ii) Let $e_F \widetilde{\in} X$. Suppose $F_A \widetilde{\subseteq} G_B$. Therefore we have $F_A \widetilde{\subseteq} G_B$ which implies that $F_A \widetilde{\cap} Q_C \widetilde{\subseteq} G_B \widetilde{\cap} Q_C$, $Q_C \widetilde{\in} \tau$. Now $F_A^* = \{e_F \widetilde{\in} X: F_A \widetilde{\cap} Q_C \widetilde{\notin} \widetilde{I}$ for every $Q_C \widetilde{\in} \tau\} \widetilde{\subseteq} \{e_F \widetilde{\in} X: G_B \widetilde{\cap} Q_C \widetilde{\notin} \widetilde{I}$ for every $Q_C \widetilde{\in} \tau\} = G_B^*$. Therefore $F_A^* \widetilde{\subseteq} G_B^*$.

(iii) Let $e_F \widetilde{\in} (F_A \widetilde{\cup} G_B)^*$. Now $(F_A \widetilde{\cup} G_B)^* = \{e_F \widetilde{\in} X: (F_A \widetilde{\cup} G_B) \widetilde{\cap} Q_C \widetilde{\notin} \widetilde{I}$ for every $Q_C \widetilde{\in} \tau\}$
$\Rightarrow \{e_F \widetilde{\in} X: (F_A \widetilde{\cap} Q_C) \widetilde{\cup} (G_B \widetilde{\cap} Q_C) \widetilde{\notin} \widetilde{I}$, for every $Q_C \widetilde{\in} \tau\}$
$\Rightarrow \{e_F \widetilde{\in} X: (F_A \widetilde{\cap} Q_C) \widetilde{\notin} \widetilde{I}$ or $(G_B \widetilde{\cap} Q_C) \widetilde{\notin} \widetilde{I}$, for every $Q_C \widetilde{\in} \tau\}$
$\Rightarrow \{e_F \widetilde{\in} X: F_A \widetilde{\cap} Q_C \widetilde{\notin} \widetilde{I}$, for every $Q_C \widetilde{\in} \tau\}$ or $\{e_F \widetilde{\in} X: G_B \widetilde{\cap} Q_C \widetilde{\notin} \widetilde{I}$, for every $Q_C \widetilde{\in} \tau\}$
$\Rightarrow e_F \widetilde{\in} F_A^* \widetilde{\cup} G_B^*$. Therefore $((F_A \widetilde{\cup} G_B)^* \widetilde{\subseteq} F_A^* \widetilde{\cup} G_B^*$. Again we have $F_A \widetilde{\subseteq} F_A \widetilde{\cup} G_B$ and $G_B \widetilde{\subseteq}$

$F_A \tilde{\cup} G_B$. From (ii) we have $F_A^* \tilde{\subseteq} (F_A \tilde{\cup} G_B)^*$ and $G_B^* \tilde{\subseteq} (F_A \tilde{\cup} G_B)^*$. Therefore $F_A^* \tilde{\cup} G_B^* \tilde{\subseteq}$ $(F_A \tilde{\cup} G_B)^*$. Hence $(F_A \tilde{\cup} G_B)^* = F_A^* \tilde{\cup} G_B^*$.

(iv) We have $F_A \tilde{\cap} G_B \tilde{\subseteq} F_A$ and $F_A \tilde{\cap} G_B \tilde{\subseteq} G_B$. From (ii) we have $(F_A \tilde{\cap} G_B)^* \tilde{\subseteq} F_A^*$ and $(F_A \tilde{\cap} G_B)^* \tilde{\subseteq} G_B^*$. Therefore $(F_A \tilde{\cap} G_B)^* \tilde{\subseteq} F_A^* \tilde{\cap} G_B^*$.

(v) Let $e_F \tilde{\in} F_A^*(\tilde{J})$. Then $Q_C \tilde{\cap} F_A^* \tilde{\notin} \tilde{J}$ for every $Q_C \tilde{\in} \tau$. Since $\tilde{I} \tilde{\subseteq} \tilde{J}$. Then $Q_C \tilde{\cap} F_A^* \tilde{\notin} \tilde{I}$ for every $Q_C \tilde{\in} \tau$. Hence $e_F \tilde{\in} F_A^*(\tilde{I})$. Thus $F_A^*(\tilde{J}) \tilde{\subseteq} F_A^*(\tilde{I})$.

