# BRAID GROUPS AND SOMATOTOPIC MAPS: COUPLING MATH & SENSATIONS

Arturo Tozzi

Center for Nonlinear Science, Department of Physics, University of North Texas, Denton, Texas, USA

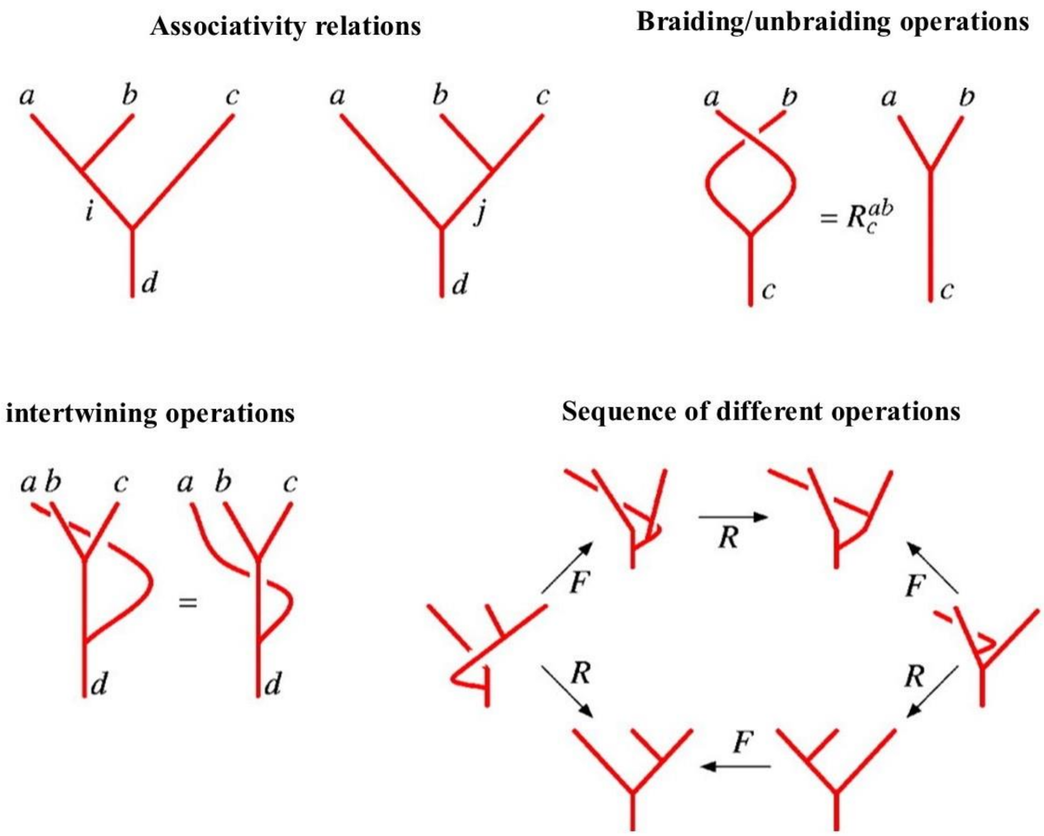
Computational Intelligence Laboratory, University of Manitoba, Winnipeg, Canada

ASL Napoli 2 Nord, Distretto 45, Caivano, Naples, Italy

tozziarturo@libero.it
arturo.tozzi@alsnapoli2nord.it

In this brief manuscript, we show how the sensitive ascending nervous fibers connecting peripheral receptors to cortical areas can be described in terms of braid groups. The group structure and generator operations of braids suggest a fresh approach to long-standing questions concerning human sensation and perception.

A braid is a collection of strands between two parallel planes (Artin 1947). The operation of composition allows braids to be joined to achieve new ones. Braids are termed isotopic when, keeping the endpoints fixed, they can be twisted into each other without cutting the strands. Given a set of braids with a fixed number of strands, its group structure is provided by generator operations such as, e.g., crossing, composition of elementary braids, and so on. The **Figure below** (modified from Brennen and Pachos 2007) illustrates some of the fusion rules, associativity relations, braiding rules of braid groups.

