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On Geoengineering the Albedo Solution to Global Warming And Identifying Key Parameters

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Abstract

A solar geoengineering global warming model is developed with a re-radiation factor. The model is shown to be consistent with the Planck's parameter. The re-radiation factor is important in quantifying the relative global warming impact of the albedo effect compared to that of greenhouse gases (GHG). The potential reverse forcing due to a change in the Earth's global albedo compared to GHGs, is illustrated. Results of modeling support solar geoengineering solutions with two key parameters from modeling; an albedo-GHG and a Planck-Albedo feedback parameter. Using these, it is concluded that a 1.5% solar geoengineering change in the global albedo could result in a significant resolution to the global warming problem. Feasibility is discussed.

1 Introduction

Solar geoengineering is vital in global warming as results can reverse trends and reduce the probability of a tipping point from occurring. In this paper, a geoengineering model that uses a re-radiation factor, which helps to quantify differences between changes in the global albedo versus greenhouse gas forcing is developed. The re-radiation parameter is obtained initially in the absence of warming feedbacks with a unique value of 0.612 (or β=0.887). The re-radiation factor is a redefined variable taken from the effective emissivity constant of the planetary system. An application of the model is provided between two different time periods (1950 and 2019). In 2019 the re-radiation parameter takes into account GHG change and feedback effects. Then, the Planck's feedback parameter is used to verify model consistency. The model illustrates a reasonable way to view the Earth's energy budget; simplifies estimates without the need for doubling theory, provides a number of useful insights in climatology sensitivity estimates and provides practical solar geoengineering calculation for global warming mitigation [1]. Specifically, a 1.6 albedo-GHG factor along and a handy Planck-Albedo parameter (having a convenient value of $1W/m^2/^oK/\Delta\%$albedo) is obtained in modeling results. These values greatly simplify solar geoengineering [2, 3] calculations. Using these values, we exemplify a global warming albedo solution and discuss feasibility [1].

2. Data and Method

To introduce the re-radiation engineering model, we will often refer to the Planck parameter and its associated variables that play a key role in development and verifying this model. Therefore, we provide an overview in Appendix A which also includes a unique way to assess its value using an albedo approach (see A.1).

2.1 The Re-radiation Global Warming Model

In geoengineering, we are working with absorption, we define

$$P_{Total} = \sigma T_S^4 = \sigma\left(\frac{T_{TOA}}{\beta}\right)^4 \text{ and } P_\alpha = \sigma T_\alpha^4 = \sigma\left(\beta T_S\right)^4 \quad (1)$$

The definitions of T_{TOA}, T_S and β are provided in Appendix A (Eq. A-1, A-2, A-3). We consider a time when there is ***no feedback issues*** causing warming trends. Then by conservation of energy, the equivalent power re-radiated from GHGs in this model is dependent on P_α

$$P_{GHG} = P_{Total} - P_\alpha = \sigma T_S^4 - \sigma T_\alpha^4 \quad (2)$$

To be consistent with Eq. A-1, $T_\alpha = T_{TOA}$, since typically $T_\alpha \approx 255^oK$ and $T_s \approx 288^oK$, then in keeping with a common definition of Beta (see Eq. A-4) for the moment $\beta \approx T_\alpha/T_s \approx T_{TOA}/T_S$.

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This allows us to write the dependence

$$P_{GHG} = \sigma T_S^4 - \sigma T_\alpha^4 = \frac{\sigma T_\alpha^4}{\beta^4} - \sigma T_\alpha^4 = \sigma T_\alpha^4 \left(\frac{1}{\beta^4} - 1 \right) \tag{3}$$

We note that when $\beta^4 = 1$, there are no GHG contributions as required. We now define a re-radiation parameter $f = \beta^4$. We know that some fraction of the blackbody radiation is re-radiated by the GHGs, so f is a re-radiation parameter. That is, the energy, P_{GHG}, must be some fraction of P_α so that its dependence is also

$$P_{GHG} = f\,P_\alpha = f\,\sigma T_\alpha^4 \tag{4}$$

Once absorption occurs, initial temperature rise occurs to the Earth, and then part of this energy is reradiated back to Earth by GHGs. It is important in geoengineering to view this as part of the albedo effect. This is a key difference in how we view the total effect from short wavelength absorption with the inclusion of re-radiation [2]. Now in order for this to be true, we require from Equations 3 and 4

$$P_{GHG} = \sigma T_\alpha^4 \left(\frac{1}{f} - 1 \right) = f\,\sigma T_\alpha^4 \tag{5}$$