(vi) Assume that $e_F \tilde{\notin} cl(F_A)$. Then there exists $Q_C \tilde{\in} \tau$ such that $Q_C \tilde{\cap} F_A = \tilde{\varphi} \tilde{\in} \tilde{I}$. Hence $e_F \tilde{\notin} F_A^*$. Thus $F_A^* \tilde{\subseteq} cl(F_A)$.

(vii) Clearly $F_A^* \tilde{\subseteq} cl(F_A^*)$. So let $e_F \tilde{\in} cl(F_A^*)$. Then $Q_C \tilde{\cap} F_A^* \neq \tilde{\varphi}$ for every $Q_C \tilde{\in} \tau$. Hence there exists $e'_F \tilde{\in} Q_C \tilde{\cap} F_A^*$. Thus $e'_F \tilde{\in} Q_C$ and $e'_F \tilde{\in} F_A^*$. It follows that $Q_C(e'_F) \tilde{\cap} F_A \tilde{\notin} \tilde{I}$ for every $Q_C(e'_F) \tilde{\in} \tau$. This implies that $Q_C \tilde{\cap} F_A \tilde{\notin} \tilde{I}$ for every $Q_C \tilde{\in} \tau$. So $e_F \tilde{\in} F_A^*$. This means that $cl(F_A^*) \tilde{\subseteq} F_A^*$. Therefore $F_A^* = cl(F_A^*)$.

Definition 3.4. Let (X, τ, E) be an intuitionistic fuzzy soft topological space and let $\tilde{I}$ be an intuitionistic soft ideal on (X, E). The intuitionistic fuzzy soft closure operator of an intuitionistic fuzzy soft set F_A in $(X, \tau, \tilde{I})$ is defined as $cl^*(F_A) = F_A \tilde{\cup} F_A^*$.

Theorem 3.2. Let (X, τ, E) be an intuitionistic fuzzy soft topological space and let $\tilde{I}$ be an intuitionistic soft ideal on (X, E). Then if $F_A \tilde{\subseteq} G_B$, then $cl^*(F_A) \tilde{\subseteq} cl^*(G_B)$.

Proof. Since $cl^*(F_A)(x) = (F_A \tilde{\cup} F_A^*)(x) = \max\{F_A(x), F_A^*(x)\} \tilde{\subseteq} \max\{G_B(x), G_B^*(x)\} = cl^*(G_B)(x)$. Hence $cl^*(F_A) \tilde{\subseteq} cl^*(G_B)$.

Definition 3.5. Let $(X, \tau, \tilde{I})$ be an intuitionistic fuzzy soft ideal topological space and let cl^* : $P(X) \to P(X)$ be the intuitionistic fuzzy soft closure operator. Then there exists a unique intuitionistic fuzzy soft topology over X finer than τ, called the $*$-intuitionistic fuzzy soft topology, denoted by $\tau^*(\tilde{I})$ or τ^*, given by

$$\tau^*(\tilde{I}) = \{F_A \text{ intuitionistic fuzzy soft: } cl^*(F_A) = F_A\}.$$

Example 3.2. Let $(X, \tau, \tilde{I})$ be an intuitionistic fuzzy soft ideal topological space. Then

1. If $\tilde{I} = \{\tilde{\varphi}\}$. Then $F_A^* = cl(F_A)$. Hence $cl^*(F_A) = cl(F_A)$ and $\tau^* = \tau$.

2. If $\tilde{I} = \{\tilde{1}\}$. Then $F_A^* = \tilde{\varphi}$. Hence $cl^*(F_A) = F_A$ and $\tau^* = P(X)$.

3. If $\widetilde{I} \tilde{\subseteq} \widetilde{J}$. Then $F^*_A(\widetilde{J}) \tilde{\subseteq} F^*_A(\widetilde{I})$. Hence $\tau^*(\widetilde{I}) \tilde{\subseteq} \tau^*(\widetilde{J})$.

