A BRIEF SURVEY ON FIBREWISE GENERAL TOPOLOGY

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Abstract

We present some recent results in Fibrewise General Topology with special regard to the theory of Tychonoff compactifications of mappings. Several open problems are also proposed.

1. Introduction

Mapping are more general object than of topological spaces. In fact, it is evident that any space can be trivially identified with the continuous mapping of that space to a single-point space.

Since 50's, this simple fact suggested the idea to consider properties for mappings instead of the traditional ones for spaces in order to obtain more general statements.

First steps in this direction were moved by Whyburn [W_1, W_2] and Dickman [D], but only in 1975, Ul'janov [U] introduced the notion of *Hausdorff mapping* (formerly called *separable*) to study the Hausdorff compactifications of countable character.

Later, Pasynkov [P_3] generalized and studied in a systematic way to the continuous mappings various other notions and properties concerning spaces like the separation axioms T_0, T_1, T_2, $T_{3\frac{1}{2}}$, the *regularity*, the *complete regularity*, the *normality*, the *compactness* and the *local compactness*. A considerable part of these new definitions and constructions is based on the notion of *partial topological product* (briefly PTP) introduced and studied by Pasynkov in [P_1]. The main properties of PTP's, included an analogous for mappings of the Embedding Lemma, are proved in detail in [P_2].

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Some weaker separation axioms for continuous mappings such as *semiregularity* and *almost regularity* were introduced and studied in [CN]. The problem of their productivity was investigated in [N_1].

Let us note that Pasynkov's papers ([P_3], in particular) have inspirated James to give a slightly different approach to the same topic (see [J]).

The generalization to mappings of notions originally defined for spaces belongs to the more general branch of the *Fibrewise General Topology* (sometimes called *General Topology of mappings*) and, from a categorial point of view, it means to pass from the study of the property of the category **Top** to those of the category $\mathbf{Top}_Y$ whose objects are the continuous mappings into some fixed space Y and whose morphisms are the continuous functions commutating the triangular diagram of two objects.

Because a property $\mathcal{P}_Y$ of $\mathbf{Top}_Y$ can be considered as a generalization of some corresponding property $\mathcal{P}$ of **Top** it must coincide with $\mathcal{P}$ when Y is a single-point space, that is when every object $f : X \to Y$ of $\mathbf{Top}_Y$ can be identified with the space X, i.e. with an object of **Top**.

In rare case, the analogous $\mathcal{P}_Y$ in $\mathbf{Top}_Y$ of some property $\mathcal{P}$ of **Top** is quite evident. For example, to $\mathcal{P}$ = compactness corresponds $\mathcal{P}_Y$ = perfectness. In other cases (e.g. for the separation axioms T_0 and T_1) the property $\mathcal{P}_Y$ can be obtained by requiring that the corresponding property $\mathcal{P}$ holds on every fibre of the mappings, but for many properties for mappings, it is necessary to give new definitions which are more complex than the corresponding ones for the spaces.

The notion of compactification (i.e. perfect extension) of a continuous mapping was given first by Whyburn in 1953 [W_1, W_2].

It is worth mentioning that in [N_2] it is presented a filter based method which allows us to build perfect extension of every function (not necessarily continuous) between two arbitrary topological spaces.

However, the first general definition of compactification for mappings analogous to the well-known notion for spaces, was given by Pasynkov in [P_3]. In fact, in that paper, using techniques based on PTP's, a method is described to obtain Tychonoff (i.e. completely regular, T_0) compactifications of Tychonoff mappings between arbitrary spaces, and it is proved that the poset $TK(f)$ of all the Tychonoff compactifications of a Tychonoff mapping $f : X \to Y$ admits a maximal compactification $\beta f : \beta_f X \to Y$ which is the exact analogous, in $\mathbf{Top}_Y$, of the Stone–Čech compactification of a Tychonoff space.