This dependence leads us to the solution of the quadratic expression

$$f^2 + f - 1 = 0 \text{ yielding } f_1 = 0.618034 = \beta^4 ,\ \beta = (0.618034)^{1/4} = 0.88664 \tag{6}$$

This is very close to the common value estimated for β (Appendix A) and this has been obtained through energy balance in the planetary system providing a completely self-determining assessment without approximations. In Section 2.6, we double check this model in another way by balancing energy in and out and in Section 3 we will apply the modeling to demonstrate its capability.

2.2 ***Re-radiation Model Applied to Two Different Time Periods***

Global warming can be modeled by looking at two different time periods. We can model the radiation for 1950 consistent with our model in Eq. 2 and 4

- we will assume no feedback issues causing a warming trend in 1950 so that from our model

$$P_{Total\_1950} = P_\alpha + P_{GHG} = P_\alpha + f_1 P_\alpha = P_\alpha (1 + f_1) = 1.618\,P_\alpha \tag{7}$$

where $P_\alpha = S_o \{0.25x(1 - Albedo)\}$ and S_o=1361W/m^2. We can use the value 1.618 (Eq. 6), since we assume no changes in GHG and feedback issues in 1950, making it a good reference number for geoengineering estimates. We can term this as a 1950 albedo-GHG value. Since its value is related to the re-radiation parameter, it changes due to variations in our climate system. However, its 1950 value in our equilibrium model is constrained by the energy balance discussed in Section 2.3 and Eq. 5.

In 2019 due to global warming trends, this model is more complex and harder to separate out terms. However, it can still be similarly modeled as

$$P_{Total\,2019} = P_{\alpha'} + P_{GHG'+Feedback} \approx P_{\alpha'} + f_2 P_{\alpha'} \tag{8}$$

Here, $P_{GHG'+Feedback}$ includes GHGs and its increase with feedbacks such as water-vapor, lapse rate effect and other changes such as an increase in snow-ice albedo variations that are hard to separate out. That is, some of this feedback is related to GHG forcing increases and some is related to albedo change. $P_{\alpha'}$ represents the 2019 point in time with its albedo due to prior changes in UHI absorption, cloud absorption, ice and snow melting, and so forth that can be discerned. The model does not demand rigid accountability in its application (see Sec.3). We note that f, a measure of the emissivity, is ***not*** constant, but must change since the amount of GHGs changes. However, f_2 is not as accurate in terms of the actual emissivity value but is an approximation that in perhaps rigorous assessment could be determined.

To be clear, f is just a fractional parameter related to the emissivity. In 1950 it was a function of the GHGs (with no feedbacks). In 2019, it is more complex and according to Eq. 8, must include feedbacks if $P_{\alpha'}$ can be determined.

The model is also constrained relative to f_1 as described in Section 2.3.2. However, it is primarily related to GHG re-radiation since $P_{GHG} \approx P_{GHG'+Feedback}$ (see results in Section 3).

2.3 Balancing P_{out} and P_{in}

Although Eq. 7 with, f_1 has the uniquely defined value found in Eq. 6. This should also result from balancing the energy in and out of our global system.

2.3.1 Balancing P_{out} and P_{in} in 1950

To balance the energy in with the energy out in 1950 with no global warming imbalance we can still start with Eq. 7. In equilibrium the radiation that leaves must balance what comes in P_α so that

$$\begin{aligned} Energy_{Out} &= (1-f_1)P_\alpha + (1-f_1)P = (1-f_1)P_\alpha + (1-f_1)\{P_\alpha + f_1 P_\alpha\} \\ &= (1-f_1)\{2P_\alpha + f_1 P_\alpha\} = 2P_\alpha - f_1 P_\alpha - f_1^2 P_\alpha = Energy_{In} = P_\alpha \end{aligned} \tag{9}$$

This is consistent with Eq. 6 so that in 1950, the value f solves the same quadratic equation as expected

$$f_1^2 + f_1 - 1 = 0 \ \text{ yielding } f_1 = 0.618 \tag{10}$$

Interestingly, this also says that

$$P_\alpha = f_1 P_{Total\_1950} \quad or \quad P_\alpha = f_1(P_\alpha + f_1 P_\alpha) \quad or \quad 1 = f_1(1+f_1) \tag{11}$$

The RHS of Eq. 11 is Eq. 10 and Eq. 6. This illustrates why f_1 is unique. It is the fractional amount of total radiation that is in equilibrium. As a final check, results will show in Section 3 and Table 1, that the value f_1 provides reasonable results.