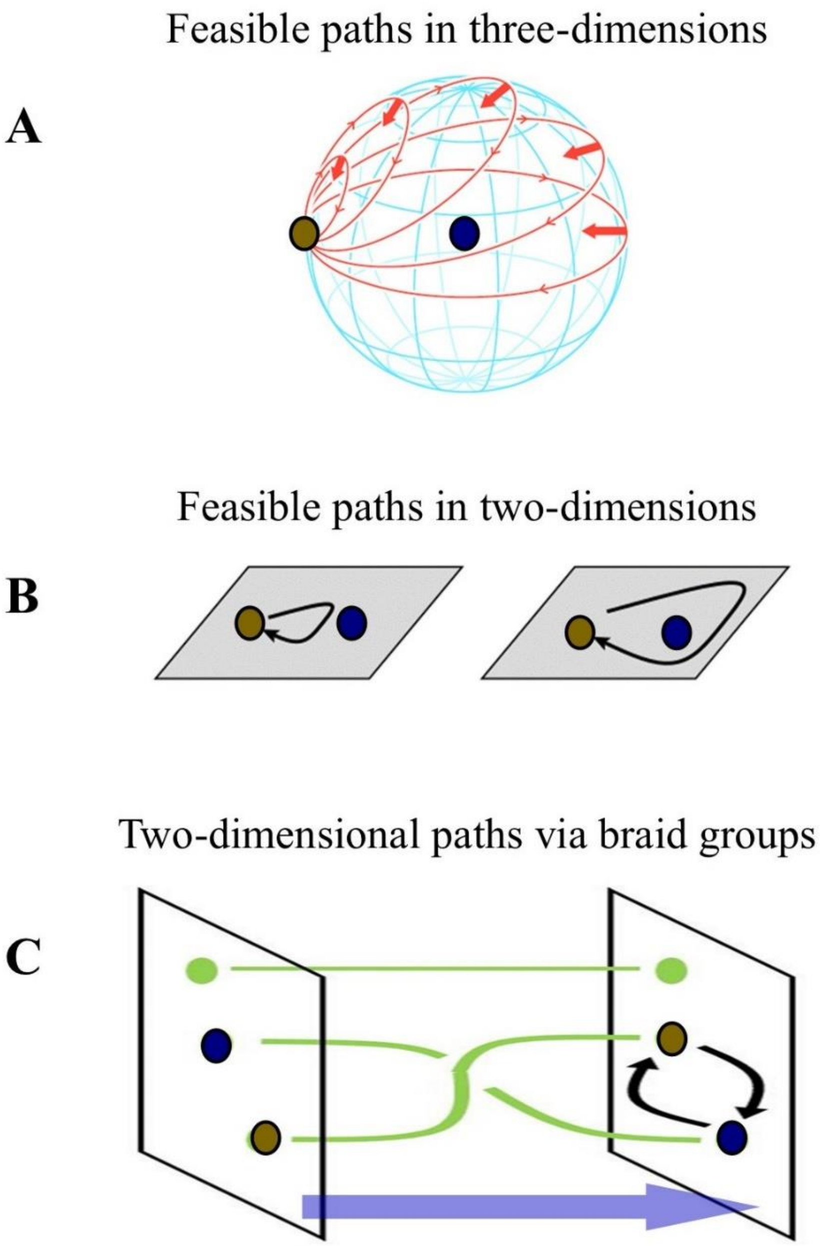


The paths followed by two or more particles are different depending the dimension of the phase space where their movements take place. The **Figure below** illustrates different possible swaps between particles moving in different dimensions.