Let us note that Künzi and Pasynkov [KP] have completely described the set $TK(f)$ of all the Tychonoff compactifications of a Tychonoff mapping $f : X \to Y$ by means of presheaves of the rings $C^*\left(f^{-1}(U)\right)$ with U open set of Y.

Recently, Bludova and Nordo [BN] have shown that if a mapping $f : X \to Y$ is *Hausdorff compactifiable* (i.e. it has some Hausdorff compactification) then there exists the greatest (called "maximal") compactification $\chi f : \chi_f X \to Y$ in the set

$HK(f)$ of all the Hausdorff compactifications of f.

An extension to mappings of the notion of *H-closedness* (see [PoW] or [CGNP] for a complete survey) was given in [CFP] by Cammaroto, Fedorchuk and Porter, while some other generalizations to the mappings of the concepts of realcompactness and Dieudonné completeness were introduced and studied in [IP], [BuP], [MuP] and [P_4].

The notion of *perfect compactification* given by Skljarenko in [S] was recently generalized to the mappings in [NP] and, in [CaP], it was proved that both the maximal realcompactification υf and the Dieudonné completion μf of a Tychonoff mapping f are perfect extensions of f.

2. Extension to mappings of notions for spaces

Throughout all the paper, the word "space" will mean "topological space" on which, no separation axiom is assumed and all the mappings will be supposed continuous unless otherwise specified. If X is a space, $\tau(X)$ will denote the family of all open sets of X.

For terms and undefined concepts we refer to [E].

For any fixed space Y, we consider the category $\mathbf{Top}_Y$ where

$$Ob(\mathbf{Top}_Y) = \{f \in C(X,Y) : \quad X \in Ob(\mathbf{Top})\}$$

is the class of the *objects* and, for every pair $f : X \to Y$, $g : Z \to Y$ of objects,

$$M(f,g) = \{\lambda \in C(X,Z) : \quad g \circ \lambda = f\}$$

is the class of the *morphisms* from f to g, whose generic representant is denoted for short by $\lambda : f \to g$.

A morphism $\lambda : f \to g$ from $f : X \to Y$ to $g : Z \to Y$ will be called *surjective* (resp. *closed*, *dense*) if $\lambda(X) = Z$ (resp. $\lambda(X)$ is closed in Z, $\lambda(X)$ is dense in Z).

If $\lambda : f \to g$ is a surjective morphism, we will say that g is the *image* of f (*by the morphism* λ) and we will write that $g = \lambda(f)$.

Moreover, we say that a morphism $\lambda : f \to g$ from $f : X \to Y$ to $g : Z \to Y$ is an *embedding* (resp. a *homeomorphism*) if so is the function $\lambda : X \to Z$.

A mapping $g : Z \to Y$ is said an *extension* of $f : X \to Y$ if there exists some dense embedding $\lambda : f \to g$ (as usual, we shall identify X and f by $\lambda(X)$ and $g|_{\lambda(X)}$ respectively).

A morphism $\lambda : g \to h$ between two extensions $g : Z \to Y$ and $h : W \to Y$ of a mapping $f : X \to Y$ will be called *canonical* if $\lambda|_X = id_X$.

Let us introduce some notions and basic facts about partial products [P_1, P_2].

Given two spaces Y, Z and an open set O of Y, we consider the set $P = (Y \setminus O) \cup (O \times Z)$ and the map $p : P \to Y$ defined by $p|_{Y \setminus O} = id_{Y \setminus O}$ and $p|_{O \times Z} = pr_O$ where $pr_O : O \times Z \to O$ denotes the projection of $O \times Z$ onto O.

We will call *elementary partial topological product* (briefly EPTP) *of Y and fibre Z relatively to the open set O* and we will denote it by $P(Y, Z, O)$, the space generated on P by the basis $\mathcal{B}(Y, Z, O) = p^{-1}(\tau(Y)) \cup \tau(O \times Z)$.
The mapping $p : P \to Y$ above defined will be called the *projection* of the EPTP $P = P(Y, Z, O)$ and it is routine to prove that it is a continuous, onto, open mapping.

