A paradigm designed for comparative analyses pertaining to multidimensional psychometrics

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Abstract

This paper is an attempt to construct a novel approach to comparing two measured entities on a multidimensional basis, so as to facilitate a psychometric result. It may be said that the most prominent psychological phenomena that this approach is applicable to, are the 5 nascent personality traits set forth by Carl Jung, and Hofstede's 6 cultural dimensions. In any event, the mathematical approaches outlined in the paper may be applied to any multivariate analysis, wherein subjects are assessed on a multiplicity of quantities that are approximately invariant to one another (thus constituting different dimensions).

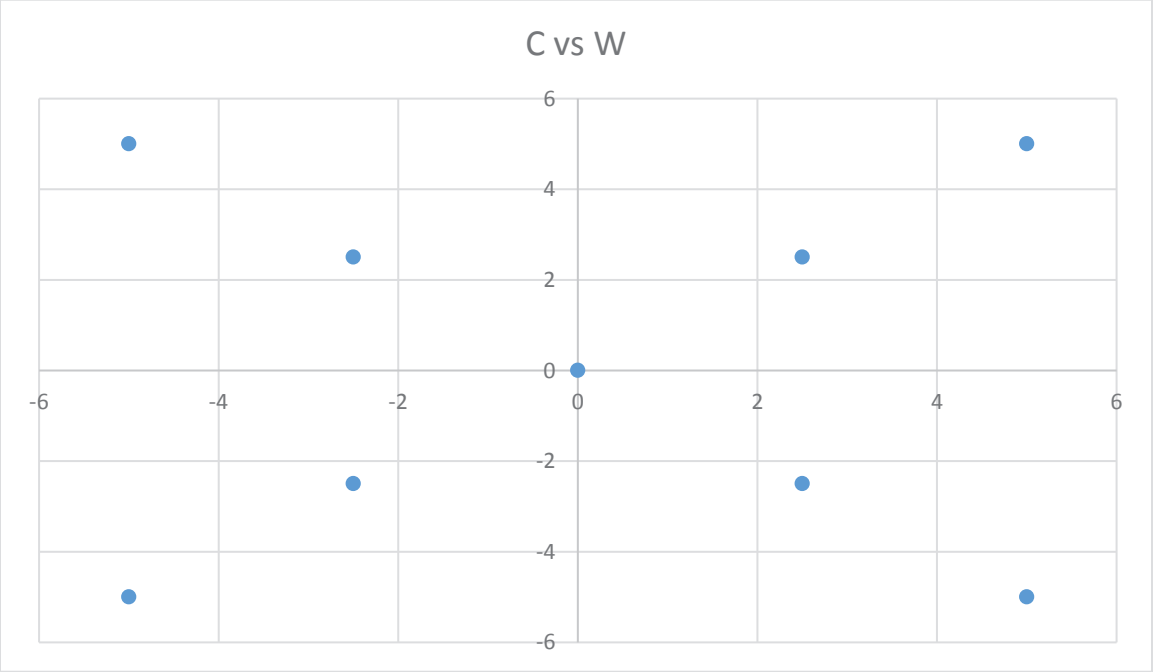
1) A Psychological Interpretation of Dimensionality

A psychometric factor analysis will often reveal multiple behavioral determinants to which a behavior is attributable. Two classical examples are the presumed contributory factors to intelligence (the Falconer Model) and the 5 Jungian personality traits (agreeableness, conscientiousness, neuroticism, openness and extroversion). The latter of which, however, is a dimensional analysis as opposed to the former. This stems from the fact that a personality is a manifestation of an individual's behavior across multiple, mutually invariant dimensions. In order to characterize their personality in this regard, one would have to use a minimum of 5 descriptions/factors. This may appear to be an analogy to the mathematical definition of a dimension: the minimal number of coordinates necessitated by the representation of an entity in that dimensionality. The Falconer model, contrarily, deconstructs intelligence into 3 constituents (heritability, shared and individual environments), some of which can be argued to be interdependent.

Another example of the above is the invocation of Hofstede's cultural dimensions. Power-distance, uncertainty avoidance and indulgence indices can all be conceived of as parameters used to compare or differentiate two cultural groups (countries, for instance).