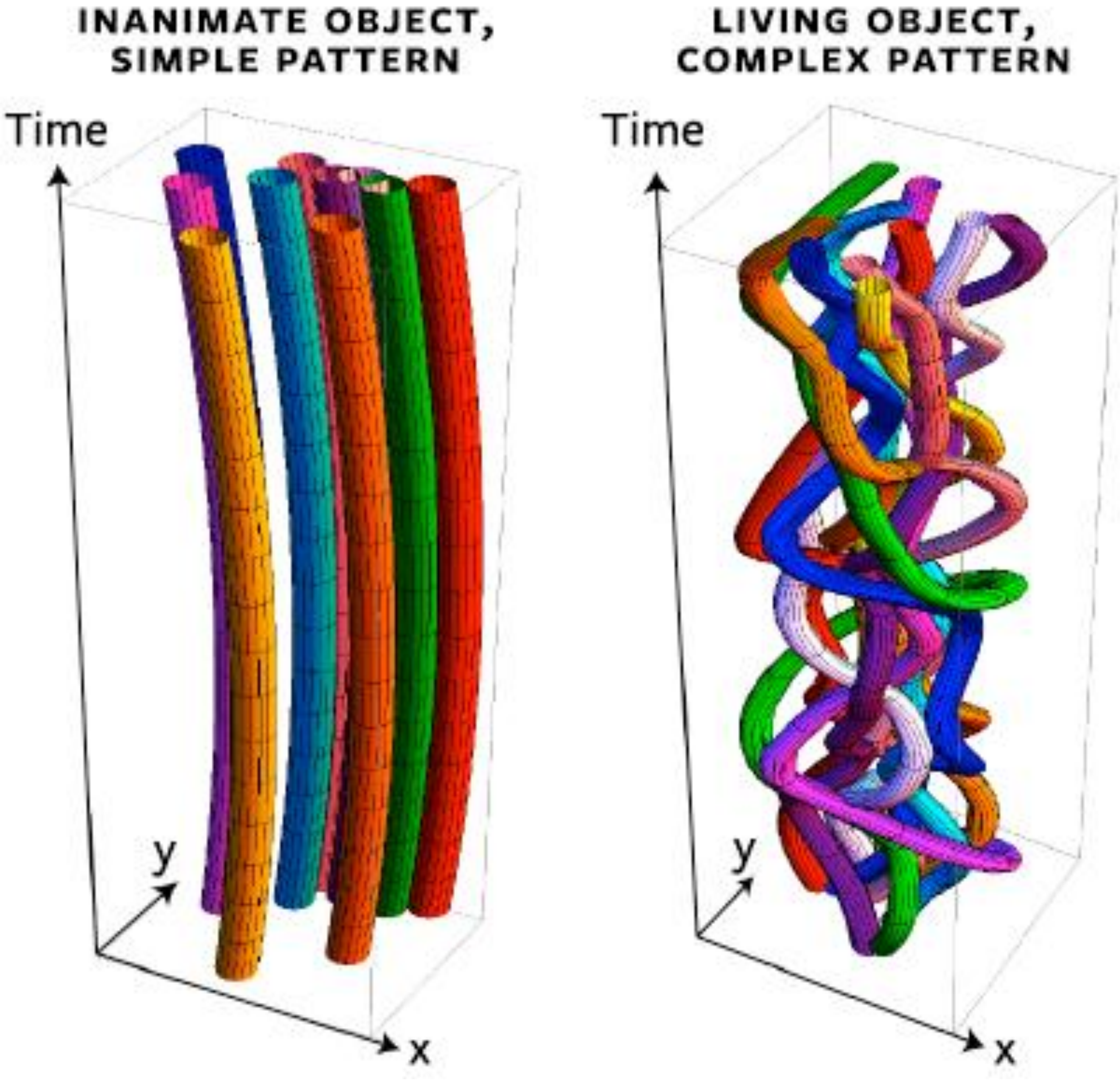
**Figure A**: when a particle loops around another inside a three-dimensional manifold, the loops are highly constrained, because they can shrink to a single point.

**Figure B**: two topologically distinct paths may occur inside a two-dimensional manifold. On the left, the path shrinks to a single point as in the three-dimensional case. On the right, a particle loops around another, so that the loop is get caught by the other particle. This means that the loop on the right cannot shrink to a single point: a non-trivial path occurs here, topologically distinct and less constrained than the path depicted in the left figure.