2.3.2 Warming Imbalance in 2019

The re-radiation parameters f_1 and f_2, are connected and from Eq. 7 and 8 we have

$$f_2 = f_1 + \left(\frac{P_{2019}}{P_{\alpha'}} - \frac{P_{1950}}{P_\alpha}\right) = f_1 + \Delta f \tag{12}$$

In this way f_2 is a function of f_1=0.618 and the differences in the global warming residuals that is identified in Eq. 12 as Δf.

3.0 Results and Discussion

Since the re-radiation parameter is fixed for $f_1=0.618$, to obtain T_{1950}=13.89°C (287.038°K), the only adjustable parameter left in our model is the Earth's albedo. This value requires an albedo value of 0.3008 (see Table 1) to obtain the correct value T_{1950}. This albedo numbers is reasonable and similar to values cited in the literature [4].

In 2019, the average temperature of the Earth is T_{2019}=14.84°C (287.99°K). Here we are not sure of the albedo since it likely changed due to UHI increase, snow and ice melting and cloud coverage changes. The IPCC value in AR5 [5] is 0.294118 (100/340) is given in AR5 [6]. However, this would represent a 3% change since 1950 which may be an overestimation. In our assessment, we will assume a low middle value of about 1.2% change. Another reason for this choice will become apparent in the resulting analysis. Then, the f_2 parameter is adjusted to *0.6311* to obtain T_{2019}. Results are provided in Table 1. The results yield P_{Total_1950}=*384.935 W/m²* and P_{Total_2019}=*390.055* W/m².

Table 1 Model results

Year	T(°K)	T_α(°K)	f_1, f_2	α, α'	P_α, $P_{\alpha'}$ (W/m²)	$P_{GHG'+feedback}$ P_{GHG} (W/m²)	P_{Total} (W/m²)
2019	287.991	254.83	0.63114	29.719	239.131	150.925	390.056
1950	287.041	254.51	0.6180	30.08	237.903	147.032	384.935
Δ2019-1950	**0.95**	0.328	**1.311%**	0.361 **(1.2%)**	**1.228**	3.893	**5.12**

From Table 1

$$\Delta P_{Total} = P_{2019} - P_{1950} = 5.121 W/m^2 \qquad (13)$$

and

$$\Delta T_{Total} = T_{2019} - T_{1950} = 0.95°C \qquad (14)$$

which is the observed surface temperature increase since 1950.

Table 1 summarizes model results for the specified albedos and observed Earth's surface temperatures. To show model consistency, the forcing change 5.121 W/m^2, resulting in a 0.95^oK rise, should agree with what is expected from Planck's feedback parameter. From A-14 and Eq. 6, it is evident that

$$\beta^4 \Delta R_{LWR\_S} = 5.12 \text{ x } \beta^4 = 3.164 W/m^2 \qquad (15)$$

This equation illustrates the consistency of the re-radiation model. Then, Planck's feedback parameter (3.3 W/m^2 $/^oK$) temperature rise is in agreement with what is observed by equilibrium modeling

$$3.164 W/m^2 \text{ x } (1/3.3)^oK/W/m^2 = 0.959^oK \text{ at } T_s \qquad (16)$$

3.1 Why the Re-radiation Parameter is Significant

In Table 1, the measure of $\Delta f = 1.45\%$ fractional increase is mainly due to re-radiation change and associated feedbacks. This is significant. From Eq. 7, 8 and 12 we can illustrate this key characteristic of climate change

$$\Delta f = \left(\frac{P_{2019}}{P_{\alpha'}} - \frac{P_{1950}}{P_{\alpha}}\right) = \left(\frac{P_{GHG'+F}}{P_{\alpha'}} - \frac{P_{GHG}}{P_{\alpha}}\right) \qquad (17)$$

Therefore, f is an estimate of climate re-radiation and Δf an estimate of climate emissivity change and confounded with feedback effects. It is a measure of GHG forcing increase and the feedback relative to the initial 1950 radiation, and is generally helpful in looking at how our climate is working.