It is evident that the EPTP $P(Y, Z, \emptyset)$ is simply Y and that $P(Y, Z, Y)$ coincides with the usual product space $Y \times Z$.

Now, let Y be a topological space, $\{Z_\alpha\}_{\alpha \in \Lambda}$ be a family of spaces and $\{O_\alpha\}_{\alpha \in \Lambda}$ be a family of open sets of Y. For every $\alpha \in \Lambda$, let $P_\alpha = P(Y, Z_\alpha, O_\alpha)$ be the EPTP of base Y and fibre Z_α relatively to O_α and $p_\alpha : P_\alpha \to Y$ be its projection. We will call *partial topological product* (PTP for short) *of base Y and fibres $\{Z_\alpha\}_{\alpha \in \Lambda}$ relatively to the open sets $\{O_\alpha\}_{\alpha \in \Lambda}$* the *fan product* of the spaces $\{P_\alpha\}_{\alpha \in \Lambda}$ relatively to the mappings $\{p_\alpha\}_{\alpha \in \Lambda}$, i.e. the subspace

$$P = \left\{ t = \langle t_\alpha \rangle_{\alpha \in \Lambda} \in \prod_{\alpha \in \Lambda} P_\alpha : \quad p_\alpha(t_\alpha) = p_\beta(t_\beta) \quad \forall \alpha, \beta \in \Lambda \right\}$$

and we will denote it by $P(Y, \{Z_\alpha\}, \{O_\alpha\}; \alpha \in \Lambda)$.

For every $\alpha \in \Lambda$, the restriction $\pi_\alpha = pr_\alpha|_P : P \to P_\alpha$ of the α-th canonical projection pr_α will be called the α-th *short projection*, while the *fibrewise product of the mappings* $\{p_\alpha\}_{\alpha \in \Lambda}$, i.e. the continuous mapping $p : P \to Y$ defined by $p_\alpha \circ \pi_\alpha = p$ for any $\alpha \in \Lambda$ will be said the *long projection* of the PTP $P(Y, \{Z_\alpha\}, \{O_\alpha\}; \alpha \in \Lambda)$.

In case $O_\alpha = Y$ for every $\alpha \in \Lambda$, the PTP $P(Y, \{Z_\alpha\}, \{O_\alpha\}; \alpha \in \Lambda)$ coincides (up to homeomorphisms) with the usual product $Y \times \prod_{\alpha \in \Lambda} Z_\alpha$, if $|O_\alpha| = 1$ for every $\alpha \in \Lambda$, the PTP $P(Y, \{Z_\alpha\}, \{O_\alpha\}; \alpha \in \Lambda)$ coincides with the usual Tychonoff product $\prod_{\alpha \in \Lambda} Z_\alpha$ of its fibres, while if $O_\alpha = \emptyset$ for any $\alpha \in \Lambda$, the PTP $P(Y, \{Z_\alpha\}, \{O_\alpha\}; \alpha \in \Lambda)$ is simply (homeomorphic to) the space Y.

Definitions. A mapping $f : X \to Y$ is said to be T_0 [P$_3$] if for every $x, x' \in X$ such that $x \neq x'$ and $f(x) = f(x')$ there exists a neighborhood of x in X which does not contain x' or a neighborhood of x' in X not containing x.

A mapping $f : X \to Y$ is said to be *Hausdorff* (or T_2) [U, P$_3$] if for every $x, x' \in X$ such that $x \neq x'$ and $f(x) = f(x')$ there are two disjoint neighborhoods of x and x' in X.

We will say that $f : X \to Y$ is *compact* if it is perfect (i.e. it is closed and every its fibre is compact).

A mapping $f : X \to Y$ is said to be *completely regular* [P$_3$] if for every closed set

F of X and $x \in X \setminus F$ there exists a neighborhood O of $f(x)$ in Y and a continuous function $\varphi : f^{-1}(O) \to [0,1]$ such that $\varphi(x) = 0$ and $\varphi(F \cap f^{-1}(O)) \subseteq \{1\}$.