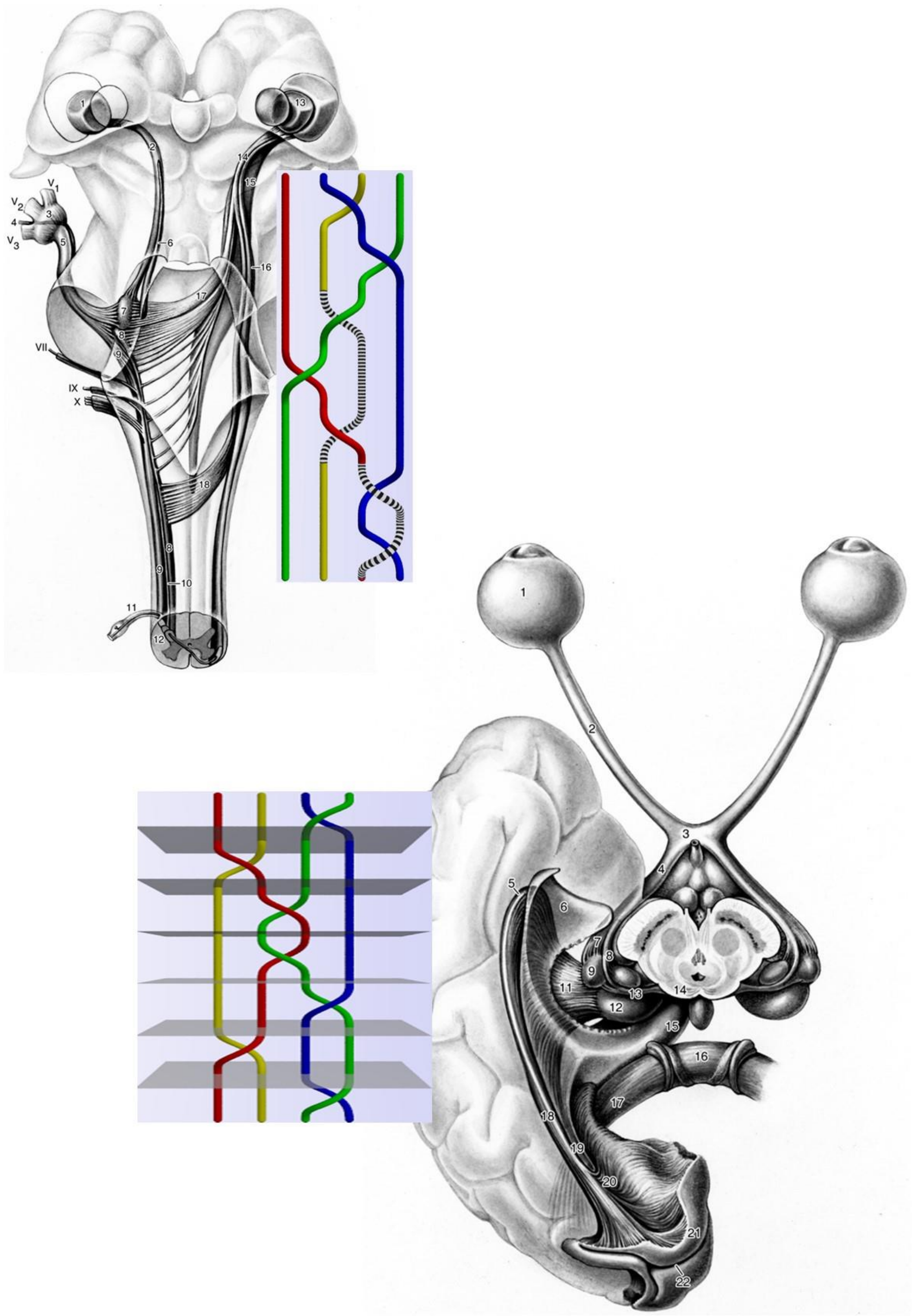
**Figure C**. The events of loops get caught by other particles can be described in terms of braid groups. Indeed, particles' paths form braids in 2+1 dimensions. Note that the swap between the blue and the red particles may occur clockwise or counterclockwise: in case of a clockwise swap, we state that an undercrossing occurs; in case of a counterclockwise swap, we state that an overcrossing occurs.