3.2 The Albedo-GHG Factor

We can look at an important ratio, the power created by the albedo effect compared to GHGs in 1950. The initial radiation is P_α which heats the Earth to 254.51^oK, and then according to Eq. 7 and Table 1, the energy increased by P_{GHG} is due to re-radiation fP_α and the ratio is

$$\left\{\frac{P_\alpha + P_{GHG}}{P_{GHG}} = \frac{P_\alpha + fP_\alpha}{fP_\alpha} = \frac{1+f_1}{f_1} = \frac{1.62}{0.62} = 2.62\right\}_{1950} \quad and \quad \left\{\frac{1+f_2}{f_2} = 2.58\right\}_{2019} \qquad (18)$$

We note the ratio is reduced in 2019 as P_{GHG} increases along with feedbacks with re-radiation increases. In the limit as f_2 approaches a maximum of unity, its minimum value is 2.

In this engineering view, we can look at a change in albedo forcing compared with a change in GHGs. The variation in the energy due to an average albedo change and its re-radiation is

$$\Delta P_{\bar{\alpha}} = \Delta P_{\alpha'} + f_2 \Delta P_{\alpha'} = 1.631\ \Delta P_{\alpha'} \qquad (19)$$

The average change in GHGs can be written in terms of Δf and the absorbed energy that GHGs receive from solar absorption is

$$\Delta P_{\overline{GHG'}} = \Delta f\ P_{GHG'} = 1.311\%\left(f_2\ P_{\alpha'}\right) = 0.827\% P_{\alpha'} \qquad (20)$$

This resulting ratio is from Table 1

$$\frac{\Delta P_{\bar{\alpha}}}{\Delta P_{\overline{GHG'}}} = \frac{\Delta P_{\alpha'}}{\Delta f}\frac{(1+f_2)}{P_{\alpha'}\ f_2} = \frac{1.228 W/m^2}{0.0131}\frac{1.631}{239.1 W/m^2\ 0.631} = 1.01 \qquad (21)$$

Note that this ratio is of course dependent on the 2019 albedo 1.2% change, selected here to obtain unity for illustrative purposes. The ratio $\Delta P_{\alpha'}/\Delta f_\alpha$ is a key aspect of climate change. In 2019, if we have knowledge of values, we can assess which is the dominant part of the warming trend. It also provides us with a measure of solar reversibility

$$\Delta P_{\alpha'} \geq \Delta f \ \frac{P_{\alpha'} f_2}{(1+f_2)} 1.02 \approx 1.21 W/m^2 \qquad (22)$$

This ratio is dependent on the change in the albedo compared with a GHG change. This does not include the potential for a transient climate response (TCR). It is perhaps not the best way to assess geoengineering estimates. True values of $\Delta\alpha$ and Δf are not easily obtained in 2019. However, it avoids CO_2 doubling estimates, which are also difficult to evaluate. Furthermore, it suggest that if we are worried that a ΔP_α change could create excess GHGs, we can estimate that as long as we are greater than this value in Eq. 22, the change is beneficial by comparison.

We can simplify things further to provide a more productive approach. In reverse solar geoengineering of a global warming solution, it is helpful to have simple reliable values. In this view, the 1.6 albedo-GHG factor (which is reasonably accurate) is an important engineering number. It provides one of the significant values needed in reverse albedo forcing that takes into account the initial absorption change followed by re-radiation. Another important engineering value is described by a Planck-albedo parameter.