2) Two-dimensional comparison

Imagine a stereotypical content model, with axes that delineate characteristics of warmth (compassion, altruism and inclusiveness) and competence (success, ability and quantifiable merit).



One may carry out a controlled observation and assessment of any number of subjects on the dimensional parameters *C* and *W*.

Pursuant to this, should one choose to compare any two subjects, one may elect to call upon the distance interval between their dimensional attributes in 2-dimesional Euclidean space;

In the form (W, C);

If two subjects are characterized by the indices

(W_1, C_1) and (W_2, C_2) respectively,

$$D1 \leftrightarrow 2^2 = [W_2 - W_1]^2 + [C_2 - C_1]^2$$

Or;

$$D1 \leftrightarrow 2 = \sqrt{[W_2 - W_1]^2 + [C_2 - C_1]^2}$$

With regards to the stereotypical content model, therefore, the distance interval [characterizing the similarity/dissimilarity shared by two subjects] between the indices

(W_1, C_1) and (W_2, C_2)

wherein

$W_1 = 5$

$W_2 = -5$

$C_1 = 5$

$C_2 = -5$

(5, 5) and (-5, -5)

$$D1 \leftrightarrow 2 = \sqrt{[W_2 - W_1]^2 + [C_2 - C_1]^2}$$

$$D1 \leftrightarrow 2 = \sqrt{[10]^2 + [-10]^2}$$

$$D1 \leftrightarrow 2 = \sqrt{200}$$

$$D1 \leftrightarrow 2 = 10\sqrt{2}$$

Interpretation:

$D1 \leftrightarrow 2$ provides a comparative measure by which one may gauge whether or not two subjects are relatively similar. If the distance interval between one pair of subjects exceeds that of another (in 2-dimensional analysis), the former pair may so be classified as being more disparate than the latter.

Alternative: Scalar calculations;

If one were to, say, average the scalar distances (absolute differences between the individual dimensional scores of two subjects); the same result could not be yielded.

For instance, with

(W_1, C_1) and (W_2, C_2)

Processing both independent parameters yields;

$$|W2 - W1| + |C2 - C1|$$

This isn't effective whatsoever, and is therefore not an accurate mechanism of comparison.

3) Relative Similarity/Differences

It may be noted, that the distance interval only provides an absolute two-dimensional disjunction between two subjects, without necessary regard to the relative size of either subject's scores.

This is analogous to an absolute uncertainty in measurement; wherein the interval isn't representative of a proportion of the subject's 'size' in either dimension.

To resolve this;

(W_1, C_1) and (W_2, C_2)

wherein

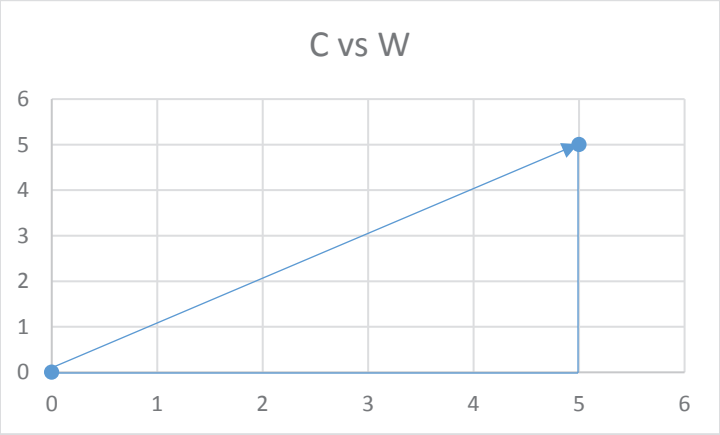
$W_1 = 0$

$W_2 = 5$

$C_1 = 0$

$C_2 = 5$

(0, 0) and (5, 5)



One may opt to characterize the differences between two subjects in relativity to either parameter *C* or *W*.