A completely regular, T_0 mapping is called *Tychonoff* (or $T_{3\frac{1}{2}}$) [P_3].

Remark. It is easy to verify that all the previous properties in $\mathbf{Top}_Y$ coincide with the corresponding ones in **Top** provided $|Y| = 1$ and that every continuous mapping $f : X \to Y$ has such a property *iff* both the spaces X and Y have the corresponding properties (in particular, they are $\mathcal{P}-$functions in the sense of [CN]).

Definition. A restriction $f|_{X'} : X' \to Y$ to $X' \subseteq X$ of a mapping $f : X \to Y$ is said a *closed restriction* of f, if X' is a closed subset of X.

Obviously (see for example [PoW]), every closed restriction of a compact mapping is compact too.

Most well-known statements which hold in the category **Top** have correspondent ones (and hence generalizations) in $\mathbf{Top}_Y$. The following properties are essentially given in [P_3] (detailed proofs can be found in [N_4]).

PROPOSITION 2.1. *Every image $\lambda(f)$ of a compact mapping $f : X \to Y$ is compact too.*

PROPOSITION 2.2. *Every closed restriction $f|_{X'}$ of a compact mapping $f : X \to Y$ is compact too.*

PROPOSITION 2.3. *Every compact restriction $f|_{X'}$ of a Hausdorff mapping $f : X \to Y$ is a closed restriction of f.*

PROPOSITION 2.4. *Let λ and μ be morphisms from a mapping $f : X \to Y$ to a Hausdorff mapping $g : Z \to Y$ and D be a dense subset of X. Then if $\lambda|_D = \mu|_D$, the morphisms λ and μ coincide.*

PROPOSITION 2.5. *Every morphism $\lambda : f \to g$ from a compact mapping $f : X \to Y$ to a Hausdorff mapping $g : Z \to Y$ is perfect.*

3. Compactification of mappings

Let $f : X \to Y$ be a mapping. We say that a mapping $c : X^c \to Y$ is a *compactification* of f (in $\mathbf{Top}_Y$) if it is a compact (= perfect) extension of f.

This approach to the notion of the compactification of a mapping was proposed by Whyburn [W_2], but in the most general situation, this notion was studied first by Pasynkov in [P_3].

Remark. A different variant of compactifications of mappings was examined by Uljanov [U]. But, it is a common opinion that Uljanov's definition is not natural for non-surjective mappings because, in that case, a compact mapping is not its own compactification.

Definitions. Let $c : X^c \to Y$ and $d : X^d \to Y$ be two compactifications of a mapping $f : X \to Y$ (in $\mathbf{Top}_Y$). We say that:

- *c is projectively larger than d* (relatively to f) and we write that $c \geq_f d$ (or $c \geq d$, for short) if there exists some canonical morphism $\lambda : c \to d$.

- *c is equivalent to d* (relatively to f) and we write that $c \equiv_f d$ (shortly, $c \equiv d$) if there exists a canonical homeomorphism $\lambda : c \to d$.

The following useful result is given in [BN].

PROPOSITION 3.1. *Let $c : X^c \to Y$ and $d : X^d \to Y$ be two Hausdorff compactifications of a mapping $f : X \to Y$. Then $c \equiv_f d$ if and only if $c \geq_f d$ and $d \geq_f c$.*

Definition. A Hausdorff mapping $f : X \to Y$ will be called *Hausdorff compactifiable* if it has some Hausdorff compactification (in $\mathbf{Top}_Y$).

In [BN], it is noted that the class of all Hausdorff compactifications of any Hausdorff compactifiable mapping $f : X \to Y$ forms a set modulo the equivalence $\equiv_f$.

Definition. If $f : X \to Y$ is a Hausdorff compactifiable mapping, $HK(f)$ will denote the set of all equivalence classes of Hausdorff compactifications of f.