Theorem 3.3. Let $(X, \tau, \widetilde{I})$ be an intuitionistic fuzzy soft ideal topological space. Then

$$\beta(\widetilde{I}, \tau) = \{F_A - G_B: F_A \tilde{\in} \tau, G_B \tilde{\in} \widetilde{I}\}.$$

is an intuitionistic fuzzy soft basis for the intuitionistic fuzzy soft topology $\tau^*(\widetilde{I})$.

Proof. Since $\tilde{1} \tilde{\in} \tau$, $\tilde{\varphi} \tilde{\in} \widetilde{I}$. Then $\tilde{1} - \tilde{\varphi} \tilde{\in} \beta$, hence $\tilde{1}$, $\tilde{\varphi} \tilde{\in} \beta$ and $\tilde{\cup}_{j \tilde{\in} J}(F_{Aj} - G_{Bj}) = \tilde{1}$. Also let $(F_A - G_B)$, $(F'_A - G'_B) \tilde{\in} \beta$ such that $e_F \tilde{\in} (F_A - G_B) \tilde{\cap} (F'_A - G'_B) = (F_A \tilde{\cap} F'_A)\text{-}(G_B \tilde{\cup} G'_B) \tilde{\in} \beta$. Thus β is an intuitionistic fuzzy soft basis for the intuitionistic fuzzy soft topology $\tau^*(\widetilde{I})$.

Corollary 3.1. Let $(X, \tau, \widetilde{I})$ be an intuitionistic fuzzy soft ideal topological space. Then $\tau \tilde{\subseteq} \beta(\widetilde{I}, \tau) \tilde{\subseteq} \tau^*(\widetilde{I})$.

Proof. Immediate from Theorem 3.1 (v) and Theorem 3.3.

4. Compatibility of intuitionistic fuzzy soft ideals with intuitionistic fuzzy soft topology

Definition 4.1. Let $(X, \tau, \widetilde{I})$ be an intuitionistic fuzzy soft ideal topological space. We say that the intuitionistic fuzzy soft topology τ is compatible with the intuitionistic fuzzy soft ideal $\widetilde{I}$, denoted by $\tau \sim \widetilde{I}$, if the following holds for every intuitionistic fuzzy soft set F_A: if for every intuitionistic fuzzy soft point e_F there exists $Q_C \tilde{\in} \tau$ such that $Q_C \tilde{\cap} F_A \tilde{\in} \widetilde{I}$, then $F_A \tilde{\in} \widetilde{I}$.

Theorem 4.1. Let $(X, \tau, \widetilde{I})$ be an intuitionistic fuzzy soft ideal topological space and let $\tau \sim \widetilde{I}$. Then the following are equivalent:

(i) For every intuitionistic fuzzy soft set F_A, $F_A \tilde{\cap} F^*_A = \tilde{\varphi}$.

(ii) For every intuitionistic fuzzy soft set F_A, $(F_A - F^*_A)^* = \tilde{\varphi}$.

(iii) For every intuitionistic fuzzy soft set F_A, $(F_A \tilde{\cap} F^*_A)^* = F^*_A$.

Proof. (i) ⇒ (ii) Let F_A an intuitionistic fuzzy soft set. Since $(F_A - F^*_A) \tilde{\cap} (F_A - F^*_A)^* = \tilde{\varphi}$. Then $(F_A - F^*_A)^* = \tilde{\varphi}$ by (i).

(ii) ⇒ (iii) Let F_A an intuitionistic fuzzy soft set. Since $F_A = (F_A - (F_A \tilde{\cap} F^*_A)) \tilde{\cup} (F_A \tilde{\cap} F^*_A))$. Then $F^*_A = (F_A - (F_A \tilde{\cap} F^*_A)) \tilde{\cup} (F_A \tilde{\cap} F^*_A))^* = (F_A - (F_A \tilde{\cap} F^*_A))^* \tilde{\cup} (F_A \tilde{\cap} F^*_A)^* = \tilde{\varphi} \tilde{\cup} (F_A \tilde{\cap} F^*_A)^* = (F_A \tilde{\cap} F^*_A)^*$.