If that were the case;

$$D1 \leftrightarrow 2 = \sqrt{[W_2 - W_1]^2 + [C_2 - C_1]^2}$$

$$\frac{r}{w}d1 \leftrightarrow 2 = \frac{\sqrt{[W_2 - W_1]^2 + [C_2 - C_1]^2}}{W_2 - W_1}$$

$$\frac{r}{c}d1 \leftrightarrow 2 = \frac{\sqrt{[W_2 - W_1]^2 + [C_2 - C_1]^2}}{C_2 - C_1}$$

Wherein $\frac{r}{1}d1 \leftrightarrow 2$ denotes the ratio of the two-dimensional difference in assessment between two subjects and their differences with regard to any dimension.

In the case above,

$$\frac{r}{w}d1 \leftrightarrow 2 = \frac{\sqrt{[W_2 - W_1]^2 + [C_2 - C_1]^2}}{W_2 - W_1}$$

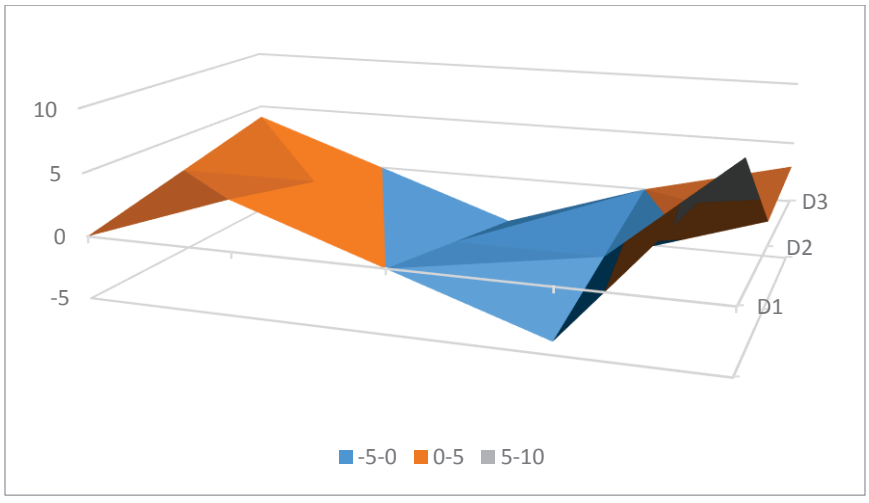
$$\frac{r}{w}d1 \leftrightarrow 2 = \sqrt{2}$$

$$\frac{r}{c}d1 \leftrightarrow 2 = \frac{\sqrt{[W_2 - W_1]^2 + [C_2 - C_1]^2}}{C_2 - C_1}$$

$$\frac{r}{w}d1 \leftrightarrow 2 = \sqrt{2}$$

Since $\frac{r}{w}d1 \leftrightarrow 2 = \frac{r}{w}d1 \leftrightarrow 2$, one may infer that the subjective differences in measurement are equally attributable to either parameter i.e. the two subjects' characteristics differ equally in both dimensions.

4) N-dimensional generalizations



For more complex analyses, such as a multidimensional analysis of cultures (given Hofstede's research paradigm), one may opt to generalize the distance interval between two points in n dimensional space [note: this is a ubiquitous mathematical fact].

$$D1 \leftrightarrow 2 = \sqrt{[W_2 - W_1]^2 + [C_2 - C_1]^2}$$

For two subjects *A* and *B* assessed on n dimensions;

$$DB \leftrightarrow A = \sqrt{\sum_{v=1}^{N} (x_{vB} - x_{vA})^2}$$

wherein

$$x_{vB} - x_{vA}$$

Denotes the absolute difference in personality/character between subjects B and A on the dimension v.

Consequently, for Hofstede's 6 cultural dimensions; the aggregate disjunction between two subjects *A* and *B* will translate into;

$$DB \leftrightarrow A = \sqrt{\sum_{v=1}^{6} (x_{vB} - x_{vA})^2}$$

$$DB \leftrightarrow A$$
$$= \sqrt{(x_{1B} - x_{1A})^2 + (x_{2B} - x_{2A})^2 + (x_{3B} - x_{3A})^2 + (x_{4B} - x_{4A})^2 + (x_{5B} - x_{5A})^2 + (x_{6B} - x_{6A})^2}$$

Using mathematics to conceive of personality or psychological phenomena in terms of their determinants is thus helpful, for it constitutes a conduit by which to demystify its complexity and reduce it to comparable terms.