3.3 Planck-Albedo Parameter and a Simplified Reverse Forcing Solution

The albedo changes and ΔP_α in Table 1, are: $\%\Delta\alpha = 1.6\%$ and 1.638W/m^2, respectively. We note that we can define a unique handy Planck-albedo parameter $\lambda_{\%\Delta\alpha} = \Delta P_\alpha / \%\Delta albedo$. To illustrate from Table 1

$$\lambda_{\%\Delta\alpha} = 1.024 \text{ W/m}^2/\Delta\%\text{albedo} \qquad (23)$$

This parameter can also be expressed per degree (noting the 0.95^{o}K change in Table 1)

$$\lambda_{\%\Delta\alpha\Delta T} \approx 1 W/m^2 / \Delta\% albedo / {}^\circ K \qquad (24)$$

The helpful parameter [3] is featured here as a modeling tool. We term it the Planck-albedo parameter, since it relates to blackbody (P_α) absorption. A simple numeric example is given in the conclusion to illustrate how it provides helpful estimates along with the albedo-GHG factor. This interesting parameter simplifies from the basic assessments of the two different time periods (see also Eq. A-8) as

$$\lambda_{\%\Delta\alpha} = \frac{(\Delta E_o)_\alpha}{\frac{\alpha_1 - \alpha_2}{\alpha_1} 100} = \frac{E_o(\alpha_1 - \alpha_2)}{\frac{\alpha_1 - \alpha_2}{\alpha_1} 100} = E_o \alpha_1 / 100 \approx 1 W/m^2 / \%\Delta albedo \qquad (25)$$

where E_o=340 W/m^2 and when α_1 is 0.294118, the value 1.000W/m^2/Δ%albedo is obtained. We note the value 29.4118% (100/340) is given in AR5 [6]. The parameter's relationship to λ_α is

$$\lambda_\alpha = \lambda_{\%\Delta\alpha\Delta T} \ x \ \%\Delta\alpha \qquad (26)$$

and appropriate feedback parameters could including the re-radiation albedo-GHG factor in 2019 [2], for example

$$\lambda_\alpha^\dagger = \lambda_{\%\Delta\alpha\Delta T} \ x \ \%\Delta\alpha \ \ (1+f_2) \qquad (27)$$

The albedo-GHG and the Planck-Albedo feedback parameter may be combined in order to provide a simple solar geoengineering solution estimate

$$\Delta P_{\text{Rev}\_S} = -\lambda_{\%\Delta\alpha\Delta T} \ \ \%\Delta\alpha \ (1+f_2) \ A/T = \Delta P_T (1+f_2) \ A/t \qquad (28)$$

and from A-14 $\Delta P_{\text{Rev}\_LWR} = \beta^4 \Delta P_{\text{Rev}\_S}$ the temperature reduction is

$$\Delta T_{\text{Rev}} = -\Delta P_{\text{Rev}\_LWR} \frac{1}{\lambda_o} \qquad (29)$$

Here ΔP_{Rev} is the reverse forcing, A is an estimate of the anticipated GW amplification reduction, t is an attempt to include a transient response, an allowance for the climate system to equilibrate, which has not occurred at the predictive time, ΔP_T is the reverse forcing from the target area. A suggested transient value of t is 1.25 [13]. The equation provides a fairly simple and practical way to estimate ΔP_{Rev}. An example is provided in the conclusion.

4.0 Conclusion

In this paper, we provided a re-radiation global warming model. The model shows consistency with the Planck parameter. We noted that the re-radiation parameter increased by about 1.45% due to global warming from 1950 to 2019, illustrating the warming from a different perspective. From the model, a helpful albedo-GHG parameter was quantified having a value of 1.6.

We also found an engineering factor that we termed the Planck-albedo parameter, which is about $\lambda_{\%\Delta\alpha\Delta T} \approx 1W/m^2/\Delta\%albedo/°K$. These findings can be helpful in quickly estimating the effect of an albedo change on global warming and in assessing λ_α. These results support solar geoengineering solutions [3, 7-9].

For example, Feinberg 2020 [2] suggested a goal of 1.5% geoengineering albedo change. Using Equation 26, with a decrease in water-vapor feedback anticipated, we might use a value of A≈2 [10], then

$$\Delta P_{Rev\_S} = -1\text{W/m}^2\text{/\% x 1.5\% x }(1+f_2)\text{ x 2/1.25} = -3.84\text{ Watt/m}^2 \quad (29)$$

One can multiply this by β^4 to compare with IPCC models or to relative to our results in Table 1 with a forcing of 5.12 W/m^2. Equation 29 expressed in terms of reverse temperature warming results is then

$$\Delta T_{Rev} = -0.72°K \quad (30)$$

This would indicate a significant resolution to the current warming trend. As one might suspect, a 1.5% albedo change requires a lot of modified area. Feasibility is discussed in more detail in Feinberg's 2020 [2]. Results of this paper indicate the required area of change, if proper hotspots are targeted, is 3.4-17 times smaller than the estimates of the area of urbanization. Other solar geoengineering solutions have been proposed [7-9].