(iii) ⇒ (i) Let F_A an intuitionistic fuzzy soft set and let $F_A \tilde{\cap} F_A^* = \tilde{\varphi}$. Then $F_A^* = (F_A \tilde{\cap} F_A^*)^* = (\tilde{\varphi})^* = \tilde{\varphi}$. Hence $F_A \tilde{\in} \tilde{I}$.

Corollary 4.1. Let $(X, \tau, \tilde{I})$ be an intuitionistic fuzzy soft ideal topological space and let $\tau \sim \tilde{I}$. Then $(F_A^*)^* = F_A^*$.

Proof. Let F_A an intuitionistic fuzzy soft set. Since $F_A^* = (F_A \tilde{\cap} F_A^*)^* \tilde{\subseteq} (F_A^*)^*$ from Theorem 4.1(iii). But we have $(F_A^*)^* \tilde{\subseteq} F_A^*$. Thus $(F_A^*)^* = F_A^*$.

Theorem 4.2. Let $(X, \tau, \tilde{I})$ be an intuitionistic fuzzy soft ideal topological space. Then the following are equivalent:

(i) $\tau \sim \tilde{I}$.

(ii) For every intuitionistic fuzzy soft set F_A, $F_A \tilde{\cap} F_A^* = \tilde{\varphi}$, $F_A \tilde{\in} \tilde{I}$.

(iii) For every intuitionistic fuzzy soft set F_A, $F_A - F_A^* \tilde{\in} \tilde{I}$.

(iv) For every τ^*-intuitionistic fuzzy soft closed set F_A, $F_A - F_A^* \tilde{\in} \tilde{I}$.

(v) For every intuitionistic fuzzy soft set F_A, if F_A contains no non null intuitionistic fuzzy soft set G_B with $G_B \tilde{\subseteq} G_B^*$ then $F_A \tilde{\in} \tilde{I}$.

Proof. (i) ⇒ (ii) Let F_A an intuitionistic fuzzy soft set such that $F_A \tilde{\cap} F_A^* = \tilde{\varphi}$. Then for every $e_F \tilde{\in} F_A$ and $e_F \tilde{\notin} F_A^*$ we have $Q_C \tilde{\cap} F_A \tilde{\in} \tilde{I}$ for every $Q_C \tilde{\in} \tau$. Thus $F_A \tilde{\in} \tilde{I}$ by (i).

(ii) ⇒ (iii) Let F_A an intuitionistic fuzzy soft set. Since $(F_A - F_A^*) \tilde{\cap} (F_A - F_A^*)^* = (F_A \tilde{\cap} F_A^*) \tilde{\cap} (F_A - F_A^*)^* \tilde{\subseteq} (F_A \tilde{\cap} F_A^*) \tilde{\cap} F_A^* = \tilde{\varphi}$. Then $F_A - F_A^* \tilde{\in} \tilde{I}$ by (ii).

(iii) ⇒ (iv) Let F_A an τ^*-intuitionistic fuzzy soft closed set. Then $F_A - F_A^* \tilde{\in} \tilde{I}$ by (iii).

(iv) ⇒ (i) Let F_A an intuitionistic fuzzy soft set and assume that for every $e_F \tilde{\in} F_A$ there exists $Q_C \tilde{\in} \tau$ such that $Q_C \tilde{\cap} F_A \tilde{\in} \tilde{I}$. Then $e_F \tilde{\notin} F_A^*$. Hence $F_A \tilde{\cap} F_A^* = \tilde{\varphi}$ and since $F_A \tilde{\cup} F_A^*$ is τ^*-intuitionistic fuzzy soft closed set, we have $(F_A \tilde{\cup} F_A^*) - (F_A \tilde{\cup} F_A^*)^* \tilde{\in} \tilde{I}$ by (iv). Hence $(F_A \tilde{\cup} F_A^*) - (F_A \tilde{\cup} F_A^*)^* = (F_A \tilde{\cup} F_A^*) - F_A^* = F_A \tilde{\in} \tilde{I}$. Thus $\tau \sim \tilde{I}$.