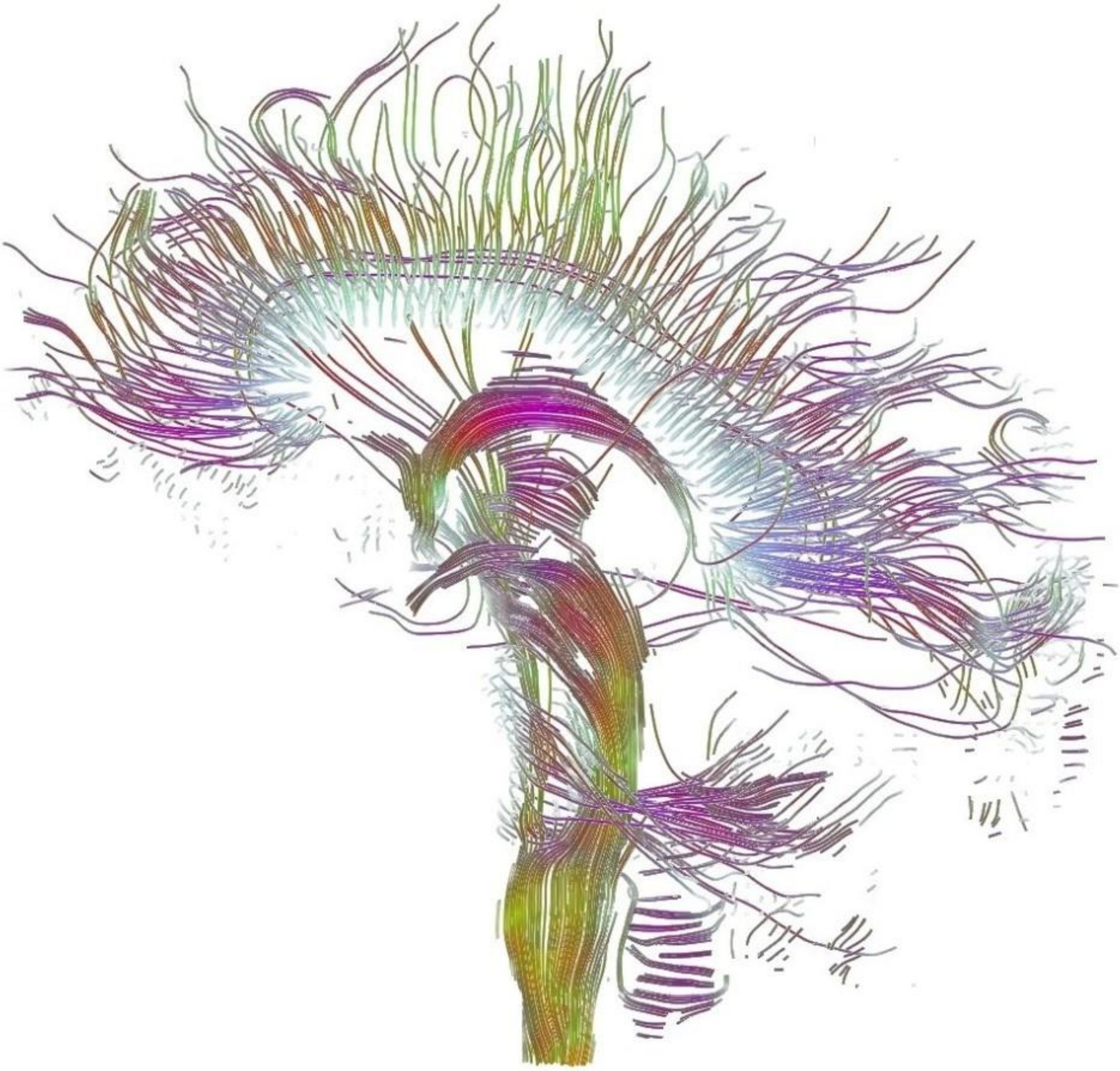
Braids have been (rarely) used to describe physical and/or biological issues. Here we provide an example. In the **Figure** below (Tegmark, 2015: http://nautil.us/issue/29/scaling/life-is-a-braid-in-spacetime-rp), the motion of ten accelerating particles produces different patterns in spacetime, depending on the structure we are examining. In our framework, this in an example of isotopic braids describing the evolution of a multi-particle system equipped with two ends: a beginning and an end.



Here we show how the nervous fibers that connect different structures of the peripheral and the central nervous system are shaped as braids. To provide an example, consider the sensory inputs from the external world detected by the peripheral nervous fibers: they follow a multisynaptic, ascending path towards the higher areas of the central nervous system. The **Figure below** illustrates the central connections of the trigeminal nerve in a sagittal view (upper Figure) and the retino-geniculo-cortical projection in a ventral view (lower Figure). Hypothetical examples of likely corresponding braids are illustrated. A warning is due: we are not going to build here the truthful braid group for every one of the countless ascending (and descending ) nervous paths in the mammalian brain: here we want just to provide a proof of concept. Modified from: [http://matematita.science.unitn.it/braids/summary.html](http://matematita.science.unitn.it/braids/summary.html) and Nieuwenhuys et al. (2008).