Appendix A

Overview of Planck Feedback Parameter

Estimates on Planck's feedback parameter are varied, typically between -3.8W/m^2/°K and -3.21W/m^2/°K with some values as large as -7.1W/m^2/°K [11]. The IPCC AR4 [12] lists a value of -3.21W/m^2/°K. Numerous authors have developed different expressions [11]. A typical estimate starts with

$$F_{TOA} = (1-\alpha)S_o/4 - \sigma(\beta T_S)^4 = (1-\alpha)S_o/4 - R_{LWR} \quad (A\text{-}1)$$

where S_o=1361W/m^2, F_{TOA} is the radiation budget at the top of the atmosphere, R_{LWR} is the outgoing long wave radiation (a function of surface temperature and albedo), σ is the Stefan-Boltzmann constant and β is described in this section below and is redefined in terms of a re-radiation parameter in this paper. Then the Planck parameter λ_o can be calculated as

$$\lambda_o = \partial F_{TOA}/\partial T_S = -\partial R_{LWR}/\partial T_S \quad (A\text{-}2)$$

This result is

$$\lambda_o = -4\beta^4\sigma T_S^3 = -4\beta\sigma T_{TOA}^3 \quad (A\text{-}3)$$

where β varies in the literature from 0.876 to 0.887 (averaging=0.8815) and Ts=288°K [12]. This yields -3.37W/m^2/°K<λ_o<-3.21W/m^2/°K. However, from Eq. A-3, β is often taken as the ratio

$$\beta = T_{TOA}/T_S = 255°K/288°K = 0.8854 \text{ and } \beta^4 = 0.615 \quad (A\text{-}4)$$

A common assessment uses T_{TOA}=255°K, so that λ_o =-3.33W/m^2/°K. Another expression developed by Schlesinger [6] is dependent on the albedo and surface temperature as

$$\lambda_o = S_o(1-\alpha)/T_S \quad (A\text{-}5)$$

When S_o=1361, 0.294118<α<0.3, and Ts=288 °K then -3.308W/m^2/°K >λ_o>-3.3358W/m^2/°K , respectively.

A.1 Estimating Planck's Parameter with an Albedo Method

Consider a global albedo change corresponding to 1°K rise from solar absorption letting

$$F_{TOA} = 0 = (1-\alpha)E_o - \sigma(T_S)^4 \quad \text{(A-6)}$$

where $E_o=S_o/4$. Then a 1°K change is

$$\Delta T_S = T_2 - T_1 = \left(\frac{E_o}{\sigma}(1-\alpha_2)\right)^{1/4} - \left(\frac{E_o}{\sigma}(1-\alpha_1)\right)^{1/4} = 1°K \quad \text{(A-7)}$$

Here we will use the AR5 albedo starting value of 0.294118 [6]. We find that the corresponding albedo change is 0.28299 when E_o=340W/m^2. This corresponds to

$$\Delta E_o = E_o\left\{(1-\alpha_2)-(1-\alpha_1)\right\} = E_o(\alpha_1-\alpha_2) = 3.784W/m^2 \quad \text{(A-8)}$$

Since this is for a 1°K rise, then it can also be written as

$$\lambda_{1K}\text{=3.784W/m}^2\text{/°K} \quad \text{(A-9)}$$

We note this is related to the surface value, then

$$\lambda_{1K} = -4\sigma T_s^3 \quad \text{(A-10)}$$

By comparison to above we have

$$\lambda_o\text{=}\lambda_{1K}\,\beta = -3.784\text{W/m}^2\text{/°K=-3.349W/m}^2\text{/°K} \quad \text{(A-11)}$$

This is very close to the -3.33 W/m^2/°K value obtained in the traditional manner.

It can be helpful to recall that from Eq. A-1 if we let

$$R_{LWR\_S} = \sigma(T_S)^4 \quad \text{(A-13)}$$

then

$$\beta^4 R_{LWR\_S} = R_{LWR} \quad \text{(A-14)}$$

We use this expression in showing model consistency with the Planck feedback parameter.

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