RLC Circuits and Division by Zero Calculus

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Abstract: In this paper, we will discuss an RLC circuit for missing the capacitor from the viewpoint of the division by zero calculus as a typical example.

Recall that David Hilbert:

The art of doing mathematics consists in finding that special case which contains all the germs of generality.

Key Words: Division by zero, division by zero calculus, RLC circuit, capacitor, condenser, $1/0 = 0/0 = z/0 = \tan(\pi/2) = \log 0 = 0$, $[(z^n)/n]_{n=0} = \log z$, $[e^{(1/z)}]_{z=0} = 1$.

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1 Introduction

We will consider an RCL circuit stated by the ordinary differential equation

$$L\frac{di}{dt} + Ri + \frac{1}{C}\int idi = E_0 \sin \omega t, \tag{1.1}$$

$$i = \frac{E_0}{\sqrt{R^2 + ((\omega L - (1/(\omega C))^2}} \sin(\omega t - \varphi); \tag{1.2}$$

$$\varphi = \arctan \frac{1}{R}\left(\omega L - \frac{1}{\omega C}\right). \tag{1.3}$$

Here, $E_0 \sin \omega t$ is a given AC voltage.

In this circuit, for the case $C = 0$ that is the capacitor (condenser) is missing, we obtain the corresponding result precisely by the division by zero

$$\frac{1}{C} = 0$$

and

$$\frac{1}{\omega C} = 0.$$

In this paper, we would like to consider this property from the viewpoints of the division by zero calculus and the physical models.

2 Division by zero calculus – definition

We would like to consider some values for isolated singular points for analytic functions. The very typical problem is to consider some value of the fundamental function $W = 1/z$ at the origin. We found that its value is zero. When the result is written as

$$\frac{1}{0} = 0,$$

it will have a serious sense, because it looks like the division by zero that has a mysteriously long history ([1, 3, 20, 32, 33, 34]). However, note that $0 \times 0 \neq 1$. We showed that our result gave great impacts widely with over 1100 items. For example, look the papers cited in the reference.

The essence is stated as follows:

For any Laurent expansion around $z = a$,

$$f(z) = \sum_{n=-\infty}^{-1} C_n (z-a)^n + C_0 + \sum_{n=1}^{\infty} C_n (z-a)^n, \tag{2.1}$$

we will define

$$f(a) = C_0. \tag{2.2}$$

For the correspondence (2.2) for the function $f(z)$, we will call it **the division by zero calculus**. By considering derivatives in (2.1), we **can define** any order derivatives of the function f at the singular point a; that is,

$$f^{(n)}(a) = n!C_n.$$

With this assumption, we can obtain many new results and new concepts.

Typically, we found a beautiful and important circle with this division by zero calculus, see [14] and [18].

However, for this assumption we have to check the results obtained whether they are reasonable or not. By this idea, we can avoid any logical problem. – In this viewpoint, **the division by zero calculus may be considered as an axiom.**

3 Physical interpretation

The capacitor (condenser) C may be realized or represented by the formula

$$C = \frac{\varepsilon S}{d},$$

where S is the surface measure of parallel plates, d is their distance and ε is the physical constant.

If d tends to infinity with fixed S and ε, then C tends to zero. However, then the circuit will be disconnect. Therefore, with this interpretation we can not consider the above typical LR circuit from the RCL circuit.

Meanwhile, from the expression

$$\frac{1}{C} = \frac{d}{\varepsilon S},$$

if $d = 0$, the circuit becomes just the LR circuit in the both senses of the differential equation and physical model. Then, if our division by zero property $1/0 = 0$ is not admitted, we will not be able to give a suitable interpretation for the LR circuit and the corresponding differential equation.

4 Remark

As a typical similar example, we recall

Ctesibios (BC. 286-222): *We consider a flow tube with some fluid. Then, when we consider some cut with a plane with its area* S *and with its velocity* v *of the fluid on the plane, by continuity, we see that for any cut plane,* $Sv = C$*;* C *: constant. That is,*

$$v = \frac{C}{S}.$$

When S tends to zero, the velocity v tends to infinity. However, for $S = 0$, the flow stops and so, $v = 0$. Therefore, this example shows the division by zero $C/0 = 0$ clearly. Of course, in the situation, we have $0/0 = 0$, trivially.

For any nonnegative function $f(x)$ with some parameter x and we will consider that the area S is given by the function $f(x)$. Then, for some point $f(x_0) = 0$, we will obtain the identity

$$v = \frac{C}{f(x_0)} = 0.$$

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