(iii) ⇒ (v) Let F_A an intuitionistic fuzzy soft set such that F_A contains no null intuitionistic fuzzy soft set G_B with $G_B \tilde{\subseteq} G_B^*$. Since $F_A \tilde{\cap} F_A^* \tilde{\subseteq} F_A^* = (F_A \tilde{\cap} F_A^*)^*$. It follows that $F_A \tilde{\cap} F_A^* \tilde{\subseteq} (F_A \tilde{\cap} F_A^*)^*$. By assumption, $F_A \tilde{\cap} F_A^* = \tilde{\varphi}$. Thus $F_A = F_A - F_A^* \tilde{\in} \tilde{I}$ by (iii).

(v) ⇒ (iii) Let F_A an intuitionistic fuzzy soft set. Since $(F_A - F_A^*) \tilde{\cap} (F_A - F_A^*)^* = \tilde{\varphi}$ and $F_A - F_A^*$ contains no null intuitionistic fuzzy soft set G_B with $G_B \tilde{\subseteq} G_B^*$. Hence $F_A - F_A^* \tilde{\in} \tilde{I}$ by (v).

Theorem 4.3. Let $(X, \tau, \tilde{I})$ be an intuitionistic fuzzy soft ideal topological space and let $\tau \sim \tilde{I}$. Then an intuitionistic fuzzy soft set is τ^*-intuitionistic fuzzy soft closed set if and only if it is the union of an τ^*-intuitionistic fuzzy soft closed set and an intuitionistic fuzzy soft set in $\tilde{I}$.

Proof. Let F_A be a τ^*-intuitionistic fuzzy soft closed set. Then $cl^*(F_A) = F_A$ and $F_A \tilde{\cup} F_A^* = F_A$. Hence $F_A^* \tilde{\subseteq} F_A$. Thus $F_A = (F_A - F_A^*) \tilde{\cup} F_A^*$, $F_A - F_A^* \tilde{\in} \tilde{I}$ from Theorem 4.2 and F_A^* is τ -intuitionistic fuzzy soft closed set. Conversely, let $F_A = G_B \tilde{\cup} I_E$, where G_B is τ- intuitionistic fuzzy soft closed set and $I_E \tilde{\in} \tilde{I}$. Then $F_A^* = (G_B - I_E)^* = G_B^* \tilde{\subseteq} cl(G_B) = G_B \tilde{\subseteq} F_A$. Hence $F_A \tilde{\cup} F_A^* = F_A$. Thus $cl^*(F_A) = F_A$. It follows that F_A is a τ^*-intuitionistic fuzzy soft closed set.

5. Intuitionistic fuzzy soft I-open set

Definition 5.1. Let $(X, \tau, \tilde{I})$ be an intuitionistic fuzzy soft ideal topological space. An intuitionistic fuzzy soft set F_A on (X, E) is said to be intuitionistic fuzzy soft I-open set if $F_A \tilde{\subseteq} int(F_A^*)$, where F_A^* is the intuitionistic fuzzy soft local function of F_A.

Definition 5.2. Let $(X, \tau, \tilde{I})$ be an intuitionistic fuzzy soft ideal topological space. An intuitionistic fuzzy soft set F_A on (X, E) is said to be

(i) Intuitionistic fuzzy soft semi-I-open set if $F_A \tilde{\subseteq} cl^*(int(F_A)$.

(ii) Intuitionistic fuzzy soft pre-I-open set if $F_A \tilde{\subseteq} int(cl^*(F_A)$.

(iii) Intuitionistic fuzzy soft α-I-open set if $F_A \tilde{\subseteq} int(cl^*(int(F_A)))$.

(iv) Intuitionistic fuzzy soft β-I-open set if $F_A \tilde{\subseteq} cl(int(cl^*(F_A))$.

An intuitionistic fuzzy soft subset A of an intuitionistic fuzzy soft ideal topological space $(X, \tau, \tilde{I})$ is said to be intuitionistic fuzzy soft-I-closed set (resp. intuitionistic fuzzy soft semi-I-closed set, intuitionistic fuzzy soft pre-I-closed set, intuitionistic fuzzy soft α-I-closed set, intuitionistic fuzzy soft β-I-closed set) if its complement is intuitionistic fuzzy soft -I-open set (resp. intuitionistic fuzzy soft semi-I-open set, intuitionistic fuzzy soft pre-I-open set, intuitionistic fuzzy soft α-I-open set, intuitionistic fuzzy soft β-I-open set).