So, by 3.1, it follows that $(HK(f), \geq)$ is a poset and, for any pair of Hausdorff compactifications $c, d \in HK(f)$ we can write $c = d$ instead of $c \equiv_f d$, that is we do not distinguish between equivalent Hausdorff compactifications.

In [P$_3$], Pasynkov erroneously indicated that it is proved in [U] that every Hausdorff compactifiable mapping $f : X \to Y$ has a maximal one.
This fact was also used in several following papers like [BuP], [CFP], [IP], [KP], [MuP], [M$_2$], [P$_4$], etc. but it is not correct because the Ul'janov's definition is different from the currently used one (given by Pasynkov in [P$_3$]) that does not include the surjectivity.
Anyway the existence of the maximal Hausdorff compactification was actually proved by Bludova and the author as direct consequence of the following more general result.

THEOREM 3.2. [BN] *For any Hausdorff compactifiable mapping $f : X \to Y$, $(HK(f), \geq)$ is a complete upper semilattice*

The projective maximum of $(HK(f), \geq)$, i.e. the *maximal Hausdorff compactification* of f, will be denoted by $\chi f : \chi_f X \to Y$.

From this and by 2.4 and 2.5, it follows – in particular – that for any Hausdorff compactification $bf : X^b \to Y$ of a Hausdorff compactifiable mapping $f : X \to Y$ there exists a unique perfect canonical morphism $\lambda_b : \chi f \to bf$.

In [P_3], Pasynkov proved that every Tychonoff mapping has a Tychonoff compactification.

Since it is easy to show that a Tychonoff mapping is Hausdorff, Proposition 3.1 allow us to give the following:

Definition. For any Tychonoff mapping $f : X \to Y$, we will denote by $TK(f)$ the set of all Tychonoff compactifications of f up to the equivalence $\equiv_f$.

For a mapping $f : X \to Y$, let us denote by $C^*(f)$ the family of all the *partial mappings* on f, i.e. of all the continuous bounded real-valued mappings $\varphi : f^{-1}(O_\varphi) \to I_\varphi$ defined from the inverse image by f of an open set O_φ of Y to a compact subset I_φ of the real line $\mathbb{R}$.

Definitions. A subfamily $\mathcal{C} = \{\varphi : f^{-1}(O_\varphi) \to I_\varphi\}$ of $C^*(f)$ is said to be:

- *separating the points of f* if for every $x, x' \in X$ such that $f(x) = f(x')$ there exists some $\varphi \in \mathcal{C}$ such that $x, x' \in f^{-1}(O_\varphi)$ and $\varphi(x) \neq \varphi(x')$.
- *separating the points from the closed sets of f* if for any closed set F of X and every $x \in X \setminus F$ there exists some $\varphi \in \mathcal{C}$ such that $x \in f^{-1}(O_\varphi)$ and $\varphi(x) \notin cl_{I_\varphi}(F \cap f^{-1}(O_\varphi))$.

It is shown in [P_3] (see [N_4] for a more detailed proof) that every Tychonoff compactification $bf : X^b \to Y$ of a Tychonoff mapping $f : X \to Y$ is uniquely determinated by a subfamily $\mathcal{C} = \{\varphi : f^{-1}(O_\varphi) \to I_\varphi \;:\; O_\varphi \in \tau(Y)\}$ of $C^*(f)$ separating the points and the points from the closed sets of f and that bf coincides with a particular restriction of the long projection $p_\mathcal{C} : P_\mathcal{C} \to Y$ of the PTP $P_\mathcal{C} = P(Y, \{O_\varphi\}, \{I_\varphi\}; \varphi \in \mathcal{C})$.

Thus, the notion of PTP plays in the category $\mathbf{Top}_Y$ the same role that the notion of product space has in the category **Top** and, as matter of fact, they coincide when $|Y| = 1$.