The **Figure below** (modified from http://www.sci.utah.edu/~gk/DTI-data/ ) suggests that also the anatomical nervous structures studied by tractography can be described in terms of braids.



We stated that the sensitive nervous paths are multi-synaptic, i.e. anatomical/functional intermediate structures are located along the road connecting the peripheral receptors with the cortical areas. If we term "braid group" the whole tract between peripheral receptors and cortical areas, we could term "knots" the intermediate anatomical/functional structures.
In algebraic topology, a well-established link does exist between braids and knots. Is it always possible to transform a knot into a closed braid: for example, Alexander theorem states that every knot or link in three-dimensional Euclidean space is the closure of a braid (Alexander 1923). Nevertheless, the correspondence between knots and braids is not one-to-one, because a single knot may have many braid representations. The Markov theorem provides the moves to relate closed braids representing the same knot type (Birman 1974).

When the generators are the elementary braids, the relations can be of two kinds: one is typical of the braids groups and the other is a commutative relation between distant crossings (see: http://matematita.science.unitn.it/braids/summary.html )

We conclude that the intricate nervous paths detectable in the nervous system can be described in terms of braid and knot theory. The next step is to answer to the crucial question: WHAT FOR? Indeed, the overwhelming complexity of the mammalian nervous system makes it very difficult to get the proper generators/operations to mimic anatomical structures with mathematical braids: given this objective obstacle, why should we describe the nervous paths in terms of braid groups? What are the (methodological, philosophical, ontological, explanatory, medical) advantages?

Here follows a list of the advantages, if we consider nervous paths in terms of braid groups and claim that the external message follows peculiar nervous paths which can be described in the mathematical words of braid groups:

1) If a link does exist between the anatomical conformation of the braids and the brain activity, this means that different braiding rules give rise to different nervous functions.

2) Braid groups might explain the differences among the different cues. According to our framework, different braids conformations lead to different computational processes/properties of every sensory cortex. Therefore, fully different perceptions such as the olfactive, auditory, tactile, visive ones could be explained by the distinct configurations of the braids that link the external receptor to the corresponding sensitive cortex.

3) The correlation between somatotopic maps is preserved from the periphery to the brain. This could be explained by the occurrence of isotopic braids: the strand between the starting and the ending structure (e.g., the peripheral receptor and the corresponding cortical projection) is the same, therefore it carries the same message. Note that we do not require that the external message stands in 1:1 relationship with the corresponding cortical area: due to the braid rules, the two extremities of the braids can be surjective, bijective, etc.

4) The conformation of the fibers of the cortical somatotopic maps is correlated with the conformation of the peripheral fibers. Who comes first during the embryonal development: the brain sensitive areas or the peripheral nervous fibers? If the brain comes first, our braid theory suggests that the brain builds perceptions before the peripheral production of sensations (à la Kant). If peripheral nervous structures come first, our braid theory suggests that what we perceive is exactly what does exist in the external environment (à la Hume). The first option à la Kant is the right one, because it has been experimentally demonstrated that, during embryonic development, axons from the midbrain thalamus build columnar connections to the cortex in the absence of sensory input: therefore, a somatosensory map is sketched out before actual sensory input begins to refine the details (Antón-Bolaños et al., 2019).

5) In case of brain diseases, instead of looking for alterations in the central nervous systems, pathological features could be looked for in the braid conformation of the peripheral nervous fibers. In sum, brain alterations might depend on the anatomical configuration of nerve fibers.

6) In a much more speculative approach, we may consider the cortical layer as a two-dimensional manifold, instead of a three-dimensional one. This suggests that brain activities between two cortical areas could be equipped with the above-mentioned topologically distinct paths occurring inside a two-dimensional manifold between two particles. This leads us in the ream on the abelian and non-abelian anyons, intermediate particles between the Fermi Dirac statistics of fermions and the Bose Einstein statistics of bosons. But that's another story…

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