Theorem.5.1. Let $(X, \tau, \widetilde{I})$ be an intuitionistic fuzzy soft ideal topological space. Then the following statements hold:

1) Every intuitionistic fuzzy soft open set is intuitionistic fuzzy soft α-I-open set,

2) Every intuitionistic fuzzy soft-I-open set is intuitionistic fuzzy soft pre-I-open set,

3) Every intuitionistic fuzzy soft α-I-open set is intuitionistic fuzzy soft semi-I-open set,

4) Every intuitionistic fuzzy soft α-I-open set is intuitionistic fuzzy soft pre-I-open set,

5) Every intuitionistic fuzzy soft semi-I-open set is intuitionistic fuzzy soft β-I-open set,

6) Every intuitionistic fuzzy soft pre-I-open set is intuitionistic fuzzy soft β-I-open set.

Proof. 1) Let F_A be an intuitionistic fuzzy soft open set. Then, we have $F_A = int(F_A)$ since, $F_A \widetilde{\subseteq} int(F_A) \widetilde{\subseteq} cl^*(int(F_A))$. But F_A is intuitionistic fuzzy soft open set then $F_A = int(F_A) \widetilde{\subseteq} int(cl^*(int(F_A)))$. This shows that F_A is intuitionistic fuzzy soft α-I-open set.

2) Let F_A be an intuitionistic fuzzy soft-I-open set. Then, we have $F_A \widetilde{\subseteq} int(F_A^*)$, but $F_A^* \widetilde{\subseteq} cl^*(F_A)$, then $F_A \widetilde{\subseteq} int(cl^*(F_A))$. This shows that F_A is intuitionistic fuzzy soft pre-I-open set.

3) Let F_A be an intuitionistic fuzzy soft α-I-open set. Then, we have $F_A \widetilde{\subseteq} int(cl^*(int(F_A))) \widetilde{\subseteq} cl^*(int(F_A))$ This shows that F_A is intuitionistic fuzzy soft semi-I-open set.

4) Let F_A be an intuitionistic fuzzy soft α-I-open set. Then, we have $F_A \widetilde{\subseteq} int(cl^*(int(F_A))) \widetilde{\subseteq} int(cl^*(F_A))$ This shows that F_A is intuitionistic fuzzy soft pre-I-open set.

5) Let F_A be an intuitionistic fuzzy soft semi-I-open set. Then, we have $F_A \widetilde{\subseteq} cl^*(int(F_A)) \widetilde{\subseteq} cl(int(cl^*(F_A)))$. This shows that F_A is intuitionistic fuzzy soft β-I-open set.

6) Let F_A be an intuitionistic fuzzy soft pre-I-open set. Then, we have $F_A \widetilde{\subseteq} int(cl^*(F_A)) \widetilde{\subseteq} cl(int(cl^*(F_A)))$. This shows that F_A is intuitionistic fuzzy soft β-I-open set.

Example 5.1. Let $F_A = \{F(e_1) = \{(a, 0.4, 0.5), (b, 0.3, 0.7)\}, F(e_2) = \{(a, 0.7, 0.2), (b, 0.5, 0.4)\}\}$, $\tau = \{\widetilde{\varphi}, \widetilde{1}, F_A\}$ and $\widetilde{I} = \{\widetilde{\varphi}\}$. Therefore $(X, \tau, \widetilde{I})$ is an intuitionistic fuzzy soft topological space. Let $G_B = \{G(e_1) = \{(a, 0.6, 0.4), (b, 0.7, 0.2)\}, G(e_2) = \{(a, 0.8, 0.1), (b, 0.9, 0.1)\}\}$ be an intuitionistic fuzzy soft set. Then $cl^*(int(G_B)) = cl^*(F_A)$. Since $F_A^* = \{e_1 = \{(a, 0.6, 0.5), (b, 0.7, 0.3)\}, e_2 = \{(a, 0.3, 0.8), (b, 0.5, 0.6)\}\}$. Therefore $cl^*(F_A) = F_A \widetilde{\cup} F_A^* = \{e_1 = \{(a, 0.6, 0.5), (b, 0.7, 0.3)\}, e_2 = \{(a, 0.7, 0.2), (b, 0.5, 0.4)\}\}$. Therefore $cl^*(int(G_B)) = cl^*(F_A) = \{e_1 = \{(a, 0.6, 0.5), (b, 0.7, 0.3)\}, e_2 = \{(a, 0.7, 0.2), (b, 0.5, 0.4)\}\}$. Hence $int(cl^*(int(G_B))) = \widetilde{1}$. Therefore $G_B \widetilde{\subseteq} int(cl^*(int(G_B)))$. i.e. G_B be an intuitionistic fuzzy soft α-I-open set but not intuitionistic fuzzy soft open set.