In [P_3], it is also proved that for any Tychonoff mapping $f : X \to Y$ there exists, in $(TK(f), \geq)$, a maximal Tychonoff compactification $\beta f : \beta_f X \to Y$ that is determinated by all the whole family $C^*(f)$ and characterized by some extension properties very similar to that of the Stone-Čech compactification.
We have, in fact, the following:

THEOREM 3.3. *For any Tychonoff compactification $bf : X^b \to Y$ of a Tychonoff mapping $f : X \to Y$, the following conditions are equivalent:*

(1) $bf = \beta f$;
(2) for every $U \in \tau(Y)$ and $\varphi \in C^*(f^{-1}(U))$ there exists a unique extension $\widetilde{\varphi} \in C^*((bf)^{-1}(U))$;
(3) for every compact Tychonoff mapping $k : Z \to Y$ and every morphism $\lambda : f \to k$ there exists a morphism $\widetilde{\lambda} : bf \to k$ which extends λ.

Moreover, Theorem 3.2. allow us to obtain as immediate consequence the following:

THEOREM 3.4. [BN] *The poset $(TK(f), \geq)$ of all Tychonoff compactifications of a Tychonoff mapping $f : X \to Y$ is a complete upper semilattice whose projective maximum is βf.*

PROPOSITION 3.5. [P_3] *For any Tychonoff compactification $bf : X^b \to Y$ of a Tychonoff mapping $f : X \to Y$ there exists a unique (perfect) canonical morphism $\mu_b : \beta f \to bf$ such that $\mu_b(\beta_f X \backslash X) = X^b \backslash X$.*

Let us observe that if $|Y| = 1$, X is a Tychonoff space, the domain $\beta_f X$ of βf coincides with the Stone Čech compactification βX of X, the domain X^b of bf is a generic compactification of X and $\lambda_b : \beta_f X \to X^b$ becomes the usual quotient map (see for example [Ch]).

In general, for a Tychonoff mapping $f : X \to Y$, we have

$$TK(f) \underset{\neq}{\subset} HK(f)$$

that is, unlike the corresponding case for spaces, there exist Hausdorff compactification which are not Tychonoff or, equivalently, there are compact Hausdorff mapping which are not Tychonoff. In fact, it was proved in [Cb] (see also [HI]) that it is possible to build a perfect ($\equiv$ compact) mapping defined on a regular T_0 but non Tychonoff space onto a Tychonoff space (that is the property $T_{3\frac{1}{2}}$ is not an inverse invariant by perfect mappings) and since it is proved in [P_3] that if a mapping and its range are both completely regular, its domain is too, it follows directly that such a mapping can not be completely regular and hence Tychonoff.

This is the reason why it is necessary to study the classes of Tychonoff and Hausdorff compactifiable mappings separately.

4. Open problems.

It seems that the following questions might be interesting. Some of these problems are published for the first time.

Problem 1. It is well-known that the poset $K(X)$ of all Hausdorff compactification of a Tychonoff space X can be completely characterized in terms of the families of $C^*(X)$ that separete points and points from closed sets of X (see, for example, [Ch]).
Is it possible to obtain such a similar characterization for the set $HK(f)$ (the set $TK(f)$) of all Hausdorff (Tychonoff) compactification of a Hausdorff compactifiable (Tychonoff) mapping f in terms of the separating families of $C^*(f)$?

Problem 2. Magill has proved in [M] (see also [Ch]) that the posets $K(X)$ and $K(Y)$ of all Hausdorff compactifications of two locally compact spaces X and Y are isomorphic if and only if their Stone-Čech remainders $\beta X \setminus X$ and $\beta Y \setminus Y$ are homeomorphic.
Is it possible to find a definition of locally compact mapping that allow us to obtain a Magill-type theorem for mappings ?

Problem 3. *Is it possible to obtain analogous in* $\mathbf{Top}_Y$ *of properties like the countably compactness, the paracompactness and the pseudocompactness ?*

Problem 4. *Is there a consistent definition of metrizable mapping which extends the corresponding notion for spaces and allow us to obtain general metrization theorems ?*

Problem 5. *Is it possible to extend to mappings other kind of Tychonoff extension properties like the m-boundedness* (see [PoW]) *?*

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