Remarks on the circle arising from Laurent expansion

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Abstract. We consider the notable circle for the arbelos arising from Laurent expansion appeared in [1], [2, 3] in detail.

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In [1], [2, 3], we have considered a notable circle, which is denoted by δ in Figure 1, arising from Laurent expansion under the definition of division by zero calculus [4]. In this remark we consider the properties of the circle in detail.

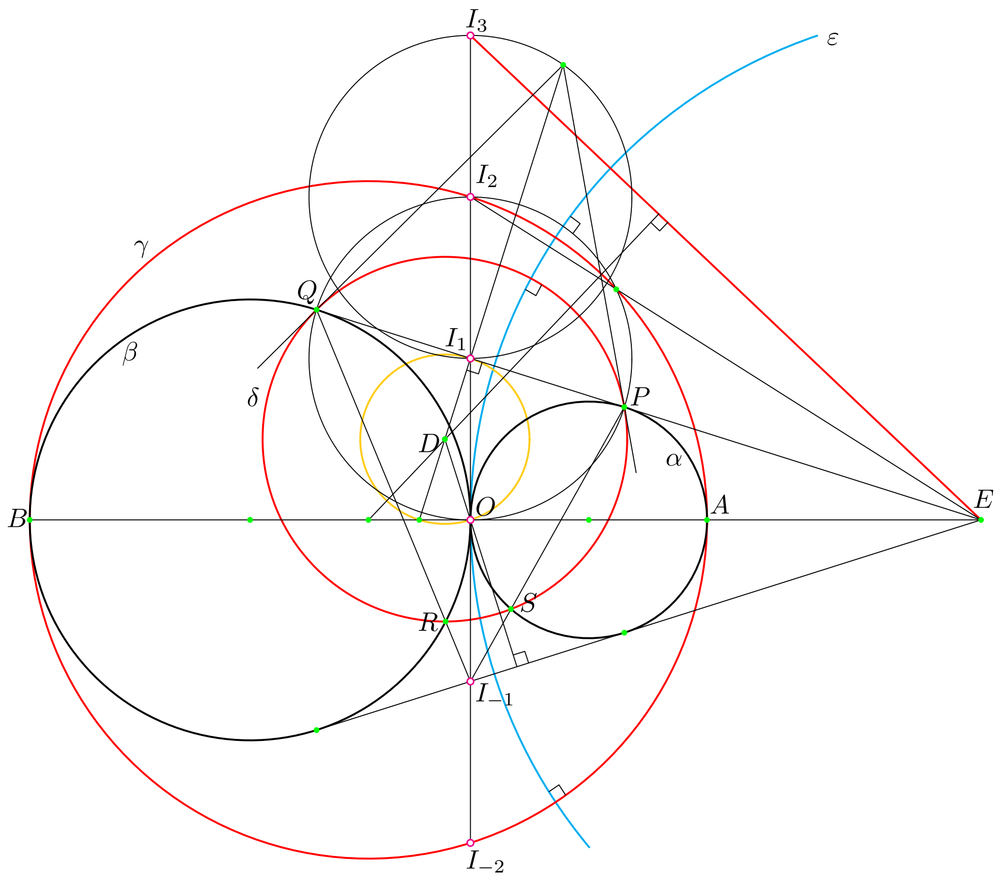

Figure 1.

For a point O on the segment AB, we consider an arbelos configuration consisting of three circles α, β and γ of diameters AO, BO and AB, respectively, where $|AO| = 2a$ and $|BO| = 2b$. The radical axis of α and β is called the axis. We use a rectangular coordinate system with origin O such that the farthest point on α from AB has coordinates (a, a). The point of coordinates $(k\sqrt{ab}, 0)$ is denoted by I_k, where $I_0 = O$. The external common tangents of α and β meet in the point of coordinates $(2ab/(b-a), 0)$, which is denoted by E. The axis meets the two common tangents in the points $I_{\pm 1}$ and the circle γ in the points $I_{\pm 2}$.

Let the line EI_1 touch α and β at P and Q, respectively, and let the lines PI_{-1} and QI_{-1} meet α and β again in S and R, respectively. The four points have coordinates

$$P : \left(2r_{\rm A}, 2r_{\rm A}\sqrt{\frac{a}{b}}\right),\ Q : \left(-2r_{\rm A}, 2r_{\rm A}\sqrt{\frac{b}{a}}\right),$$

$$R : \left(\frac{-2ab}{a+9b}, \frac{-6b\sqrt{ab}}{a+9b}\right),\ S : \left(\frac{2ab}{9a+b}, \frac{-6a\sqrt{ab}}{9a+b}\right),$$

where $r_{\rm A} = ab/(a+b)$, and lie on the circle of center D and radius given by

$$D : \left(\frac{a-b}{4}, \frac{\sqrt{ab}}{2}\right),\quad \frac{\sqrt{a^2+18ab+b^2}}{4}.$$

It seems that this circle has never been considered in the long history of studying the arbelos. However the circle has recently been discovered by using Laurent expansion under the definition of division by zero calculus [1], [2, 3], [4]. We denote the circle by δ.

The circle δ makes the same angle with the circles α and β. The line DI_1 is the perpendicular bisector of PQ, and DI_1 and the two tangents of δ at P and Q meet in a point on the circle of diameter I_1I_3, whose coordinates equal

$$\left(\frac{4ab(b-a)}{(a+b)^2}, \frac{\sqrt{ab}(a^2+10ab+b^2)}{(a+b)^2}\right).$$

The line DI_1 passes through the midpoint of the segment joining O and the center of γ, and this point and O and I_1 lie on the circle of radius $(a+b)/4$ and center D. The line DO is perpendicular to the line EI_{-1}.

The line EI_3 has an equation $3(a-b)x - 2\sqrt{ab}y + 6ab = 0$ and is the radical axis of the circles γ and δ. The perpendicular from D to this line passes through the center of γ. Let ε be the circle of center E passing through O. It is orthogonal to any circle touching α and β at points different from O, and is also orthogonal to δ and the circle of diameter OI_2. The circle of diameter OI_2 and γ and the line EI_2 meet in the point of coordinates

$$\left(\frac{2ab(b-a)}{a^2-ab+b^2}, \frac{2ab\sqrt{ab}}{a^2-ab+b^2}\right).$$

REFERENCES

[1] H. Okumura, A mystery circle arising from Laurent expansion, submitted on 2020-09-07, `https://vixra.org/abs/2009.0052`.

[2] H. Okumura, S. Saitoh, Division by zero calculus and Euclidean geometry - revolution in Euclidean geometry, submitted on 2020-10-28, `https://vixra.org/abs/2010.0228`.

[3] H. Okumura, S. Saitoh, Remarks for the twin circles of Archimedes in a skewed arbelos by Okumura and Watanabe, Forum Geom., **18** (2018) 97–100.

[4] S. Saitoh, Division by zero calculus (draft).