Example 5.2. Let $F_A = \{F(e_1) = \{(a, 0.8, 0.2), (b, 0.8, 0.1)\}, F(e_2) = \{(a, 0.6, 0.3), (b, 0.6, 0.4)\}\}$, $G_B = \{G(e_1) = \{(a, 0.2, 0.8), (b, 0.2, 0.8)\}, G(e_2) = \{(a, 0.4, 0.6), (b, 0.4, 0.6)\}\}$, $\tau =\{\widetilde{\varphi}, \widetilde{1}, F_A, G_B\}$ and $\widetilde{I} = \{\widetilde{\varphi}\}$. Then $(X, \tau, \widetilde{I})$ be an intuitionistic fuzzy soft topological space. Let $H_C = \{H(e_1) = \{(a, 0.6, 0.4), (b, 0.3, 0.7)\}, H(e_2) = \{(a, 0.9, 0.1), (b, 0.2, 0.8)\}\}$ be an intuitionistic fuzzy soft set. Since H_C^* is intuitionistic fuzzy soft closed in τ. Hence $cl^*(H_C) = H_C \widetilde{\cup} H_C^* =\{e_1 = \{(a, 0.4, 0.6), (b, 0.7, 0.3)\}, e_2 = \{(a, 0.1, 0.9), (b, 0.8, 0.2)\}\}$. Therefore $H_C \widetilde{\subseteq} int(cl^*(H_C))$. H_C is an intuitionistic fuzzy soft pre-I-open set. Also $H_C \widetilde{\subseteq} cl(int(cl^*(H_C)))$. Hence H_C is an intuitionistic fuzzy soft β-I-open set but not intuitionistic fuzzy soft α-I-open set.

Example 5.3. Let $F_A = \{F(e_1) = \{(a, 0.4, 0.6), (b, 0.3, 0.6)\}, F(e_2) = \{(a, 0.7, 0.3), (b, 0.5, 0.4)\}\}$, $\tau =\{\widetilde{\varphi}, \widetilde{1}, F_A\}$ and $\widetilde{I} = \{\widetilde{\varphi}\}$. Then $(X, \tau, \widetilde{I})$ be an intuitionistic fuzzy soft topological space. Let $G_B =\{G(e_1) = \{(a, 0.6, 0.4), (b, 0.7, 0.2)\}, G(e_2) =\{(a, 0.3, 0.6), (b, 0.5, 0.4)\}\}$ be an intuitionistic fuzzy soft set. Then $int(G_B) = F_A$. Hence $F_A^* = \{(e_1) = \{(a, 0.6, 0.4), (b, 0.7, 0.2)\}, (e_2) = \{(a, 0.3, 0.6), (b, 0.5, 0.4)\}\}$. Therefore $cl^*(F_A) = F_A \widetilde{\cup} F_A^* =\{e_1 = \{(a, 0.6, 0.4), (b, 0.7, 0.2)\}, e_2 = \{(a, 0.7, 0.3), (b, 0.5, 0.4)\}\}$. Therefore $cl^*(int(G_B)) = cl^*(F_A)$. Hence $G_B \widetilde{\subseteq} cl^*(int(G_B))$ i.e. G_B be an intuitionistic fuzzy soft semi-I-open set but not intuitionistic fuzzy soft α-I-open set.

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